

Inelastic interactions between nuclei at high energies

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A theory of nucleus-nucleus collisions has been developed for kinetic energies substantially in excess of the binding energy. The very high pressure produced in the compound system as a result of the coalescence of the two colliding nuclei is the reason for the subsequent hydrodynamic expansion of the nuclear medium. The energy and angular distributions of the reaction products are investigated. The charge distribution is also determined in the case where the nucleon and ion components of the reaction products are predominant. A solution is found for the expansion into vacuum of a sphere in which the initially uniformly distributed material is initially at rest and at an ultrarelativistic temperature.

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1. INTRODUCTION

Progress in the technology of acceleration of multiply charged ions^[1] has substantially contributed to recent developments in this important field of research in nuclear physics. The complexity of the colliding systems, i. e., the accelerated ion and the target nucleus, gives rise to a variety of possible reaction channels specific for this category of processes. Let E_1 be the kinetic energy of the incident nucleus per nucleon. For $E_1 \sim 1-10$ MeV, the nucleon binding energy in the initial systems, and the Coulomb barrier, which impedes the approach of the two particles, may still play an appreciable role. This, of course, leads to an increase in the fraction of reactions involving the transfer or capture of individual nuclear particles during the interaction.^[2] However, these values do not, in principle, represent the limit of experimental possibilities, and there is a promising tendency for E_1 and the atomic weight of the colliding systems to increase. The physical picture may then be expected to undergo a substantial change, and the predominant mechanism responsible for most of the interaction cross section turns out to be relatively simple.

To avoid unnecessary detail with very little bearing on the essence of the situation, we shall confine our attention to a head-on collision between two identical nuclei and, unless stated to the contrary, we shall carry out our analysis in the center-of-mass system (c.m.s.). Suppose that the kinetic energy per nucleon in this system is E_0 , the mass number of the nuclei is A , and the atomic number is Z . For sufficiently high values of E_0 , we can neglect electric forces and assume that, as the nuclei approach one another, an overlap between the spatial distributions of the initially cold medium will appear from a certain instant of time onward. To obtain an approximate measure of the strong interaction which results in this situation, let us estimate the mean free path. The cross section σ_{nn} for the interaction between the initial elementary particles, i. e., nucleons, is known from experiment (see, for example,^[3]) and is of the order of the pion Compton wavelength $\hbar/m_\pi c$, whereas the role of the various constraints imposed by the Pauli principle decreases with increasing E_0 . Very approximately, therefore, we have

$$1/\sigma_{nn}n \sim \hbar/m_\pi c, \quad (1)$$

where n is the density of nucleons in nuclear matter (the possible creation of new particles will require a more careful analysis, and in the discussion below we shall interpret n as the spatial density of baryon charge). The mechanism and the possibility of a theoretical description of the phenomenon are very dependent on the relative free path given by (1) and the nuclear radius

$$R = r_0 A^{1/3}. \quad (2)$$

It is well known that

$$r_0 \sim \hbar/m_\pi c \cong 1.4 \cdot 10^{-13} \text{ cm}$$

(this has already been used above in estimating n). We thus find that, in the case in which we are interested here¹⁾

$$1/\sigma_{nn}nR \sim A^{-2/3} \ll 1. \quad (3)$$

Since the mean free path is short, the initial stage is the coalescence of the nuclei into a "compound system." It is, however, important to emphasize the difference between this system and the usual compound nuclei formed, say, by nucleon capture. In cold or relatively low-temperature nuclei there is no appreciable pressure, and such nuclei exhibit no noticeable tendency to expand. On the other hand, a very high pressure is produced during the formation of the system in which we are interested. Thus, the most conservative estimates, which do not take into account the compression of the medium during coalescence, show that the pressure is proportional to the total internal energy $E = 2AE_0$. This results in the expansion of the compound system into vacuum. The condition given by (3) enables us to consider the second stage, i. e., expansion, in hydrodynamic terms. It is not clear whether any systematic theory would be capable of providing a detailed quantitative description of the "coalescence stage." The essential feature is that the entropy of the system increases from zero to some maximum value S .

The formulation of the problem is thus quite close to the suggestion put forward at one time, on Fermi's initiative,^[8] for the description of collisions between relativistic strongly-interacting elementary particles,

namely, hadrons. These ideas were extended further in an interesting paper by Landau.^[9] Without going into the various aspects of this complex problem, we shall merely note some of the features of the difference between the nuclear case and the "elementary interaction" between two hadrons. The initiating interaction between the two initial particles may in itself be capable of creating "hadron matter" in macroscopic amounts, but this entire question is, to some extent, shrouded in doubt. On the other hand, during the coalescence of heavy nuclei, the number of particles is known to be macroscopically large because of the nucleons that are present right from the beginning. This enhances the credibility of the above thermodynamic and hydrodynamic conditions (see above).

Strictly speaking, only the second stage of the process, i. e., the expansion stage, will be subjected to theoretical analysis. Let us begin with a few preliminary remarks on the physics of the phenomena involved in this process. The expansion of matter into vacuum occurs with near-sonic or ultrasonic velocities, so that viscous friction and thermal conductivity can hardly be expected to lead to an appreciable increase in entropy. If the resulting adiabatic motion ($S = \text{const}$) of the continuous medium is to be treated in a simplified mechanistic way, the internal energy of the liquid will, so to speak, play the role of potential energy. This provides a clear physical picture of why the overall character of the motion of the individual elements of the medium (the fluid particles) depends on the order of magnitude of the velocities communicated to them.

In the case of inelastic collisions between nuclei, the physically interesting region is the relatively extensive nonrelativistic region $E_0 \ll m_n c^2$ (m_n is the nucleon mass), which is even more accessible to current experimental possibilities. We shall write the nonrelativistic energy of a fluid particle in the form of the sum

$$Mv^2/2 + \varepsilon$$

i. e., the sum of kinetic and potential energies ($M = \text{const}$ by definition and ε is the internal energy). In very approximate calculations, we can initially ignore the energy of interaction with the ambient fluid (i. e., the work done by pressure), and suppose that the velocity v increases due to the reduction in ε during the adiabatic expansion. The latter leads to a subsequent reduction in pressure, so that the assumption that the interaction between the fluid particles is small will become increasingly valid. The net result is that $Mv^2 \gg \varepsilon$, i. e., the fluid particles become "freed" and execute inertial motion with $v \approx \text{const}$. This condition also determines the validity of the assumption that the true particles of the medium have negligible thermal velocities (due to cooling on expansion), as compared with the translational velocity v of the fluid. Thus, the final velocity distribution of the particles, i. e., the reaction products, is predetermined while, on the other hand, the hydrodynamic conditions which demand that the mean free path is small in comparison with the linear dimensions of the entire system may still be valid. Essentially, this stage is actually reached in a rela-

tively short time $t \gtrsim l/u_0$, where l represents the linear dimensions of the system before the onset of expansion, and u_0 is the initial velocity of sound.

The foregoing general ideas lose their validity as we enter the ultrarelativistic region $E_0 \gg m_n c^2$. The single expression

$$\varepsilon / (1 - v^2/c^2)^{1/2}$$

cannot be divided into "kinetic" and "potential" components in an entirely natural fashion. We note that the idea of a "freed" liquid particle is not altogether consistent with the general character of relativistic relationships. It is clear, for example, that the reduction in ε should be compensated by a reduction in the denominator. The fluid continues to accelerate and, in reality, the pressure [for which at ultrarelativistic temperatures one usually employs the equation of state given by (30)] remains effective. The only process capable of terminating the reduction in ε during expansion, and of stabilizing the velocity, is the formation of individual particles in the hadron matter, the rest masses of which begin to dominate all the contributions to the internal energy. Here again we return to the situation where the energy and angular distribution of the reaction products are predetermined and, correspondingly, the equation of state for the medium changes and departs from (30). In the opinion of Pomeranchuk^[10] and Landau,^[9,11] this occurs at temperatures $T \sim m_\pi c^2$.

2. COLLISIONS OF NONRELATIVISTIC NUCLEI

We shall suppose below that the change in the internal state of the medium during expansion is described by the Poisson adiabatic curve^[12]:

$$pV^\gamma = \text{const.} \quad (4)$$

If we recall that $dE = -pdV$ and integrate, we can write the basic relationships in the following form, which is particularly convenient for subsequent calculations:

$$\gamma = \frac{2\nu+3}{2\nu+1}, \quad w = \frac{2\nu+1}{2} u^2, \quad dw = (2\nu+1)u du. \quad (5)$$

In simple cases, the parameter γ is the ratio of specific heats, but this is not essential; w is the enthalpy per unit mass. Moreover, for adiabatic (isentropic) flow

$$s \propto n \propto \rho \propto u^{2\nu+1} \quad (6)$$

where ρ is the density of the spatial mass distribution and s is the entropy per unit volume. For the so-called simple (self-similar) rarefaction wave, we have

$$u + \frac{v}{2\nu+1} = u_0, \quad (7)$$

where u is the local velocity of sound (see, for example,^[13]) and

$$v_{\text{max}} = (2\nu+1)u_0 \quad (8)$$

is the limiting value of the velocity of free expansion of the medium which is initially at rest in vacuum.

To obtain an estimate for the preliminary compression of nuclear matter, we shall suppose that the coalescence of nuclei occurs gradually. Initially, in the region of space where the two media have come into contact, the liquid undergoes intensive "boiling" but, outside this region, it remains cold. Since, prior to collision, the product of the nucleon momentum by its velocity is $2E_0$, we find that the momentum transported out of the ambient space through unit area on the separation boundary per unit time is

$$p=2n_0E_0 \quad (9)$$

where n_0 is the usual equilibrium density of the baryon charge at zero temperature. The momentum transfer specified by (9) is obviously equivalent to a pressure p . After the medium has been brought to the boil, the pressure in the medium is approximately given by

$$p=\frac{2}{3}nE_0 \quad (10)$$

which is the equation of state for an ideal gas.²⁾ Assuming that mechanical equilibrium is established more rapidly than thermal equilibrium in the neighborhood of the separation boundary, we can equate the expressions given by (9) and (10). This yields

$$n/n_0 \approx 3 \quad (11)$$

prior to the onset of free expansion. We emphasize that the result given by (11) is insufficient to determine both the longitudinal and transverse size of the figure at the very beginning of the hydrodynamic stage. The medium may undergo some flow in the transverse plane which is perpendicular to the x axis during the coalescence of the nuclei, and this is not impeded by external pressure. More accurate estimates of the radial size $L > R$ reached in this direction are difficult because of the highly nonequilibrium character of the coalescence stage (see also the Introduction).

Let us now consider the adiabatic stage of the expansion process. The initial configuration can be schematically represented by a disk of thickness $2l$. It is natural to assume that

$$l \ll L. \quad (12)$$

In the first approximation, therefore, the hydrodynamic flow can be looked upon as one-dimensional. The symmetry of the problem enables us to confine our attention to the region $x > 0$. In addition to the coordinate measured from the center of symmetry, it will occasionally be useful to use the variable $x' = x - l$. The edge of the distribution of matter moves forward with the velocity given by (8). As long as $t < l/u_0$, the situation is no different from the solution of the well-known problem on the expansion of a half-space into vacuum. Against the flow, we have the propagation of a simple wave up to the "weak discontinuity" $x' = -u_0 t$ (i. e., the point at which the sonic signal reaches at this time; see, for example, ^[13]). When $t > l/u_0$, the weak discontinuity

moves in the positive direction of the x axis, and the relative size of the region occupied by the simple wave decreases rapidly. The space on the other side of the weak discontinuity, where the appropriate value of the so-called general integral of hydrodynamic equations³⁾ is reached, begins to play the dominant role. In principle, a general analytic expression can be obtained for it for integral values of ν .^[13,14]

In the case of an ideal gas of elementary particles, we have $\gamma = 5/3$ and $\nu = 1$. The corresponding general solution can be written in the form

$$\chi(w, v) = \frac{1}{u} \left\{ F_1 \left(u + \frac{v}{3} \right) + F_2 \left(u - \frac{v}{3} \right) \right\}, \quad (13)$$

where F_1 and F_2 are certain arbitrary functions. The "velocity potential" can be used for the implicit determination of the required functions $w(x', t)$ and $v(x', t)$ from the formulas

$$t = \frac{\partial \chi}{\partial w}, \quad x' = v \frac{\partial \chi}{\partial w} - \frac{\partial \chi}{\partial v} \quad (14)$$

(see, for example, ^[13], pp. 474-479). By satisfying the boundary conditions both at $x' = -l$ ($x = 0$), at which the fluid is at rest, and at the point of contact with the simple wave (7), we finally obtain

$$\chi = \frac{3}{2} \frac{l}{u} \left\{ \left(u + \frac{v}{3} \right)^2 - u_0^2 \right\}. \quad (15)$$

By substituting in (14) [see also (5)], we immediately return to the physically most interesting time $t \gg l/u_0$ (in which case, $u \ll u_0$, where u_0 is the initial velocity of sound in the originally resting medium):

$$v = \frac{x}{t}, \quad \rho \propto u^3 = \frac{l}{2t} \left(u_0^2 - \frac{v^2}{9} \right). \quad (16)$$

The velocity field given by (16) corresponds to the inertial motion of the fluid particles, and the velocity distribution of the masses remains unaltered (see also the preliminary remarks in the Introduction). In fact, at any time

$$\rho dx \propto \rho dv \propto (u_0^2 - v^2/9) dv. \quad (17)$$

If we now transform to the new variable defined by $v \propto \sqrt{\epsilon}$, $dv \propto d\epsilon/\sqrt{\epsilon}$ and normalize the expression $W(\epsilon)d\epsilon \propto \rho dx$ to the unit integral between 0 and ϵ_{\max} , we obtain the following expression for the energy distribution of the reaction products, i. e., nucleons, in the center-of-mass system (see Fig. 1):

$$W(\epsilon)d\epsilon = \frac{3/4}{(\epsilon_{\max})^{3/4}} (\epsilon_{\max} - \epsilon) \frac{d\epsilon}{\sqrt{\epsilon}}. \quad (18)$$

The presence of the cutoff point $\epsilon = \epsilon_{\max}$ in (18) is a consequence of the hydrodynamic character of the expansion stage. The energy $\bar{\epsilon}$ averaged over the entire spectrum is given by

$$\epsilon_{\max} = 5\bar{\epsilon} = 5E_0 \quad (19)$$

($\bar{\epsilon} = E_0$ follows from energy considerations).

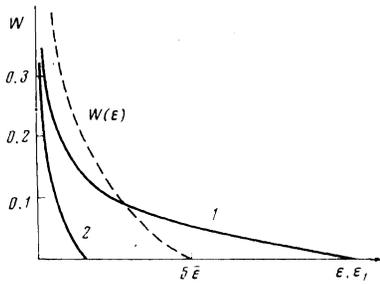


FIG. 1. The dashed curve shows the particle spectrum in the c.m.s. Curve 1 refers to particles traveling in forward directions in the laboratory system. Curve 2 refers to particles traveling in the backward directions. In the laboratory system $\epsilon_{1 \max} = (6 \pm 2\sqrt{5})\bar{\epsilon}$.

When (12) is satisfied, the angular distribution of the nucleons is confined to the forward and backward directions. It cannot be calculated in a closed form, and we shall therefore confine our attention to an estimate of the characteristic angle $\theta \approx v_y/v$, where v_y is the transverse component of the fluid-particle velocity. Its total acceleration $d\mathbf{v}/dt$ is given by the Euler equation, the transverse component of which is

$$\frac{dv_y}{dt} = -\frac{\partial p/\partial y}{\rho} \sim \frac{u^2}{L} \sim \frac{v^2 u_0^{4/3}}{L t^{2/3}} \quad (20)$$

The solution given by (16) is used here for approximate purposes and is valid for $u_0 t \lesssim L$, after which expansion enters the three-dimensional phase. Integrating up to the above limit, and recalling that $v \sim u_0$, we find that

$$\theta \sim (L/L)^{3/2} \quad (21)$$

Transforming to the laboratory system, in which one of the nuclei was at rest prior to collision, we obviously obtain $E_1 = 4E_0 = 4\bar{\epsilon}$ for the primary energy per nucleon. For most particles, the observed angle ϑ between their momenta and the collision axis has the same order of magnitude, i.e., $\vartheta \sim \theta \ll 1$. If we apply the Galilean transformation to (18), we can readily show that

$$W(\epsilon_1) d\epsilon_1 = \frac{3/8}{(5\bar{\epsilon})^{3/2}} [5\bar{\epsilon} - (\sqrt{\epsilon_1 \mp \sqrt{\bar{\epsilon}}})^2] \frac{d\epsilon_1}{\sqrt{\epsilon_1}} \quad (22)$$

where ϵ_1 is the laboratory nucleon energy, and the upper and lower signs refer to particles traveling in the forward and backward directions in this frame, respectively. The distribution given by (22) is normalized to a unit total integral evaluated over both regions, and the corresponding branches of it are shown in Fig. 1. We note that, when $\epsilon_1 \lesssim E_1 \theta^2$, the angular distribution of the nucleons becomes broad, filling the entire solid angle (in the laboratory system).

The foregoing discussion was, in fact, confined to the case $E_0 \lesssim m_\pi c^2$. When the inequality

$$m_\pi c^2 \ll E_0 \ll m_n c^2 \quad (23)$$

is satisfied, the position is modified somewhat because

of the creation of a large number of relativistic pions during the coalescence of the nuclei. In the region defined by (23), most of the mass is carried by the nucleons. On the other hand, the internal energy (less the nucleon rest mass, as is usually assumed in non-relativistic theory) resides mainly in the meson degrees of freedom, and these particles are also largely responsible for the pressure in the medium. By analogy with blackbody radiation^[12] [see also the next section and, in particular, the equation of state given by (30)], the pressure will be approximately specified by the equation

$$p = \frac{1}{3} n E_0 \quad (24)$$

Equating the pressure given by (24) to the external pressure given by (9), which, during the coalescence stage, describes the cold part of the medium for a certain interval of time, we find that

$$n/n_0 \approx 6 \quad (25)$$

The increase in the preliminary compression as compared with (11) suggests that the validity of (12) may improve.⁴⁾

Blackbody radiation and other similar ultrarelativistic modifications of matter correspond to $\gamma = 4/3$ and $\nu = 5/2$. In principle, for fractional values of ν , there is no closed general analytic solution of the equations of one-dimensional hydrodynamics that are analogous to (13) and (14). However, for large times t , the asymptotic behavior of the form given by (16) can readily be generalized to fractional values of ν :

$$\rho \propto u^{2\nu+1} = \frac{\Gamma(2\nu+1)}{2^{2\nu} [\Gamma(\nu+1)]^2} \frac{l}{t} \left[u_0^2 - \frac{v^2}{(2\nu+1)^2} \right]^\nu \quad (26)$$

If we use this expression to determine the energy distribution of the particles in the center-of-mass system, we have for $\nu = 5/2$,

$$W(\epsilon) d\epsilon = \frac{16/5\pi}{(\epsilon_{\max})^3} (\epsilon_{\max} - \epsilon)^{3/2} \frac{d\epsilon}{\sqrt{\epsilon}}, \quad \epsilon_{\max} = 8\bar{\epsilon} \quad (27)$$

which is valid for any reaction products with nucleons and pions predominating. The quantity ϵ_{\max} is proportional to the mass of the particles with which we are concerned. For example, $\epsilon_{n \max}/\epsilon_{\pi \max} = m_n/m_\pi$. Because the conditions for the validity of the theory are unfavorable (see the last footnote), the equation $\bar{\epsilon}_n \approx E_0$ is, in fact, satisfied only approximately. Finally, if we estimate the transverse forces in the Euler equation by analogy with the derivation of (21) from (20) and (16), we get the expression

$$\theta \sim (L/L)^{3/2} \quad (28)$$

for the effective angle at which the particles are emitted in the center-of-mass system. We shall not consider here the kinematics of the transformation to the laboratory system, since it is analogous to that discussed above for the case $E_0 \lesssim m_\pi c^2$.

3. COLLISIONS OF ULTRARELATIVISTIC NUCLEI

When

$$E_0 \gg m_n c^2 \quad (29)$$

we must use the method of relativistic hydrodynamics.^[12,9,11] We begin by writing down the basic thermodynamic relationships. In the spirit of the Landau idea^[9,11] on the nature and the probable form of the equation of state for hadron matter at ultrarelativistic temperatures $T \gg m_n c^2$, we assume that

$$p = \frac{e}{3}, \quad e = ks^{3/2}, \quad T = \frac{de}{ds} = \frac{4}{3} ks^{1/2}. \quad (30)$$

In these expressions, e is the energy per unit proper volume of the liquid particle in its rest system, s is the entropy per unit proper volume, p is the pressure, and T the temperature. This yields the following constant value for the velocity of sound:

$$u = c/\sqrt{3}. \quad (31)$$

The numerical value of k cannot be established by purely deductive means. Dimensional considerations suggest that

$$k \sim \hbar c. \quad (32)$$

The volume of the compound system produced as a result of the coalescence of the original nucleus is given by

$$V \sim R^3 \frac{m_n c^2}{E_0}. \quad (33)$$

where R is the nuclear radius and the factor $m_n c^2/E_0$ appears as a result of the Lorentz compression:

$$n/n_0 \sim E_0/m_n c^2. \quad (34)$$

Recalling also the expression given by (2), we can readily show that the temperature and entropy of the system at the time preceding the onset of adiabatic expansion are given by

$$T_0 \sim m_n c^2 \left(\frac{m_n}{m_n}\right)^{3/4} \sqrt{\frac{E_0}{m_n c^2}}, \quad S \sim A \left(\frac{m_n}{m_n}\right)^{3/4} \sqrt{\frac{E_0}{m_n c^2}}. \quad (35)$$

Let us now consider the hydrodynamic stage. Because of the geometry of the initial configuration, this stage has the character of one-dimensional flow over a certain interval of time. However, analysis shows that the increase in the influence of transverse forces gradually leads to the isotropization of the flow and its rapid transformation into the three-dimensional phase.⁵⁾ As a result, the liquid is so rapidly accelerated that it becomes concentrated largely at finite distances from the surface which is expanding with the velocity of light. One way of describing this is to say that a "cavity," i.e., a region of sharply reduced density, is produced inside the spatial distribution of matter with this peculiar geometry. We shall first describe this isotropic

part of the process and will return to the influence of the initial conditions later.

It is well known that the equations of relativistic hydrodynamics are contained in the differential conservation laws

$$\partial T^{ik}/\partial x^k = 0, \quad (36)$$

where T^{ik} is the energy-momentum tensor of the medium.^[9,11,13,15] It will be convenient to use the system of units in which $c=1$ and adopt (30) to simplify all the expressions to the case of spherical symmetry. Instead of the radial distance r , we shall use the independent variable

$$\xi = t - r \quad (37)$$

and expand into a series in powers of the reciprocal of the relativistic 4-velocity

$$\gamma = \frac{1}{\sqrt{1-v^2}} \gg 1, \quad (38)$$

retaining only the first two terms ($t/\xi \sim \gamma$, as we shall soon show). In terms of the new variables, we then obtain the following set of equations for the ultrarelativistic flow:

$$\begin{aligned} \gamma^2 \frac{\partial s}{\partial t} + 2 \frac{1}{t} \gamma^2 s + 2 \frac{\xi}{t^2} \gamma^2 s + \frac{1}{2} \frac{\partial s}{\partial \xi} + \frac{1}{2} \left[\frac{\partial \gamma^2}{\partial t} - \frac{1}{\gamma} \frac{\partial \gamma}{\partial \xi} \right] s = 0, \\ \gamma^2 \frac{\partial s}{\partial t} - \frac{1}{2} \frac{\partial s}{\partial \xi} + \frac{3}{2} \left[\frac{\partial \gamma^2}{\partial t} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial \xi} \right] s = 0. \end{aligned} \quad (39)$$

It is readily verified that all the requirements⁶⁾ are satisfied by the following very simple solution

$$s = \frac{C}{t^2 \xi^3}, \quad \gamma^2 = \frac{1}{2} \frac{t^2}{\xi^2}. \quad (40)$$

The singularity at $\xi=0$ reflects, formally, the inability of matter to propagate with velocities in excess of the velocity of light and this is, of course, the basic feature of the equations of relativistic hydrodynamics. In point of fact, the value of the general integral of these equations given by (40) is valid only up to a certain $\xi_0 > 0$. The sphere $\xi = \xi_0$ is a surface of weak discontinuity. Integrating the differential equation of its motion [it can be set up with the aid of the relativistic law of addition of velocities; relative to the fluid, the weak discontinuity always propagates with the velocity of light which, in this case, is $1/\sqrt{3}$, see (31)], we can verify that $\xi_0 \rightarrow \text{const}$ when

$$t \gg \xi_0. \quad (41)$$

To achieve a more specific physical interpretation of ξ_0 , let us consider the conservation laws. The spatial energy density is the time component

$$T^{00} = (e+p)\gamma^2 - p \approx \frac{1}{3} e \gamma^2 \quad (42)$$

of the energy-momentum tensor.^[9,13,15] Moreover, the total entropy S is a constant in the case of adiabatic

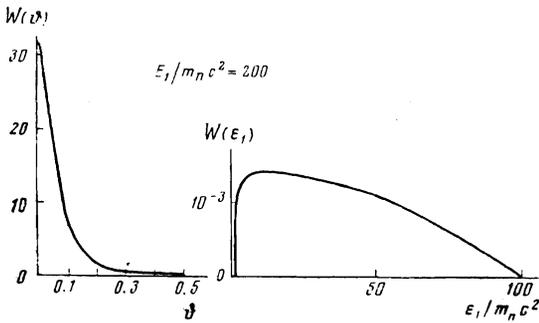


FIG. 2. These graphs were plotted for (48) replaced by an equation in which the proportionality factor was taken to be equal to unity.

flow. Using (30) and (40), we obtain

$$E = \int \frac{4}{3} k s^{1/2} \gamma^2 dx = \frac{4}{3} \cdot 4\pi t^2 k \frac{C^{3/2}}{t^2} \int_0^{\xi_0} \frac{d\xi}{\xi^3} = \frac{8}{15} \pi k \frac{C^{3/2}}{\xi_0^3},$$

$$S = \int s \gamma dx = 4\pi t^2 \frac{C}{t^2} \int_0^{\xi_0} \frac{d\xi}{\xi^2} = \frac{2^{3/2}}{3} \pi \frac{C}{\xi_0^2} \quad (43)$$

which enable us to calculate ξ_0 and the arbitrary constant C :

$$\xi_0 = \frac{2}{5} \left(\frac{3}{\pi} \right)^{1/2} k \frac{S^{3/2}}{E} = \frac{2}{5} \left(\frac{3}{\pi} \right)^{1/2} V^{1/2},$$

$$C = \frac{2^{3/2}}{125} \left(\frac{3}{\pi} \right)^2 k^3 \frac{S^3}{E^3} = \frac{2^{3/2}}{125} \left(\frac{3}{\pi} \right)^2 S V. \quad (44)$$

Consequently [see (33), and we return to ordinary units]

$$\xi_0 \sim V^{1/2} \sim R (m_n c^2 / E_0)^{1/2}. \quad (45)$$

When the temperature is reduced to $T \sim m_n c^2$, the individual particles are finally formed, and the relativistic acceleration mechanism ceases to operate (see also the preliminary remarks at the end of the Introduction). To estimate the corresponding time t , let us return to (40). It is clear that the volume in which the medium is concentrated is $\sim (ct)^2 \xi_0$. Moreover, $\gamma \sim ct / \xi_0$. We may therefore conclude that the order of magnitude of the "proper volume" is $(ct)^2 \xi_0 \gamma \sim (ct)^3$. In this volume, the above temperature corresponds to pion separations of the order of their Compton wavelength. Thus,

$$ct \sim \frac{\hbar}{m_n c} N_n^{1/2}. \quad (46)$$

We note that this result is similar to the well-known formula given by (2). Since $N_n \gg A$, comparison with (45) shows that the inequality given by (41) is clearly satisfied. The energy spectrum of the particles must be judged from the entropy distribution (see the original papers in [9, 11]). Its observed density is $s\gamma$ and is determined by (40). Therefore,

$$W(\epsilon) d\epsilon \propto s \gamma d\xi = s \gamma \frac{d\xi}{d\gamma} d\gamma \propto \gamma^2 d\gamma. \quad (47)$$

This distribution cuts off sharply at $\gamma = \gamma_{\max}$, which corresponds to $\xi = \xi_0$ at time given by (46). It is readily

seen that

$$\gamma_{\max} \sim \left(\frac{m_n}{m_\pi} \right)^{1/2} \left(\frac{E_0}{m_n c^2} \right)^{1/2}. \quad (48)$$

The particle-energy distribution in the center-of-mass system, normalized to unity, assumes the form

$$W(\epsilon) d\epsilon = \frac{3}{(\epsilon_{\max})^3} \epsilon^2 d\epsilon, \quad \epsilon = \frac{3}{4} \epsilon_{\max} \quad \epsilon < \epsilon_{\max} = m c^2 \gamma_{\max}. \quad (49)$$

In this expression, m is the rest mass of the particular type of particles with which we are concerned. Let us now briefly consider the kinematics of the transformation to the laboratory system. Elementary relativistic transformation yields:

$$E_1 \approx 2 \frac{E_0^2}{m_n c^2} \gg E_0,$$

where E_1 is the primary laboratory energy per nucleon in the bombarding nucleus. The particle energy and angular distribution can be found without great difficulty, but the process is laborious. Integrating it with respect to one of the variables, we find that

$$W(\theta) d\theta = \frac{(m_n c^2 / E_0)^2}{[\theta^2 + (m_n c^2 / E_0)^2]^2} \frac{d\theta}{\pi},$$

$$W(\epsilon_1) d\epsilon_1 = \frac{3/2}{\epsilon_{1\max}} \left[1 - \left(\frac{\epsilon_1}{\epsilon_{1\max}} \right)^2 \right] d\epsilon_1, \quad \epsilon_{1\max} = \frac{E_1}{E_0} \epsilon_{\max} \quad (50)$$

for the angular and energy distributions, respectively. The quantity $d\theta = 2\pi \theta d\theta$ is the solid-angle element and, as can be seen, $\bar{\theta} \sim m_n c^2 / E_0 \ll 1$, i.e., the angular distribution is confined to forward directions in the laboratory system. We emphasize these such simple distributions are valid for the great majority of particles but, strictly speaking, not for all of them. As a matter of fact, the laboratory energy has the lower bound

$$\epsilon_{1\min} = \frac{1}{2} \frac{m_n E_0}{m_n \gamma_{\max}} \ll \epsilon_{1\max},$$

which is relatively low but still ultrarelativistic. In the "soft" part of the spectrum adjacent to $\epsilon_{1\min}$, the particles are emitted at relatively large angles right up to the maximum possible

$$\theta_{\max} = \frac{m_n c^2}{E_0} \gamma_{\max}, \quad 1 \gg \theta_{\max} \gg \theta.$$

These details of the ultrarelativistic distributions are illustrated in Fig. 2.

One further remark must be introduced in connection with the foregoing. The general principles of solution of this kind of hydrodynamic problem would appear to enable us to say that the region of space $\xi < \xi_0$ cannot be absolutely "empty." It should contain the simple wave which is in direct contact with vacuum. Since in the equation of state given by (30) the edge of the distribution of matter (strictly speaking, it, too, is a weak discontinuity) always moves with the velocity of light, the radial size ξ_0 of the simple wave will also remain constant in the ultrarelativistic limit which we have considered. To estimate it, therefore, we must return to

an earlier stage in the expansion process.

The one-dimensional Landau-Khalatnikov theory^[9,11,16] is valid for $ct \ll \xi_0$. The initial configuration was characterized by the longitudinal size $l \sim R m_n c^2 / E_0$ [see also (33)]. When $ct > l\sqrt{3}$, both weak discontinuities move in the same, positive, direction. It is readily shown that, in the one-dimensional relativistic simple wave (self-similar, see, for example, [13]), we have

$$\xi_0 \propto t^\lambda, \quad \lambda = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right)^2 = 7-4\sqrt{3} \approx 0.07 \quad (51)$$

which describes the distance ξ_0 between the weak discontinuities as a function of time. Thus,

$$\xi_0 \sim l \left(\frac{ct}{l} \right)^\lambda \sim l \sim R \frac{m_n c^2}{E_0} \quad (52)$$

if, for the purpose of very approximate calculations, we neglect the effect of the small exponent, and take into account the short duration of the entire one-dimensional phase of the expansion process. Comparison with (45) then yields

$$\xi_0 \ll \xi_0. \quad (53)$$

We may, therefore, neglect the contributions of the energy and entropy of the simple wave, and this was taken into account in the derivation of the formulas considered below.

4. CHARGE DISTRIBUTION OF REACTION PRODUCTS

The fact that the individual particles (hadrons) have certain discrete quantum numbers, i.e., different "charges," enables us to derive a number of interesting relationships.

The equilibrium character of the resulting electric charge distribution is clear even from (46). Immediately after the formation of the individual hadrons, the free path $\sim \hbar/m_n c$ is still small in comparison with the linear dimensions of the entire system, so that hydrodynamics and thermodynamics remain valid, as before, for an appreciable length of time even after transition to the region $T \ll m_n c^2$ in which we have a Boltzmann gas with a practically constant number of particles.⁷⁾ Under these conditions, elastic interactions between the particles, including charge-transfer processes, are sufficiently effective.

We shall base our analysis on the principle of isotopic invariance (see, for example, [17-19]). When nuclei with the same number of protons and neutrons coalesce, the initial state is completely isotropic in isotopic space, with all the ensuing consequences. In particular, all pions (π^+ , π^0 , π^-) are then created in equal numbers. However, in practice, sufficiently heavy nuclei have a neutron excess $A-2Z$. Using the analogy with thermodynamics, and the statistics of rotating bodies,^[12] we can adhere to the point of view that, in equilibrium, a fluid particle rotates as a whole in isotopic space with angular velocity Ω . The Boltzmann

distribution then contains the factor $\exp\{\hbar\Omega\tau/T\}$, which includes the component τ of the particle isospin along the rotation axis. Consequently,

$$\frac{N_p}{N_n} = \frac{N_{\pi^+}}{N_{\pi^-}} = \frac{N_{\pi^0}}{N_{\pi^-}} = \exp\left\{\frac{\hbar\Omega}{T}\right\} \approx 1 + \frac{\hbar\Omega}{T},$$

$$N_{\pi^+} = \frac{N_{\pi^-}}{3} \left(1 + \frac{\hbar\Omega}{T}\right), \quad N_{\pi^0} = \frac{N_{\pi^-}}{3}, \quad N_{\pi^-} = \frac{N_{\pi^-}}{3} \left(1 - \frac{\hbar\Omega}{T}\right) \quad (54)$$

where N_p is the number of protons and N_n is the number of neutrons among the reaction products, and the other subscripts refer to pions of the appropriate type.

The validity of relationships such as those given by (54) does not depend on the presence of other particles. Let us suppose now that antibaryons can be practically neglected, and baryons are represented only by protons and neutrons. Conservation of the baryon charge $2A$ of the entire system then yields

$$N_p = A \left(1 + \frac{\hbar\Omega}{2T}\right), \quad N_n = A \left(1 - \frac{\hbar\Omega}{2T}\right). \quad (55)$$

Let us now apply the conservation of electric charge $N_p + N_{\pi^+} - N_{\pi^-} = 2Z$. We have

$$\frac{\hbar\Omega}{T} = -2 \frac{A-2Z}{A+1/2 N_{\pi^+}}, \quad \frac{A-N_p}{A-2Z} = \frac{1}{1+1/2 N_{\pi^+}/A}. \quad (56)$$

Thus, after the reaction, the neutron excess $A-N_p$ decreases in comparison with its original value $A-2Z$, and hence the pion fraction contains more negative pions than positive pions.

We note that the above formulas are even more valid for $E_0 \lesssim m_n c^2$ when, roughly speaking, there is not enough energy for antinucleon creation. Even in the absence of pions, the relative neutron excess is not large enough to enable us to assume that $\hbar/T \ll 1$, as above. For the region defined by (23), we can readily show that, very approximately,

$$N_{\pi^-} \sim S \sim \left(\frac{E_0}{m_n c^2} \right)^{1/4} A. \quad (57)$$

In the ultrarelativistic limit $E_0 \gg m_n c^2$, the situation can, at least in principle, become modified by the creation of baryon pairs. However, under these conditions, since $N_{\pi^-} \sim S$ [see (35) and [9,11]], the neutron excess in the nucleon fraction is negligible compared with the initial excess.

We leave on one side the somewhat obscure question as to whether the specific "charge" of the so-called strange particles (for example, kaons) affects their yield in such nuclear reactions.

5. DISCUSSION

Let us now briefly review the conclusions of the theory of collisions between energetic nuclei, which refer to the energies of the individual particles after interaction. Their mean value is, as a rule, of the order of the temperature T_0 of the resulting compound system. However, the shape of the energy spectra of the reaction products does not in itself exclude the possibility

that the mechanism may be interpretable as purely thermal and "evaporative."

It is difficult to imagine, for example, that the restriction on the maximum energy of the emitted particle is due to anything other than the hydrodynamic character of the expansion of the compound system. There is a particularly sharp jump in the distribution function at $\varepsilon = \varepsilon_{\max}$ in the ultrarelativistic limit [see (49) and the explanation in text]. When we refer to the nonrelativistic case $E_0 \ll m_n c^2$, we must also emphasize the shape $d\varepsilon/\sqrt{\varepsilon}$ of the soft part of the spectrum, which is totally uncharacteristic for particle-evaporation processes in the case of the usual compound nucleus. When the necessary experimental data become available, therefore, one would hope to be able to achieve a sufficiently reliable identification of the hydrodynamic mechanism discussed in the present paper.

We must now briefly consider the specific features of collisions that are not of the head-on type. The compound system whose evolution is described by the above theory arises in the region of space where the colliding nuclei overlap. Those parts of the nuclei which do not overlap remain as relatively cold fragments, in effect, truncated on collision. They largely continue to execute inertial motion with energy E_0 per nucleon. Subsequently, the shape of a fragment in its rest system tends to an equilibrium, and the oscillations of the surface become transformed into heat. The final temperature reached in the course of this process is probably a slowly-varying function of the primary energy E_0 and is low. Consequently, the velocity of the nucleons evaporated from the fragment is also small in comparison with its translational velocity as a whole. The nucleons evaporated by this mechanism should therefore produce an additional monochromatic peak at $\varepsilon \approx E_0$ in the energy spectrum (in the center-of-mass system). This interesting feature of the phenomenon suggests that the experimental energy distributions should be even more informative.

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APPENDIX

There is undoubted fundamental interest in the expansion of matter which initially occupies uniformly a spherical volume of radius R at rest. We shall assume that the temperature is ultrarelativistic.

In the limit $t \gg R$, when the one-sided expansion away from an internal weak discontinuity $\xi = \xi_0$ has taken place, we have the general integral given by (40). On the other side, $\xi < \xi_0$, we have a spherically symmetric simple wave. To establish the shape of the singularity on the surface of the external weak discontinuity, i. e., on the boundary with vacuum, let us consider the corresponding self-similar solution (which depends only on the variable $\eta = r/t$). Detailed analysis, which we shall omit for lack of space, leads to the following natural-

looking result

$$s = \frac{C}{\xi_0^2 t^2} \left(\frac{\xi}{\xi_0} \right)^3, \quad \gamma^2 = \frac{1}{2} \frac{t^2}{\xi_0^2} \left(\frac{\xi_0}{\xi} \right)^2, \quad (A.1)$$

$$t \gg R, \quad \xi < t.$$

Here, in contrast to ξ , the coordinate ζ is measured from a different bounding surface of the light signal (see also below); ζ_0 is the position of the internal weak discontinuity on the ζ scale. In (A.1), the constants have been chosen so as to ensure that it agrees with (40) when $\xi = \xi_0$. For the conserved total energy and entropy, we now have $E = E' + E''$, $S = S' + S''$, where E' and S' are given by (43) and E'' and S'' can be calculated by analogy, using the solution (A.1) for the simple wave, and then integrating with respect to ζ between zero and ζ_0 . The result is

$$S = \frac{2^4}{3} \pi \frac{C}{\xi_0^3} \left(1 + \frac{\xi_0}{\xi} \right), \quad E = \frac{8}{15} \pi k \frac{C''}{\xi_0^3} \left(1 + \frac{5}{3} \frac{\xi_0}{\xi} \right). \quad (A.2)$$

To ensure that the "asymptotic" solution (40), in general, misses the region in which the equations of hydrodynamics are at least formally satisfied (this is discussed below with a suitable choice of the origin of time), the sphere $\xi = 0$ must be reduced to a point. It is clear, on the other hand, that, in reality, this kind of singularity at $r = 0$ occurs only when the surface of the internal weak discontinuity contracts to the origin, $t = R\sqrt{3}$. During the same time, the external weak discontinuity will move forward to a distance $R\sqrt{3}$. Subsequently, the distance

$$\xi - \xi_0 = \zeta_0 - \xi_0 = (\sqrt{3} + 1)R \quad (A.3)$$

between the singular surfaces (the spheres $\zeta = 0$ and $\xi = 0$) of the two solutions to be matched undergoes no changes because both surfaces expand with velocities strictly equal to the velocity of light. This is the necessary third condition which, together with (A.2), determines finally all three arbitrary constants C , ξ_0 , and ζ_0 . Since

$$k^3 \frac{S^3}{E^3} = V = \frac{4}{3} \pi R^3,$$

we obtain an algebraic equation of a high (sixth) degree in the ratio R/ξ_0 . Numerical solution yields

$$\xi_0 = 0.52R, \quad \zeta_0 = 3.25R, \quad \zeta_0/\xi_0 = 6.3, \quad (A.4)$$

$$C = \frac{24\sqrt{2}}{125\pi} \frac{(1 + 5\zeta_0/3\xi_0)^3}{(1 + \zeta_0/\xi_0)^3} R^3 S.$$

Returning now to ordinary units, and using the dimensionless small combinations

$$z' = 1 - \frac{r + \sqrt{3}R}{ct}, \quad z'' = 1 - \frac{r - R}{ct}$$

for the sake of brevity, we finally obtain

$$s = \frac{C}{(ct)^6} (z')^{-3}, \quad \gamma = \frac{1}{\sqrt{2}} (z')^{-1} \quad \text{for } z' > \frac{\xi_0}{ct};$$

$$s = \frac{C}{\xi_0^2 \xi_0^3} (z'')^3, \quad \gamma = \frac{1}{\sqrt{2}} \frac{\xi_0}{\xi_0} (z'')^{-1} \quad \text{for } 0 < z'' < \frac{\xi_0}{ct};$$

$$t \gg R/c. \quad (A.5)$$

Equations (A.4) and (A.5) form the solution of our problem.

If, as a result of further expansion, the medium cools down to nonrelativistic temperatures, the equation of state given by (30) will be violated and the particle energy distribution will cease to vary. We shall not consider the details of this; the necessary derivations are similar to those leading to (47) and (49). We merely note that the "weakness" of the internal discontinuity at $\xi = \xi_0$ is reflected in the continuity of the thermodynamic and hydrodynamic quantities (but not of their spatial derivatives). However, to determine the particle-energy spectrum, we must transform to the γ scale, in which case the spectral density $W(\varepsilon)$ itself exhibits a discontinuity. It is not difficult to show that

$$W_+/W_- = \xi_0/\xi_0 \quad (\text{A.6})$$

directly at the singularity $\varepsilon = \varepsilon_0 = mc^2\gamma(\xi_0)$ (the subscripts + and - indicate the values of the functions to the right and to the left of it). The final result is that the energy density $W(\varepsilon)$ increases discontinuously [see (A.4)] and thereafter decreases in accordance with the formula $W(\varepsilon)d\varepsilon \propto d\varepsilon/\varepsilon^4$, $\varepsilon > \varepsilon_0$. On the other hand, it follows from (53) that, in the case of the problem considered in Sec. 3, the initial geometry ensures that the distribution function $W(\varepsilon)$ falls to a negligible value for

$$\varepsilon = \varepsilon_0 = \varepsilon_{\max}$$

¹⁾In our previous papers^[4-7] on the macroscopic treatment of apparently unrelated nuclear phenomena, we always came across the condition $k_f R \gg 1$, where k_f is the limiting momentum of the Fermi-liquid quasiparticles. It is readily seen that the condition given by (3) reduces to a very similar criterion.

²⁾This means that we are neglecting the potential energy of the interaction between the nucleons. The assumption that an ideal gas is produced seems, at first sight, to be somewhat drastic. Nevertheless, there are reasons to suppose that it does, in fact, lead to a reasonable description of the main features of the phenomenon. It is clear from the foregoing that the resulting particle-energy distribution essentially reflects the hydrodynamic character of the process, but is not too sensitive to the particular choice of the adiabatic curve. It is also important to remember that the contribution of the interaction energy rapidly decreases during the expansion process.

³⁾In the model example corresponding to $\nu = 0$, the "joining" of the general integral to the simple wave is readily achieved exactly and in an explicit form for any time $t > l/u_0$. The fraction of energy and entropy which is asymptotically taken up by the simple wave turns out to be of the order of $l/u_0 t \ll 1$. Similar estimates are characteristic for other values of ν . It must not, however, be supposed that the fact that the simple wave is negligible for large times t is a universal feature of all hydrodynamic problems involving the free expansion of material into vacuum. In the ultrarelativistic case, the fluid is rapidly accelerated and tends to the limiting (light) velocity so that, in general, a considerable fraction of the total energy and total entropy is concentrated in the simple wave. A specific example of this is the problem solved in the Appendix.

⁴⁾Nevertheless, the narrowness of the region in which (23) is valid is a serious defect of the theory applicable to it. The condition given by (23) may not be sufficient because the nucleon and ion rest masses are not, in reality, all that different from one another.

⁵⁾This phenomenon was considered qualitatively in the original papers.^[8,11] Landau called it "lateral" or "conical" expansion. He used conservation laws to predict a time dependence of the main quantities, which is confirmed by the rigorous formula (40); see below for further details.

⁶⁾From the more formal point of view, the simplicity of this solution and the complexity of the one-dimensional Landau-Khalatnikov solution^[9,11,16] are probably connected with the three-dimensional character of real physical space. Both the hydrodynamic equations (39) and the equation of state (30), which is taken into account in their derivation (this also implicitly assumes the three-dimensional character of space), correspond to this nature of physical space. In this sense, complete concordance of the equations of hydrodynamics and thermodynamics in the "one-dimensional world" also results in an exceedingly simple solution. We shall not reproduce this solution here and merely note the following: when the one-dimensional analog of the thermodynamic relationships given by (30) is used, the equations of hydrodynamics turn out to be strictly linear and can be readily solved in general form. The solution is some explicit function of the initial conditions.

⁷⁾Transition to the Boltzmann region $T < m_\pi c^2$ is accompanied by the strong suppression of pion annihilation processes because, as the density falls, the role of triple (and higher order) collisions falls rapidly to zero.

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