The observed microwave conductivity signals show that the former is true in our samples, i.e., it is more likely that carriers are captured by defects and impurities.

4. CONCLUSIONS

The breakdown of excitons in an hf electric field was observed and investigated. A high-sensitivity method used to detect free carriers resulting from the breakdown to excitons (a microwave spectrometer was used) made it possible to study the kinetics of excitons at temperatures down to 1.3 °K. The experimental results, demonstrating constancy of the exciton concentration for a relatively long time interval, were explained by a theory allowing for the existence of the liquid exciton phase in germanium (electron–hole drops). This model was confirmed by the observation that the position of the breakdown threshold minimum on the time axis was independent of the frequency of the breakdown field. The exponential decay of the free carrier density in the absence of exciton breakdown was attributed to the Auger recombination of carriers in electron–hole drops.

The authors are grateful to A. M. Prokhorov and L. V. Keldysh for their valuable advice.

[1)]A detailed analysis of a circuit for measuring the microwave conductivity of semiconductors is given in[4].

[2)]It should be noted that complete breakdown of an exciton gas results in a fairly interesting physical situation when electron–hole drops exist in an "exciton vacuum."

[1]A. A. Manenkov, V. A. Milyaev, G. N. Mikhaĭlova, and S. P. Smolin, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 454 (1972) [JETP Lett. **16**, 322 (1972)].
[2]L. V. Keldysh, A. A. Manenkov, V. A. Milyaev, and G. N. Mikhaĭlova, Zh. Eksp. Teor. Fiz. **66**, 2178 (1974) [Sov. Phys.-JETP **39**, 1072 (1974)].
[3]A. D. MacDonald, Microwave Breakdown in Gases, Wiley, New York, 1966 (Russ Transl., Mir, M., 1969).
[4]B. V. Zubov, A. A. Manenkov, V. A. Milyaev, G. N. Mikhaĭlova, T. M. Murina, and A. S. Seferou, Fiz. Tverd. Tela (Leningrad) **18**, 706 (1976) [Sov. Phys.-Solid State **18**, 406 (1976)].
[5]Ya. E. Pokrovskiĭ and K. I. Svistunova, Fiz. Tekh. Poluprovodn. **4**, 491 (1970) [Sov. Phys.-Semicond. **4**, 409 (1970)].
[6]L. V. Keldysh, in: Éksitony v poluprovodnikakh pod red. B. M. Vula (Excitons in Semiconductors, ed. by B. M. Vul), Nauka M., 1971, p. 5.
[7]R. M. Westervelt, T. K. Lo, J. L. Shachli, and C. D. Jeffries, Phys. Rev. Lett. **32**, 1051 (1974).
[8]J. C. Hensel, T. G. Phillips, and T. M. Rice, Phys. Rev. Lett. **30**, 227 (1973).
[9]K. Betzler, B. G. Zhurkin, A. L. Karuzskii, and B. M. Balter, Preprint No. 71, Lebedev Physics Institute, Academy of Sciences of the USSR, M., 1975.
[10]P. S. Gladkov, Kandidatskaya dissertatsiya (Thesis for Candidate's Degree), Lebedev Phsyics Institute, Academy of Sciences of the USSR, M., 1972.

Translated by A. Tybulewicz

Dragging of excitons and electron-hole drops by phonon wind

V. S. Bagaev, L. V. Keldysh, N. N. Sibel'din, and V. A. Tsvetkov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted August 19, 1975)
Zh. Eksp. Teor. Fiz. **70**, 702–716 (February 1976)

Dragging of excitons by nonequilibrium short-wave phonons is studied theoretically. It is shown that in multivalley semiconductors with degenerate valence bands (such as Ge or Si) the cross section for scattering of short-wave phonons by indirect excitons may exceed by several orders of magnitude the scattering cross section calculated for the case of isotropic and nondegenerate bands. The kinetics of electron-hole drop (EHD) growth is also considered with allowance for the diffusion of excitons to the drop surface and dragging of the excitons by the phonon wind. By invoking the dragging effect one can explain a number of experiments in which the EHD "diffusion" coefficient is measured: moreover, the dragging effect may restrict the growth of the EHD. Results are presented of experimental observation of EHD motion induced by the phonon wind.

PACS numbers: 71.80.+j

Despite the appreciable progress in the understanding of the properties of electron-hole drops (EHD),[1–3] many facts connected with the kinetics of drop formation and growth and with the spatial distribution and motion remain unclear. These include experiments aimed at determining the coefficient of EHD "diffusion"[2,4–7] and the limitation of the EHD radius at temperatures close to threshold.[8,9] The EHD diffusion coefficient was measured by different workers,[2,4–7,9–12] but there are colossal discrepancies (six orders of magnitude) between the results of different measurements. In most studies the diffusion coefficient was determined by measuring the dimensions of the region occupied by the EHD. In[2,4,7], where a relatively low excitation intensity was used, a value $D < 1$ cm^2/sec was obtained for the diffusion coefficient. A value $D \approx 150$ is cited

in[5] and values and 25–500 cm^2/sec are cited in[6], depending on the experimental conditions, with an increase of D observed with increasing excitation level, while in[11] it is shown that when the excitation level is raised from 14 to 600 mW the "diffusion" coefficient increases from 0.8 to 80 cm^2/sec. The EHD diffusion coefficient was estimated from experiments on the motion of the EHD in a uniform deformation field, [9,10]; in these experiments they measured directly the EHD momentum relaxation time. The obtained EHD diffusion coefficient was $D \approx 10^{-4}-10^{-3}$ cm^2/sec, depending on the EHD radius assumed in the estimate. Since the dimensions of the EHD are sufficiently well known from light-scattering experiments, [2,3,8,9,12] it can be assumed that this value is not far from the true one. An estimate of the diffusion coefficients from mobility measurements in experiments aimed at determining the EHD charge yields approximately the same value. [11,12]

Thus, experiments on the motion of EHD[9,10] are apparently the most direct way of measuring the diffusion coefficients. The results of experiments in which the size of the region occupied by the EHD was measured[2,4–7,11,12] suggest that some EHD drift mechanism exists in which the drift velocity increases with increasing excitation intensity. This assumption is confirmed by results of studies[13,14] that have demonstrated that the region occupied by the EHD is spherical and has a sharp edge—a fact difficult to explain from the point of view of EHD diffusion. It was suggested in[11,12,15,16] that the EHD are dragged by excitons in the presence of an exciton density gradient. Estimates show, however, that this effect should be very small, especially at low temperatures, when the exciton density is negligible, whereas experiment yields a contrary result.[6,17] Dragging of EHD in an electric field by a current of free electrons and holes was considered in[18]. However, the measurements cited above were performed in the absence of an electric field, and furthermore the estimates made in[18] are not sufficiently well founded, since the concentration of the free carriers is usually much less than is assumed in that paper.

We consider in the present paper the dragging of excitons and of EHD by the phonon wind. The generation and recombination of the non-equilibrium carriers are accompanied by dissipation of energy, which goes mainly to the phonons, so that intense currents of nonequilibrium exist in the system and can drag the excitons and the EHD, acting by the same token on their spatial distributions. These currents flow both from the region where the carriers are generated, on account of their thermalization, and directly from each EHD, on account of the energy released in the nonradiative recombination channels. We note that in this case we are dealing mainly with short-wave acoustic phonons, which are generated mainly by thermalization and by nonradiative recombination, and have sufficiently large lifetimes, while the optical phonons decay very rapidly into acoustic ones. We shall consider below the process of exciton dragging by short-wave phonons and shall show that in semiconductors such as Ge and Si, owing to the peculiarities of their band structure, the cross section for the scattering of such phonons by indirect excitons is larger by several orders of magnitude than the scattering cross section calculated for nondegenerate isotropic bands. No rigorous analysis was made for EHD, but owing to the large relaxation time of the EHD momentum[9,10] we can expect the drop-dragging effect to be more appreciable. Results of an experimental observation of EHD motion under the influence of phonon wind will be presented.

The phonon-wind assumption helps explain the results of experiments on the spatial distribution of EHD[2,4–7,11–14] and makes it possible to understand the mechanism that moves the drops to the p-n junction in measurements of current pulses.[5,15,19–21] In addition, the phonon wind, propagating from each EHD and dragging excitons with it, may be the cause of the limitation, observed in[8,9,17], of the growth of the EHD radius.

THEORY

We consider the dragging of excitons by a current of nonequilibrium phonons flowing from a region where nonequilibrium carriers are generated by an external excitation source. As these phonons collide with the excitons and are absorbed or scattered by them, they drag the excitons with them. The force exerted by this "phonon wind" on the exciton can be determined in terms of the average momentum transferred to each exciton by the phonons per unit time:

$$\mathbf{f}=d\mathbf{p}/dt. \tag{1}$$

By virtue of the energy and momentum conservation laws and with allowance for the fact that the velocity of sound s is much less than the thermal velocity of the excitons v_T, the momentum transferred to the exciton in each collision should be of the order of the thermal momentum of the exciton $p_T=(2MkT)^{1/2}$, even though the average quasimomentum of the phonon is $|\overline{\hbar\mathbf{k}}| \gg p_T$ (here M is the effective mass of the exciton). Introducing a certain average effective cross section σ for the scattering of an exciton by a nonequilibrium phonon (a detailed calculation and an estimate of σ are given below, formula (20)), and denoting the phonon flux density by $\mathbf{w}$, we obtain

$$\mathbf{f}=p_T\sigma\mathbf{w}/\hbar\bar{\omega}, \tag{2}$$

where $\hbar\bar{\omega}$ is the average energy of these phonons.

Let, for example, the nonequilibrium carriers be generated on a flat sample surface (with linear dimensions $\gg L_D=(D\tau)^{1/2}$, where L_D is the exciton free diffusion length, D is their diffusion coefficient, and τ is their lifetime) in a narrow surface layer of thickness $\ll L_D$. Then, in order to find the distribution of the excitons $n(x)$ over the sample thickness it is necessary to solve the continuity equation with boundary conditions $n(x)=0$ as $x\to\infty$ and $n(x)=n(0)$ at $x=0$. In the stationary case this equation takes the form

$$\operatorname{div}\mathbf{S}+n(x)/\tau=0, \tag{3}$$

where the expression for the exciton flux density, with account taken of both the diffusion and the directional

drift of the excitons under the influence of the force f, can be written in the form

$$\mathbf{S}(x)=-D\frac{\partial n(x)}{\partial x}\mathbf{i}+\frac{D}{kT}\mathbf{f}n(x), \tag{4}$$

here D/kT is the exciton mobility and i is a unit vector along the x axis. The solution of Eq. (3) is

$$n(x)=n(0)\exp\{-x/L_{\text{eff}}\}, \tag{5}$$

where

$$L_{\text{eff}}=L_D/\{[1+(fL_D/2kT)^2]^{1/2}-fL_D/2kT\}. \tag{6}$$

Expression (5) differs from the usual solution of the diffusion equation in that L_D is replaced by L_{eff}, the latter being dependent on the phonon flux density. At $fL_D/kT \gg 1$ (a condition apparently satisfied in Ge at $w \gg 10^{-2}$ W/cm^2, in view of the estimates given above) we have $L_{eff} \approx fL_D^2/kT \sim w$.

The relation $L_{eff} \sim w$, however, is not universal but is determined by the geometry of the excited region. Thus, if the radiation is focused into a narrow strip (of width $\ll L_D$ and length $L \gg L_D$), then the phonon flux density, and with it also the force $f=p_T\sigma W/\pi r L\hbar\bar{\omega}$, decreases with increasing distance from the excited region, and the spatial distribution of the excitons takes the form

$$n(r)=\text{const}\left(\frac{r}{L_D}\right)^{\nu}K_{\nu}\left(\frac{r}{L_D}\right), \tag{7}$$

where r is the distance from the excited strip, W is the total energy carried away by the nonequilibrium phonons per unit time, and $y=fr/kT=p_T\sigma W/\pi L\hbar\bar{\omega}kT$ and $K_\nu(r/L_D)$ is the known Macdonald function. At $\nu \lesssim 1$, the distribution (7) differs little from the usual distribution that does not take into account the phonon dragging by the excitons, and has a width $\sim L_D$. At $\nu \gg 1$ we have in the region $r < \nu L_E$

$$n(r)\approx\text{const}\cdot\exp[-r^2/2\nu L_D^2], \tag{8}$$

i.e.,

$$L_{\text{eff}}=L_D(2p_T\sigma W/\pi L\hbar\bar{\omega}kT)^{1/2}\sim W^{1/2}.$$

Similarly, when the exciting radiation is focused into a spot of diameter $\ll L_D$, the decrease of the concentration with increasing distance, a sufficiently large phonon-flux power

$$W \gg 4\pi kT\hbar\bar{\omega}L_D/p_T\sigma \tag{9}$$

follows the law

$$n(r)\approx\text{const}\cdot\exp[-(r/L_{\text{eff}})^3], \tag{10}$$

where

$$L_{\text{eff}}=(3p_T\sigma L_D^2W/2\pi\hbar\bar{\omega}kT)^{1/3}\sim W^{1/3}.$$

It is seen from these expressions that at sufficiently high excitation levels the volume of the excited region always increases in proportion to W, and therefore, regardless of the geometry of the excited region, there is a certain maximum exciton density n_m that can be attained under conditions of stationary but not volume excitation.

We proceed now to calculated σ. There can be two mechanisms of momentum transfer from the nonequilibrium phonons to the excitons: phonon absorption and phonon scattering by excitons. The cross section, averaged over the Maxwellian distribution, for the absorption of an acoustic phonon of frequency ω is

$$\bar{\sigma}_{\text{abs}}(\omega)=\frac{d^2}{2\hbar\rho s^2v_T}\exp\left\{-\frac{(\hbar\omega-2Ms^2)^2}{8Ms^2kT}\right\}, \tag{11}$$

where d is the deformation potential and ρ is the density of the crystal. Thus, only phonons with energy $\hbar\omega \lesssim 2^{3/2}(Ms^2kT)^{1/2}$ are effectively absorbed. If we assume that the nonequilibrium phonons are sufficiently rapidly thermalized (within a time $\ll L_D/s \sim 10^{-7}$ sec), then at low temperatures $kT \lesssim 8Ms^2$ the absorption cross section (11) reaches values $\sim 10^{-14}$ cm^2 and the dragging effect becomes noticeable starting with fluxes $w \sim 10^{-2}$ W/cm^2. This assumption, however, is hardly justified at low temperatures. During the course of thermalization of nonequilibrium electrons and holes, the phonons produced in the main are optical and short-wave acoustic with energies $\hbar\omega \gtrsim 10$ meV. The optical longitudinal acoustic phonons seem to decay rapidly into softer transverse acoustic phonons, but further energy relaxation is already greatly hindered by the energy and quasimomentum conservation laws. Indeed, there are many experiments[22,23] that show that at low temperatures there exists in germanium a ballistic (collisionless) regime of propagation of strongly nonequilibrium ($\hbar\omega \gg kT$) phonons over distances[1)] ~ 1 cm. It is therefor more legitimate to assume that $\bar{\omega}$ is of the order of the limiting frequency of the transverse acoustic phonons. By virtue of (11), however, such phonons are not absorbed at all by the excitons, and can only be scattered by them.

Phonon scattering is a two-phonon process and its cross section, at first glance, should be rather small in comparison with the cross section (11) for single-phonon scattering. Indeed, if we use for the description of the electron-phonon interaction the usual linear-in-strain Hamiltonian

$$H_{e-\text{ph}}^{(1)}=d\varepsilon \tag{12}$$

(ε is the relative strain), then in second-order perturbation theory we obtain phonon-scattering cross section values $\sigma_{sc} \sim d^4M^5p_T/(\rho s\hbar|\mathbf{k}|)^2 s \sim (10^{-18}-10^{-19})$ cm^2; these values are too small to make the dragging effect noticeable.

In crystals of the germanium and silicon type, however, which have a multivalley conduction band and a degenerate valence band, there is a specific feature that increases the cross section for the scattering of phonons by indirect excitons by several orders of magnitude. The point is that the ground level of the exciton

in such a crystal is split into two closely lying levels.[24,25] The physical cause of this splitting can be explained very roughly in the following manner: the electric field produced by the electron entering the exciton and belonging to some definite valley of the conduction band is anisotropic, by virtue of the anisotropy of the effective masses of the electron. It therefore lowers locally (in the vicinity of the given electron) the crystal symmetry and splits the valence band. Thus, the splitting δ of the exciton level is similar in character to the splitting produced in degenerate band by relative strains, which also lower the symmetry of the crystal, and it is convenient to characterize this splitting by a certain equivalent strain[25]

$$\varepsilon_c = \delta / d_v. \tag{13}$$

We present for simplicity a purely schematic consideration, disregarding the tensor structure of the strain ε and of the deformation potentials d_v. When a true strain ε, which generally speaking does not coincide at all in direction with ε_c, is produced in a crystal, the splitting of the exciton levels is given by[25]

$$\delta E = d_v(\varepsilon_c^2 + \varepsilon^2 + c\varepsilon\varepsilon_c)^{1/2}, \tag{14}$$

where c is a constant that depends on the orientation of ε. The connection between the exciton energy and the strains, and hence with the phonons, is thus strongly nonlinear even at quite small ε. Confining ourselves to the second-order terms of interest to us, we can express the Hamiltonian of the exciton-phonon interaction in place of (12) in the form

$$H_{e\text{-ph}} = H^{(1)} + H^{(2)} = \left(d + \frac{1}{2} c d_v\right)\varepsilon + \frac{1}{2} d_v \frac{\varepsilon^2}{\varepsilon_c}\left(1 - \frac{c^2}{8}\right). \tag{15}$$

It is the large value $\varepsilon_c^{-2} = (d_v/\delta)^2 \sim 10^6$ which determines the anomalously large probabilities of the two-phonon processes in our case, compared with the case of simple nondegenerate bands. Expanding in the usual manner[23] the strains in terms of the normal vibrations (phonons), we obtain in first order of perturbation theory in $H^{(2)}$ the probability of scattering of a phonon with wave vector $\mathbf{k}$ into a state $\mathbf{k}'$ by an exciton with momentum $\mathbf{p}$:

$$w_{sc}(\mathbf{p}; \mathbf{k} \to \mathbf{k}') = \frac{2\pi}{\hbar} C \left(\frac{d_v}{2\varepsilon_c}\right)^2 \frac{\hbar^2 |\mathbf{k}|^2 |\mathbf{k}'|^2}{4\rho^2 V^2 \omega_{\mathbf{k}} \omega_{\mathbf{k}'}} \delta\left(\frac{p^2 - (\mathbf{p} + \hbar\mathbf{k} - \hbar\mathbf{k}')^2}{2M} + \hbar\omega_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'}\right). \tag{16}$$

The constant $C \sim 1$ was introduced here to take phenomenologically into account (in the mean) the differences between difference modes and the direction (c in (14), the possible scattering with intervalley transitions of the exciton, the anisotropy of the effective masses, etc.; V is the normalization valume, and $\omega_{\mathbf{k}}$ is the frequency of the phonon with wave vector $\mathbf{k}$. Using (16), we calculate now not the mean value of the scattering cross section, but directly the force $\mathbf{f}$ exerted on the exciton by the phonon flux. In accord with the definition (1),

$$\mathbf{f}(\mathbf{r}) = \int \frac{d^3k d^3k' d^3p V^2}{(2\pi)^9 \hbar^3} \hbar(\mathbf{k} - \mathbf{k}') w_{sc}(\mathbf{p}, \mathbf{k} \to \mathbf{k}') \left(\frac{2\pi\hbar^2}{MkT}\right)^{1/2} \times \exp\left\{-\frac{p^2}{2MkT}\right\} F(\mathbf{k}, \mathbf{r}). \tag{17}$$

We average here over the Maxwellian distribution for the excitons; $F(\mathbf{k}, \mathbf{r})$ is the phonon distribution function in the wave vectors at the point $\mathbf{r}$. Recognizing that the momentum $\hbar\varkappa = \hbar(\mathbf{k} - \mathbf{k}')$ transferred in the scattering is on the order of $p_T \ll |\hbar\mathbf{k}|$ and that $M|\nabla_{\mathbf{k}}\omega_{\mathbf{k}}|^2 \sim Ms^2 \ll kT$, we can integrate in (17) with respect to $\mathbf{p}$ and $\varkappa$ in explicit form:

$$\mathbf{f}(\mathbf{r}) = \frac{2CkT}{3\pi^2\hbar v_T}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2 \int \frac{d^3k}{(2\pi)^3} \nabla_{\mathbf{k}}\omega_{\mathbf{k}} \cdot \hbar\omega_{\mathbf{k}} \frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3} F(\mathbf{k}, \mathbf{r}). \tag{18}$$

We note now that the phonon energy flux density at the point $\mathbf{r}$ is of the form

$$\mathbf{w}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} \nabla_{\mathbf{k}}\omega_{\mathbf{k}} \cdot \hbar\omega_{\mathbf{k}} F(\mathbf{k}, \mathbf{r}). \tag{19}$$

Therefore, taking outside the integral sign in (18) a certain mean value of the factor that distinguishes between the integrands in (18) and (19), we can rewrite $\mathbf{f}(\mathbf{r})$ in a more compact form:

$$\mathbf{f}(\mathbf{r}) = \frac{2CkT}{3\pi^2\hbar v_T}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2 \left(\overline{\frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3}}\right) \mathbf{w}(\mathbf{r}). \tag{20}$$

Strictly speaking, in the spirit of the simplifications already made on going from (18) to (20), we have neglected also the difference that the crystal anisotropy can produce between the directions of the vectors $\mathbf{f}$ and $\mathbf{w}$. The principal uncertainty in (20) is connected with the lack of any information whatever concerning the distribution function of the nonequilibrium phonons radiated in the generation region, and hence the impossibility of determining reliably the value of $(|\mathbf{k}|^4/\omega_{\mathbf{k}}^3)$. The most natural assumption, however, is that these phonons are more or less uniformly distributed over the entire Brillouin zone in the most long-lived transverse acoustic modes.

Using (20), we can write down exact formulas for the effective drift length of the excitons. At sufficiently high power we obtain in the planar case from (6)

$$L_{\text{eff}} \approx \frac{2CL_D^2}{3\pi^2\hbar v_T}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2 \left(\overline{\frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3}}\right) w, \tag{21}$$

in the case of excitation by a narrow strip, the index of the Macdonald function is

$$\nu = \frac{C}{3\pi^3\hbar v_T}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2 \left(\overline{\frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3}}\right) \frac{W}{L} \tag{22}$$

and

$$L_{\text{eff}} = \frac{L_D M d_v}{2\pi\varepsilon_c\hbar\rho}\left[\frac{2C}{3\pi\hbar v_T}\left(\overline{\frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3}}\right)\frac{W}{L}\right]^{1/2} \tag{23}$$

and finally, in the case of excitation in a point

$$L_{\text{eff}}=L_D\left[\frac{C}{\pi^3\hbar v_T}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2\left(\frac{\overline{|\mathbf{k}|^4}}{\omega_{\mathbf{k}}^3}\right)\frac{W}{L_D}\right]^{1/3}. \tag{24}$$

The limiting exciton concentration attainable under conditions of stationary excitation is given by

$$n_m\sim\frac{\hbar v_T}{D\Delta E}\left(\frac{2\varepsilon_c\hbar\rho}{Md_v}\right)^2\left(\frac{\overline{|\mathbf{k}|^4}}{\omega_{\mathbf{k}}^3}\right)^{-1}, \tag{25}$$

where ΔE is the energy going into heat for each pair of nonequilibrium carriers produced by the excitation.

The nonequilibrium phonon fluxes produced at each of the EHD can, by dragging the excitons with them, hinder the exciton diffusion towards the surface of the drop and by the same token decrease the growth rate of the EHD. Assuming that in each recombination act an energy $\approx E_g$ (the width of the forbidden band) is released in the EHD and that the nonradiative recombination channels (say, the Auger channel) predominate, we find that the energy flux from a drop of radius R is

$$W=\frac{4\pi}{3}R^3\frac{n_0}{\tau_0}E_g, \tag{26}$$

where n_0 is the density of the electron-hole liquid and τ_0 is the lifetime of the carriers in the EHD. The energy flux density at a distance r from the center of the drop is

$$\mathbf{w}(\mathbf{r})=W\mathbf{r}/4\pi r^3\hbar\bar{\omega}. \tag{27}$$

It is convenient to express the force exerted by the phonon wind on the excitons as the gradient of a certain effective potential $U(\mathbf{r})$, i.e., $\mathbf{f}=-\operatorname{grad}U(\mathbf{r})$. Then, using (20), (26), and (27), we obtain

$$U(\mathbf{r})=\frac{2C}{9\pi^2}\frac{kT}{\hbar}\left(\frac{Md_v}{2\varepsilon_c\hbar\rho}\right)^2\left(\frac{\overline{|\mathbf{k}|^4}}{\omega_{\mathbf{k}}^3}\right)\frac{n_0E_g}{v_T\tau_0}\frac{R^3}{r}=\frac{R^3kT}{R_0^2r}, \tag{28}$$

where

$$R_0=\frac{3\pi}{(2C)^{1/2}}\frac{2\varepsilon_c\hbar\rho}{Md_v}\left[\left(\frac{\overline{|\mathbf{k}|^4}}{\omega_{\mathbf{k}}^3}\right)^{-1}\frac{\hbar v_T\tau_0}{n_0E_g}\right]^{1/3}. \tag{29}$$

We consider now the spatial distribution of the excitons $n(\mathbf{r})$ in the vicinity of a drop with allowance for the potential $U(\mathbf{r})$, and the diffusion of the excitons in the drop. It is necessary for this purpose to solve the continuity equation with the corresponding boundary conditions. Since we are dealing with the solution of the problem in a small region near an individual drop (it is assumed that the drop dimensions are small in comparison with the distance between them), within which the quasi-stationary distribution of the excitons is established within a very short time, it follows that this equation goes over into the stationary diffusion equation div $\mathbf{S}=0$, where $\mathbf{S}(\mathbf{r})$ is defined by E2. (4). The boundary conditions are $n(r\to\infty)=n$, where n is the average exciton density in the volume of the crystal, and

$$S(R)=[n(R)-n_T(R)]v_T, \tag{30}$$

where $S(R)$ is the exciton flux density on the surface of the drop, $N(R)$ is the exciton gas density near the surface of the drop,

$$n_T(R)=g\left(\frac{M_dkT}{2\pi\hbar^2}\right)^{3/2}\exp\left\{-\frac{\Delta}{kT}+\frac{2\alpha}{n_0RkT}\right\} \tag{31}$$

is the thermodynamic-equilibrium exciton gas density over a drop of radius R, and $V_T=(kT/2\pi M)^{1/2}$ is the thermal velocity of the excitons; In (31), Δ is the work function of the excitons from the EHD as $R\to\infty$, M_d is the effective mass of the density of states of the excitons, α is the coefficient of surface tension of the electron-hole liquid, and g is the multiplicity of the degeneracy of the ground state of the exciton. Solving the diffusion equation with allowance for the boundary conditions, we find the exciton flux on the surface of a drop of radius R, expressed in terms of the average exciton density

$$S(n,R)=4\pi R^2\left[1+\frac{v_TR}{D}\frac{kT}{U(R)}(1-e^{-U(R)/kT})\right]^{-1}[ne^{-U(R)/kT}-n_T(R)]v_T. \tag{32}$$

Expression (32) enables us to write down an equation that describes the kinetics of the EHD growth and recombination, with allowance for the diffusion of the excitons to the surface of the drop and to their dragging by the phonons:

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3n_0\right)=S(n,R)-\frac{4}{3}\pi R^3\frac{n_0}{\tau_0}. \tag{33}$$

In the stationary case, the left-hand side of (33) is equal to zero. Then solving (32) and (33) simultaneously, we obtain

$$n=e^{U(R)/kT}\left\{n_T(R)+\frac{n_0R^2}{3D\tau_0}\frac{kT}{U(R)}\left[1+\frac{D}{v_TR}\frac{U(R)}{kT}-e^{-U(R)/kT}\right]\right\}. \tag{34}$$

This expression gives the radius of the EHD that are in equilibrium with an exciton gas of density n.

It is easy to verify that at $R\ll R_0$ Eqs. (32) and (34) go over into the corresponding expressions obtained without allowance for the effect of exciton dragging by the phonons.[17] When R approaches R_0, however, the density of the excitons that are in equilibrium with drops of radius R begins to increase very rapidly ($\sim\exp(R^2/R_0^2)$), in other words, the growth of the EHD radius slows down sharply with increasing excitation level, and the dependence of the radius on the temperature becomes very weak (since $R_0\sim T^{1/4}$), in qualitative agreement with the results of[8,9,17]. Substitution of the numerical values shows that in the temperature region $T\gtrsim3^\circ$K the right-hand side of (34) is well approximated by the first term, so that

$$n\approx n_T(R)\exp\frac{U(R)}{kT}. \tag{35}$$

Thus, $U(R)$ represents the effective decrease of the work function of the EHD, which becomes noticeable at $R\approx R_0$ and increases in proportion to R^2. The lack of information on the distribution function of the phonons emitted by the EHD does not make it possible to estimate reliably the value of R_0. Substituting then in (29), by

way of estimate, $|\bar{\mathbf{k}}| \sim 10^8$ cm^{-1} and $\bar{\omega}_{\mathbf{k}} \sim 10^{13}$ sec^{-1}, and using for all the remaining quantities more or less well known values ($\rho \approx 5$ g/cm^3, $E_g \approx 0.74$ eV, $d_v \approx 4$ eV, $2\delta = 2\varepsilon_c d_v \approx 1$ meV, $M \approx 10^{-27}$ g, $v_T \approx 5\times 10^5$ cm/sec, $n_0 \approx 2\times 10^{17}$ cm^{-3}, and $\tau_0 \approx 4\times 10^{-5}$ sec), we obtain $R_0 \approx 1.5\times 10^{-3}$ cm, which is close to the limiting value $\sim 10^{-3}$ which we observed for the EHD radius,[8,9,17] although this agreement should not be given too large a significance. The point is that the quantities n_m and R_0 are determined, in essence, by the same combination of the parameters. Using (25) and (29), we obtain

$$R_0 \sim \left(\frac{n_m}{n_0}\frac{\tau_0}{\tau}\frac{\Delta E}{E_g}\right)^{1/2} L_D, \tag{36}$$

therefore a measurement of n_m would make it possible to determine also R_0. We know of no direct experiments of this kind, but the numerous communications[27-29] reporting that an exciton density $n \gtrsim 10^{15}$ has been attained under conditions of stationary optical excitation suggest, after comparison with (36) (assuming $\Delta E \sim 1$ eV) that the action of the "phonon wind" on the excitons is strongly exaggerated[2)] in the foregoing estimates. If we assume by way of estimate $|\bar{\mathbf{k}}| \sim 3\times 10^7$ cm^{-1}, then at these values of $|\mathbf{k}|$ we have $\omega_{\mathbf{k}}/|\mathbf{k}| \approx s_t \approx 3\times 10^5$ cm/sec (s_t is the velocity of the transverse sound) and $\overline{(|\mathbf{k}|^4/\omega_{\mathbf{k}}^3)} \approx |\mathbf{k}|/s_t^3 \approx 10^{-9}$ sec^3/cm^4; then (29) yields $R_0 \sim 1.5\times 10^{-2}$ cm. This is in reasonable agreement with the value of R_0 determined from (36) at $n_m \sim 10^{15}$ cm^{-3}.

We make a few more remarks concerning the results. Although the "phonon wind" greatly hinders the growth of the EHD by condensation of the exciton from the gas phase, this does not exclude in any way the possibility of observing drops with $R \gg R_0$; for example, if the carrier generation takes place directly in the drop, or if a carrier density $n \gtrsim n_0$ is produced immediately in some part of the sample. Nor do the conclusions contradict the results of[30-32], in which EHD with a radius of hundreds of microns were observed. The point is that these drops were produced in uniaxially deformed samples and had significantly lower carrier densities n_0, and as a consequence[31] larger lifetimes ($\sim 5\times 10^{-4}$ sec). However, as is easily seen from (29), a decrease of n_0 by one order of magnitude and an equal increase of τ_0 increase R_0 by one order. In addition, if the strain produced in the samples greatly had exceeded ε_c, then the cross section for the scattering of phonons by excitons should decrease strongly.

It is presently impossible to investigate sufficiently fully the dragging of EHD by a flux of strongly-nonequilibrium phonons, owing to the lack of detailed calculations of the energy and of the spectrum of the excitations of the EHD, with account taken of the real band structure in weakly deformed germanium and silicon. The experiments[10] show, however, that in the case of small strains the dependence of the energy per particle pair in the EHD on the strain is strongly nonlinear. We can point to at least one mechanism that produces the strong nonlinearity in this case. It is known[33] that the dependence of the energy of the holes on the strain in these crystals has schematically a structure of the type of (14), provided we replace $\delta = d_v\varepsilon_c$ in this formula by the kinetic energy. Inasmuch as only holes with energies close to the Fermi energy ε_{Fh} take part in the phonon absorption and scattering, by virtue of the Pauli principle, we can introduce in analogy with (15) also a nonlinear hole-phonon interaction Hamiltonian

$$H_{h-ph} = d\varepsilon + \tilde{c}\frac{d_v^2}{\varepsilon_{Fh}}\varepsilon^2.$$

By a procedure similar to the derivation of (20), but with the Fermi degeneracy taken into account, we can obtain for the volume density of the force $n_0\mathbf{f}(\mathbf{r})$ (n_0 is the EHD density, $\mathbf{f}(\mathbf{r})$ is the force acting on one electron-hole pair), due to the scattering of the phonons by holes, the expression

$$n_0\mathbf{f}(\mathbf{r}) = \frac{2\tilde{C}}{3\pi}\left(\frac{d_v^2 m_h^2}{2\pi\hbar^3\rho}\right)^2 \left(\overline{\frac{|\mathbf{k}|^4}{\omega_{\mathbf{k}}^3}}\right)\mathbf{w}(\mathbf{r}), \tag{20a}$$

where m_h is the effective mass of the hole, the constant $\tilde{C} \sim 1$, and the averaging procedure has the same meaning as in (20).

Just as for the excitons, the dragging of EHD by a current of phonons with relatively large wavelengths, $|\mathbf{k}| \lesssim n_0^{1/3}$, is determined, naturally, by the absorption process and not by the scattering. The expression for the force is then

$$n_0\mathbf{f}_{abs}(\mathbf{r}) = \frac{d^2m^2}{8\pi\hbar^2\rho}\int_{|\mathbf{k}|\leq k_0}\frac{d^3k}{(2\pi)^3}\mathbf{k}|\mathbf{k}|F(\mathbf{k},\mathbf{r}) = \frac{d^2m^2}{8\pi\hbar^3\rho s^2}\overline{(|\mathbf{k}|)}_{|\mathbf{k}|<k_0}\mathbf{w}(\mathbf{r}),$$
$$k_0 = 2\pi\left(\frac{3}{\pi}n_0\right)^{1/3}.$$

This formula takes into account the absorption of the phonons by the electrons bound in the EHD (m is the electron effective mass). To find the total force it is necessary to add a similar term for the holes.

THE EXPERIMENT

The dragging of excitons and EHD by the phonon wind can be detected by the displacement of the region occupied by the excitons or EHD under the influece of a current of nonequilibrium phonons produced with the aid of an additional excitation source. The experiments were performed with the setup used by us previously to measure the scattering of light by EHD.[34] A block diagram of the setup is shown in Fig. 1. The excitation source was a laser of ~10 mW power, operating at a wavelength 1.52 μ. The radiation of this laser was focused into a spot of diameter ~200 μ on the front surface of the sample. The exciting radiation was modu-

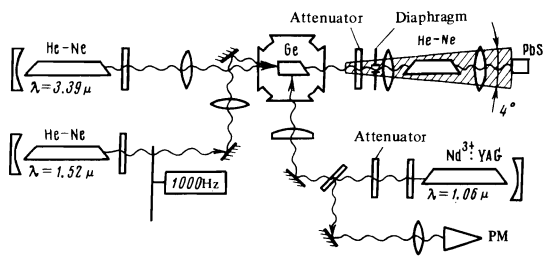

FIG. 1. Block diagram of experimental setup.

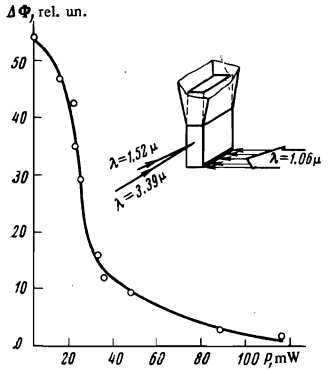

FIG. 2. Dependence of the absorption signal $\Delta\Phi$ on the power P at $x=0$ and $T=1.95\,^{\circ}\mathrm{K}$.

lated at a frequency 1 kHz. We measured the absorption of 3.39-μ laser radiation by the nonequilibrium carriers bound into excitons or EHD. This radiation was also focused on the front surface of the sample, into a spot of ~300 μ diameter. The beams of both lasers could be collocated (insert in Fig. 2) or separated by a distance x (insert in Fig. 3), with the position of the 3.39-μ laser beam remaining unchanged in all the measurements. The absorption signal was recorded with a goniometer consisting of a quantum amplifier and a PbS photoreceiver cooled to ~100 °K. This registration method made it possible to detect the EHD motion not only from the change of the absorption signal, but also from the change of the pattern of the diffraction by the edge of the region occupied by the EHD.[35]

The nonequilibrium phonons were generated upon thermalization of the nonequilibrium carriers, as well as in the nonradiative recombination of the excitons and in the EHD produced with the aid of stationary illumination of the sample by an Nd^{3+} YAG laser. This radiation was focused with a cylindrical lens on the lateral face of the sample into a narrow strip ~5 mm long, parallel to the light beams of the two other lasers (Figs. 2 and 3). The distance from this strip to the 3.39-μ laser beam was ~7 mm. The radiation of the Nd^{3+} YAG laser was attenuated with calibrated light filters and monitors with a power meter ("attenuator" and "PM" in Fig. 1, respectively).

The measurements were made on Ge samples with residual-impurity density less than 10^{12} cm^{-3}. The samples were mechanically polished and measured 15 $\times 5 \times 2$ mm. The sample plane through which the 3.39-μ radiation passed was inclined 2° to the opposite plane, to eliminate parasitic interference. Just as in the scattering measurement,[8] the samples were soldered into the bottom of the helium container of the cryostat in such a way that the working part of the sample was in vacuum while the other part was immersed in liquid helium.

The dragging of the EHD by the phonon wind was measured at $T \approx 1.95\,^{\circ}\mathrm{K}$. At this temperature almost all the nonequilibrium carriers were bound into EHD. The radii of the EHD and their concentration in the crystal, obtained from measurements of the light scattering, were under the conditions of our experiments $R=5$ μ and $N \approx 2\times 10^{7}$ cm^{-3}, respectively. Figure 2 shows the dependence of the absorption signal (which is proportional to the volume of each phase in that part of the crystal through which the probing 3.39-μ laser beam passed) on the power P of the laser radiation that produced the flux of nonequilibrium phonons, for the case when the probing and exciting laser beams were collocated ($x=0$). The absorption signal decreased with increasing power P. The connection between the observed effect and the motion of the EHD is confirmed by the next experiment.

To observe directly the EHD motion due to the phonon wind, the exciting laser spot was displaced a distance x from the probing laser spot. At $x>0.3$ mm, no absorption signal was observed with the 1.06-μ laser radiation blocked (i.e., at $P=0$). When the exciting laser beam was displaced upward relative to the probing laser beam, no absorption signal produced even when the illumination that produced the nonequilibrium phonons was turned on. When the exciting laser beam was shifted downward on the sample in such a way that it passed between the probing beam and the region where the phonons were generated (insert in Fig. 3), an absorption signal appeared at sufficiently large P and its magnitude depended on the power P and on the distance x.

Figure 3 shows the dependence of the absorption signal on the distance x at a phonon-producing laser power $P \approx 40$ mW. It is seen from the figure that the absorption signal, and hence also the number of particles bound into EHD, decreases with increasing x, in accord with expression (5). We emphasize that the spatial distribution of the excitons and the carriers bound into drops of the liquid phase is described by the same expressions (5), (7), (8), and (10), except that in the EHD case it is necessary to replace L_D in the formulas (21), (23), and (24) for L_{eff} by the quantity $(\tau_0\tau_r kT/M)^{1/2}$ (τ_r is the EHD momentum relaxation time) and L_{eff} must be expressed in terms of the force with the aid of (20) (now f is the force exerted on a pair of particles in the EHD by the phonon wind). From the slope of the straight line (Fig. 3) we obtain $L_{eff} \approx 0.5$ mm. According to (6), at sufficiently high intensity of the phonon wind acting on the EHD, we have

$$L_{\mathrm{eff}} = \tau_0 \frac{\tau_r}{M} f. \qquad (37)$$

It is seen from this formula that L_{eff} has the meaning of the distance traversed by the EHD during the lifetime

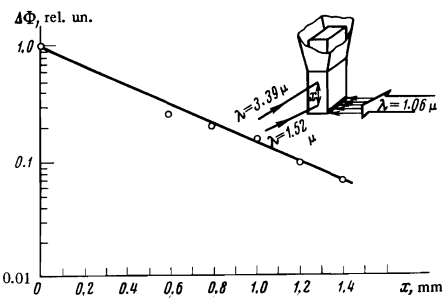

FIG. 3. Dependence of absorption signal $\Delta\Phi$ on the distance x at $P \approx 40$ mW and $T=1.95\,^{\circ}\mathrm{K}$.

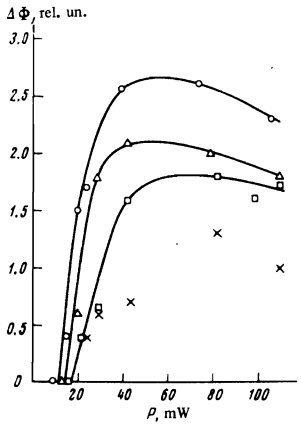

FIG. 4. Plots of the absorption signal $\Delta\Phi$ vs. the power P for different distances x: ○—$x = 0.6$ mm, △—0.8 mm, □—1.0 mm, ×—1.4 mm; $T = 1.95$ °K.

τ_0, since $v = \tau_r f/M$ is the EHD drift velocity. Assuming as an estimate that $\tau_0 = 40$ μsec and $\tau_r \approx 2\times10^{-8}$ sec,[9] we find that $v \approx 1.3\times10^3$ cm/sec and $f \approx 2.5\times10^{-17}$ dyne.

Using (20a), we can now estimate the value of $(\overline{|\mathbf{k}|^4/\omega_{\mathbf{k}}^3})$ for the phonons that interact effectively with the EHD. Assuming that half of the power $P \approx 40$ mW incident on the sample goes to generate the directed flux of nonequilibrium phonons, we obtain the energy flux density $w \approx 0.2$ W/cm^2. Substituting in (20a) $m_h \approx 0.4m_0$ (m_0 is the mass of the free electron) and the numerical values cited above for the remaining quantities, we obtain $(\overline{|\mathbf{k}^4|\omega_{\mathbf{k}}^3}) \sim 6\times10^{-10}$ sec^3/cm^4. This value does not differ greatly from the value 10^{-9} sec^3/cm^4 used above to estimate R_0 and n_m from formulas (29) and (36).

The dependence of the absorption power on the power P at different distances x are shown in Fig. 4. It is seen that the absorption signal appears only when a certain minimal power is reached, and this minimum power increases with increasing distance between the exciting and probing light beams. With further decrease of P, the absorption signal first increases, and then goes through a maximum and starts to decrease. To analyze this behavior we must use expression (5). Integrating (5) with respect to x from zero to ∞, we obtain the total number of carriers bound into drops of the condensed phase; in the stationary case this number is equal to $G\tau_0$ (G is the rate of generation). After determining in this manner the average density of the carriers bound into EHD at the point $x = 0$ and substituting it in (5) we obtain for the distribution of the liquid-phase particles (and hence also of the absorption signal) over the sample volume:

$$N_\Sigma(x) = \frac{GM}{\tau_r f}\exp\left(-\frac{M}{\tau_0\tau_r f}x\right). \qquad (38)$$

Given x, the function $N_\Sigma(x)$ (as a function of f) has a maximum at

$$f_m = Mx/\tau_0\tau_r, \qquad (39)$$

and since in our case $f \sim P$, the last formula yields the position of the maxima of the curves shown in Fig. 4, and shows that when the distance x is increased the maximum should shift towards higher powers, in agreement with the experimental results. Relation (38) has a simple physical meaning: at $f < f_m$ the EHD drift velocity is so low that during the time of motion from $x = 0$ to the observation point the EHD volume decreases strongly as a result of recombination; at $f > f_m$, owing to the appreciable velocity, the EHD spread over the large volume of the sample and this decreases the average density of the liquid-phase particles at the observation point x. The maximum value of the density at the point x is reached when the distance x is negotiated by the drops during the lifetime.

Expression (38) does not describe the cutoff of the absorption signal at small P. We recall, however, that (38) is valid only at a sufficiently high power of the phonon flux, when L_{eff} is determined by (37). At small P it is necessary to use for L_{eff} expression (6), which is valid in the general case, and from which it is seen that so long as the intensity of the phonon wind is insignificant, L_{eff} does not exceed appreciably the dimensions of the generation region. It follows from (38) that at the point $x = 0$ the absorption should decrease in inverse proportion to the power P. However, the power dependence observed by us for the absorption (Fig. 1) is stronger ($\Delta\Phi \sim p^{-1.6}$). It is possible that the decrease of the absorption at $x = 0$ is due not only to the increase of the EHD but also to the influence of the phonon wind on the kinetics of the exciton condensation and the generation of the EHD.

The diffraction measurement results agree qualitatively with the data obtained from the absorption measurements, although their quantitative reduction is difficult.

Reduction of the measurements[6, 11, 13, 14] of the spatial distribution of the EHD with the aid of expressions (10), (23), and (24) has shown that these expressions are in qualitative agreement with experiment. The force acting on a pair of particles in the EHD, estimated from these measurements, agrees within an order of magnitude with the estimate given above.

Measurements made at 4.2 °K, when all the nonequilibrium carriers produced by the exciting 1.52-μ radiation were bound into excitons, have shown that the effect of dragging of the excitons by nonequilibrium phonons is much less than in the case of EHD. A noticeable displacement of the exciton cloud is observed only at powers $P > 100$ mW. However, the sensitivity of the apparatus turned out not to be high enough for quantitative measurements. Assuming that $L_{eff} \approx L_D \approx 1$ mm at $P \approx 100$ mW, we obtain with the aid of (21) the value $(\overline{|\mathbf{k}^4|/\omega_{\mathbf{k}}^3}) \sim 6\times10^{-10}$ sec^3/cm^4, in agreement with the value estimated from the measurements of the EHD dragging.

Thus, a theoretical analysis and the results of experiments that revealed motion of EHD induced by phonon wind over distances exceeding 2 mm shows that the dragging of EHD and excitons by nonequilibrium phonons is an effect essential for the explanation of the

measurements of the spatial EHD distribution and can influence the kinetics of their growth.

We thank G. E. Pikus for a useful discussion and N. V. Znakomets for help with the experiments.

[1)]We note also that elastic scattering, say by impurities or defects, which can appreciably limit the thermal conductivity, do not play a noticeable role in our analysis, since the dragging effect, as seen from (2)–(11), is determined by the phonon frequency distribution and by the total energy flux, the latter being governed by the excitation source.

[2)]It is possible, however, that this exciton density was reached in[27–29] only in a small vicinity of the exciting light spot. In addition, it is possible that shorter-wavelength phonons interact with the excitons near the drops (the phonons move away from the drop that emits them within a time $\sim 10^{-8}$ sec).

[1]L. V. Keldysh, Tr. IX Mezhdunarodnoĭ konferentsii po fizike poluprovodnikov (Proc. 9-th Internat. Conf. on Semiconductor), Nauka, 1968, p. 1387; in: Éksitony v poluprovodnikakh (Excitons in Semiconductors) ed. by B. M. Vul, Nauka, 1971, p. 5.
[2]Ya. Pokrovskii, Phys. Status Solidi [a] 11, 385 (1972).
[3]V. S. Bagaev, Springer Tracts in Modern Phys. 73, 72 (1975).
[4]Ya. E. Pokrovskiĭ and K. I. Svistunova, Fiz. Tverd. Tela 13, 1485 (1971) [Sov. Phys. Solid State 13, 1241 (1971)].
[5]C. Benoit à la Guillaume, M. Voos, and F. Salvan, Phys. Rev. Lett. 27, 1214 (1971).
[6]R. W. Martin, Phys. Status Solidi [b] 61, 223 (1974).
[7]J. C. Hensel and T. G. Phillips, Proc. Twelfth Intern. Conf. on Physics of Semiconductors, Stuttgart, 1974, p. 51.
[8]V. S. Bagaev, N. A. Penin, N. N. Sibel'din, and V. A. Tsvetkov, Fiz. Tverd. Tela, 15, 3269 (1973) [Sov. Phys. Solid State 15, 2179 (1974)].
[9]A. S. Alekseev, T. A. Astemirov, V. S. Bagaev, T. I. Galkina, N. A. Penin, N. N. Sybeldin, and V. A. Tsvetkov, Proc. Twelfth Intern. Conf. on Physics of Semiconductors, Stuttgart, 1974, p. 91.
[10]A. S. Alekseev, V. S. Bagaev, and T. I. Galkina, Zh. Eksp. Teor. Fiz. 63, 1020 (1972) [Sov. Phys.-JETP. 36, 536 (1973)].
[11]Ya. E. Pokrovsky and K. I. Svistuna, Proc. Twelfth Intern. Conf. on Physics of Semiconductors, Stuttgart, 1974, p. 71.
[12]Ya. E. Polrovskiĭ and K. I. Svistunova, Fiz. Tverd. Tela 16, 3399 (1974) [Sov. Phys. Solid State 16, 2202 (1975)].
[13]M. Voos, K. L. Shaklee, and J. M. Worlock, Phys. Rev. Lett. 33, 1161 (1974).
[14]B. J. Feldman, Phys. Rev. Lett. 33, 359 (1974).
[15]O. Christensen and J. M. Hvam, Proc. Twelfth Intern. Conf. on Physics of Semiconductors, Stuttgart, 1974, p. 56.
[16]I. Balslev and J. M. Hvam, Phys. Status Solidi [b] 65, 531 (1974).
[17]V. S. Bagaev, N. V. Zamkovets, L. V. Keldysh, N. N. Sibel'din, and V. A. Tsetkov, Preprint FIAN No. 139 (1975).
[18]V. B. Fuks, Pis'ma Zh. Eksp. Teor. Fiz. 20, 33 (1974) [JETP Lett. 20, 14 (1974)].
[19]V. M. Asnin, A. A. Porachev, and N. I. Sablina, Pis'ma Zh. Eksp. Teor. Fiz. 11, 162 (1970) [JETP Lett. 11, 99 (1970)].
[20]C. Benoit à la Guillaume, M. Voos, F. Salvan, J. Laurant, and A. Bonnot, Compt. Rend. 272, 236B (1971).
[21]J. M. Hvam and O. Christensen, Solid State Commun. 15, 929 (1974).
[22]M. Pomeranz, R. J. von Gutfeld, Tr. IX Mezhdunarodnoĭ konferentsii po fizike poluprovodnikov (Proc. 9-th Internat. Conf. on Semiconductor Physics), Nauka, 1968, p. 732.
[23]R. C. Dynes, V. Narayanamuri, and M. Chin, Phys. Rev. Lett. 26, 181 (1971).
[24]T. P. McLean and R. Loudon, J. Phys. Chem. Solids 13, 1 (1960).
[25]G. L. Bir and G. E. Pikus, Fiz. Tverd. Tela 17, 696 (1975) [Sov. Phys. Solid State 17, 448 (1975)].
[26]R. Peierls, Quantum Theory of Solids, Oxford, 1955 (Russ. Transl., IIL, 1956).
[27]A. S. Alekseev, V. S. Bagaev, T. I. Galkina, O. V. Gogolin, and N. A. Penin, Fiz. Tverd. Tela 12, 3516 (1970) [Sov. Phys. Solid State 12, 2855 (1971)].
[28]T. K. Lo, B. J. Feldman, and C. D. Jeffries, Phys. Rev. Lett. 31, 224 (1973).
[29]G. A. Thomas, T. M. Rice, and J. C. Hensel, Phys. Rev. Lett. 33, 219 (1974).
[30]R. S. Markiewicz, J. P. Wolfe, and C. D. Jeffries, Phys. Rev. Lett. 32, 1357 (1974).
[31]J. Wolfe, R. Markiewicz, C. Kittel, and C. Jeffries, Phys. Rev. Lett. 34, 275 (1975).
[32]J. P. Wolfe, W. L. Hansen, E. E. Haller, R. S. Markiewicz, C. Kettel, and C. D. Jeffries, Phys. Rev. Lett. 34, 1292 (1975).
[33]G. L. Bir and G. E. Pikus, Simmetriya i deformatsionnye effekty v popuprovodnikakh (Symmetry and Deformation Effects in Semiconductors), Nauka, 1972, chapter V.
[34]V. S. Bagaev, N. V. Zamkovets, N. A. Penin, N. N. Sibel'din, and V. A. Tsvetkov, Prib. Tekh. Éksp. No. 2, 242 (1974).
[35]N. N. Sibel'din, V. S. Bagaev, V. A. Tsvetkov, and N. A. Penin, Preprint FIAN, No. 117, 1972.

Translated by J. G. Adashko