the constant external field:

$$d\sigma' = \frac{8Z^2e^6}{m^2}\frac{du}{u(1+u)^2}\left(\frac{\chi}{u}\right)^2\left(\frac{86}{15}+\frac{5u^2}{1+u}\right)\ln\frac{\mu}{u},$$
$$\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2}, \quad u \gg \chi. \tag{19}$$

We note that this result follows also from formula (18), which was obtained for $\xi \ll 1$ in the limit $u \gg \varkappa$, if we put $\xi\varkappa/2 = \chi$. This is explained by the fact that the time $\tau \sim E/m^2u$ of formation of the hard end of the spectrum (see[9]) is small in comparison with the period $1/\omega$ of the external wave.

The authors are deeply grateful to A. A. Sokolov for constant interest in the work.

[1)]We use the metric $(+---)$ and the system of units $\hbar = c = 1$, $\alpha = e^2 = 1/137$.

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Two-photon processes in a Coulomb field in the dipole approximation

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An expression for the amplitude of two-photon processes in a Coulomb field is derived. The expression is an analytic function of not only the photon energy, but also of the quantum numbers of the initial and final electron states. The formulas obtained are used to calculate the cross section for light scattering for both bound-bound and bound-free electron transitions.

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1. INTRODUCTION

In recent years two-photon processes in a Coulomb field have been the subject of a number of papers. Apart from the fact that these processes describe many physical phenomena, they are also of great interest as a model for investigating more complex systems.

The probability for the two-photon decay of the metastable $2s$ level of the hydrogen atom was computed in[1] by the method of approximate numerical summation of series. In[2] an approximate (semiquantitative) formula was derived for coherent light scattering from the ground state of the hydrogen atom. The cross sections for coherent $(1s \to 1s)$[3] and Raman $(1s \to 2s)$[4] light scattering and for the two-photon ionization of the $2s$ level[5] have been computed with the aid of the Schwartz–Tiemann method.

Only comparatively recently were analytic expressions for the amplitude of two-photon transitions between certain low-lying excited states of the hydrogen atom obtained with the aid of one or another integral representation of the Green function for a charged particle in the Coulomb field.[6–8] In 1967 Gavrila[9] expressed the amplitude for coherent light scattering from the $1s$ state in terms of the hypergeometric functions. The same result was independently obtained together with expressions for the two-photon $1s \rightleftarrows 2s$ transition amplitudes by Vetchinkin and Khristenko[10] and Granovskiĭ.[11] Further, Zon, Manakov, and Rappoport[12] have shown that the bound–bound and bound–free transition amplitudes can be expressed in terms of linear combinations of the hypergeometric functions. A similar result was obtained in[13] by Gorshkov and Polikanov, who used the momentum representation of the

Coulomb Green function,[8] which allowed them not to use the dipole approximation. Subsequently, Polikanov[14] expressed the cross section $\sigma_{1s\to n}$ for Raman light scattering in terms of the Appell functions. Analytic expressions and numerical results for the cross sections for the Compton scattering of light and the two-photon ionization of the $1s$ state have been obtained respectively by Gavrila[15] and Klarsfeld.[16] The Green function method has also been used by Maquet[17] to calculate the light-shifts of the hydrogen atom p states with $2 \leqslant n \leqslant 10$.

In the present paper, in contrast to the above-enumerated papers, where the expressions obtained for the amplitude were analytic functions of only the incident-photon frequency, we obtain an expression that is analytic also in the quantum numbers of the initial and final states. This allows us to compute different processes involving transitions between arbitrary states of the hydrogen atom with the aid of a single formula, and not derive for each transition its own formula, as has been suggested in previously proposed methods.[12,13] Furthermore, the obtained formulas are of interest in that they describe the contribution of second-order perturbation theory in the perturbing dipole operator in the Coulomb field. It is shown that in certain cases the dipole approximation is more accurate than might have been expected, estimating the dimensions of the atom in the initial and final states. The asymptotic form of the cross section for two-photon transitions into high-lying excited states is obtained. The cross sections for several processes are computed, using the obtained formulas. In the paper we use a system of units in which $\hbar = c = 1$.

2. THE AMPLITUDE OF TWO-PHOTON PROCESSES

In the nonrelativistic limit the amplitude of a two-photon process involving an electron in an external field is given by the well-known formula:

$$U_{1\to 2}=4\pi\alpha\mu^{-1}\langle 2|\mathbf{A}_2^*\mathbf{A}_1-\mu^{-1}(\mathbf{pA}_2^*)G(E_1\pm\omega_1)(\mathbf{pA}_1) -\mu^{-1}(\mathbf{pA}_1)G(E_1\pm\omega_2)(\mathbf{pA}_2^*)|1\rangle, \tag{1}$$

where $\mathbf{A}=\mathbf{e}(2\omega)^{-1/2}e^{\pm i\mathbf{k}\cdot\mathbf{r}}$ is the potential of the electromagnetic field corresponding to the given photon, μ is the electron mass, and $G(E)$ is the electron Green function in the external field.

If the dimensions of the system are small compared to the wavelengths of the two photons, then we can restrict ourselves in (1) to the dipole approximation. For the Coulomb field, the conditions for the validity of the dipole approximation will be

$$k_i n_j a \ll 1, \quad i,j=1,2. \tag{2}$$

Here n is the principal quantum number and a is the Bohr radius. It follows from (2) that the dipole approximation will be inapplicable in the case of high-lying excited levels, when $n \gg 1$. However, as will be shown below, the conditions (2) can be considerably relaxed.

Let us use the series expansion of the Coulomb Green function in terms of the spherical harmonics

$$G_c(E)=\sum_{l=0}^{\infty} G_l(E),$$

$$\langle \mathbf{r}_2|G_l(E)|\mathbf{r}_1\rangle = g_l(E|r_2,r_1)\sum_{m=-l}^{l} Y_{lm}\left(\frac{\mathbf{r}_2}{r_2}\right)Y_{lm}^*\left(\frac{\mathbf{r}_1}{r_1}\right).$$

To compute (1) in the dipole approximation, it is sufficient to know the following quantities:

$$Q_i(l_2,l,l_1)=\begin{pmatrix} l_2 & 1 & l \\ 0 & 0 & 0\end{pmatrix}^{-1}\begin{pmatrix} l & 1 & l_1 \\ 0 & 0 & 0\end{pmatrix}^{-1} \times\mu^{-1}\langle n_2 l_2 m_2=0|p_z G_l(E_i)p_z|n_1 l_1 m_1=0\rangle. \tag{3}$$

Separating the angle variables in (3), we easily obtain

$$Q_i(l_2,l,l_1)=\frac{-[(l_1+1)(l_2+1)]^{1/2}}{\mu}\int_0^\infty\int_0^\infty (r_1 r_2)^2\,dr_1\,dr_2 \times R_{n_2 l_2}^*(r_2)\hat{P}_{l_2 l}(r_2)g_l(E_i|r_2,r_1)\hat{P}_{l l_1}(r_1)R_{n_1 l_1}(r_1), \tag{4}$$

where

$$\hat{P}_\mu=\begin{cases} d/dr-l/r, & j=l+1 \\ d/dr+(l+1)/r, & j=l-1\end{cases},$$

R_{nl} is the radial wave function of the electron.

Below we shall write the formulas for only the case of bound–bound transitions. To describe the transitions to the continuous spectrum, it is sufficient to make the substitution $n\to i/k$ (k is the electron momentum) and change the normalization factor. Then, using the well-known expressions for R_{nl} in the Coulomb field, we can obtain

$$\hat{P}_{ji}(r)R_{nl}=\frac{2}{n^3(2j+1)!}\left[\frac{\Gamma(n+l+1)}{\Gamma(n-l)}\right]^{1/2}\left(\frac{2r}{n}\right)^{j-1}e^{-r/n} \times\sum_{p=\pm 1}C_{nljp}\,{}_1F_1\left(-n+j+1+p,2j+2,\frac{2r}{n}\right),$$

$$C_{nljp}=\begin{cases}1, & j=l-1 \\ (np-l-1)(np-l-2), & j=l+1\end{cases}. \tag{5}$$

It can be seen from (4) and (5) that the reduced matrix elements $Q(l_2,l,l_1)$ are each expressible as a linear combination of integrals of the type

$$M_i(l,p,q)=4^i\mu^{-1}(n_1 n_2)^{-l-2}[(2l+1)!]^{-2} \times\int_0^\infty\int dr_1\,dr_2(r_1 r_2)^{l+1}\exp\left(-\frac{r_1}{n_1}-\frac{r_2}{n_2}\right)g_l(E_i|r_2,r_1) \times{}_1F_1\left(l+p+1-n_1,2l+2,\frac{2r_1}{n_1}\right){}_1F_1\left(l+q+1-n_2,2l+2,\frac{2r_2}{n_2}\right), \tag{6}$$

where $l=l_1\pm 1$ is the "orbital angular momentum" of the electron in the virtual state; $p,q=\pm 1$; $i=1,2$. Substituting (5) and (6) into (4), we find:

$$Q_i(l_2,l,l_1)=\left[\frac{\Gamma(n_1+l_1+1)\Gamma(n_2+l_2+1)(l_1+1)(l_2+1)}{\Gamma(n_1-l_1)\Gamma(n_2-l_2)}\right]^{1/2} \times\sum_{p,q=\pm 1}pqC_{n_1 l_1 l p}C_{n_2 l_2 l q}M_i(l,p,q). \tag{7}$$

Thus, the computation of the amplitude (1) in the di-

pole approximation has been reduced to the evaluation of sixteen integrals of the type (6). Actually, their number can be considerably less, since nonzero contributions are made by only those for which $l \geq 0$, $n_1 - p - l \geq 1$, and $n_2 - q - l \geq 1$. If the initial or final state of the electron belongs to the discrete spectrum, then the double integral in (6) can be reduced to a single integral. For this purpose, let us, following Granovskiĭ,[11] use the convenient integral representation of $g_l(E|r_2, r_1)$ obtained by Hostler[7]:

$$g_l(E|r_2,r_1)=(-1)^{l+1}\frac{2\mu i}{(r_1r_2)^{1/2}}\int_1^\infty \frac{dt}{(t^2-1)^{1/2}}\left(\frac{t+1}{t-1}\right)^{\nu} \times\exp\{ikt(r_1+r_2)\}J_{2l+1}(2k[r_1r_2(t^2-1)]^{1/2}). \tag{8}$$

Here $k=(2\mu E)^{1/2}$, $\nu=-i\mu\alpha Z/k$, and J_n is a Bessel function. Then the integral over r_1 in (6) reduces to a table integral (see, for example, [18]), and has the following form:

$$\int_0^\infty dr'\, g_l(E|r,r')e^{-r'/n}(r')^{l+1}\,{}_1F_1\left(-p,2l+2,\frac{2r'}{n}\right) =(-1)^p 2\mu\nu r^l\int_1^\infty dt\left(\frac{t+1}{t-1}\right)^{\nu}(t^2-1)^l\left(\frac{\nu}{n}+t\right)^{-2l-2}\left(\frac{nt-\nu}{nt+\nu}\right)^p \times\exp\left(-\frac{r}{n}\frac{t+n/\nu}{t+\nu/n}\right){}_1F_1\left(-p,2l+2,\frac{2rn(t^2-1)}{n^2t^2-\nu^2}\right). \tag{9}$$

If we do not assume that the condition (2) for the applicability of the dipole approximation is satisfied in (1) for n_2, then we should consider the forbidden transitions as well. It can, however, be seen from (9) that the smallness of the matrix elements of the transitions forbidden in the dipole approximation is determined not by the ratio of the dimension of the system in the state $|2\rangle$ to the photon wavelength, but by the quantity

$$\delta=k_1a\left[\min_{1\leqslant l<\infty}\left|\frac{1}{n_2}+\frac{1}{n_1}\frac{t+n_1/\nu}{t+\nu/n_1}\right|\right]^{-1}.$$

Taking into account the fact that the condition (2) is assumed to be fulfilled for n_1, we can easily see that the smallness of δ is equivalent to the inequalities

$$|1/n_2+1/\nu|^{-1}ak_1\ll 1, \tag{10}$$

which replace the condition (2) for n_2. Physically, this is explainable by the fact that the extent to which the electron is smeared out in the virtual state is determined, on the one hand, by the dimensions of the system in the initial state (i.e., in the n_1 state) and, on the other, by ν, the "principal quantum number" of the electron in the intermediate state.

For dipole transitions the integral over r_2 in (6) also reduces to a table integral if we use (9), and, after making a change of integration variable, we obtain

$$M(l,p,q)=\frac{2^{4l+2}}{(2l+1)!}\nu^{2l+3}(n_1n_2)^l[(\nu-n_1)(n_2-\nu)]^{-2l-2} \times\int_0^1 dx\, x^{l-\nu}\left(\alpha\frac{v-x}{u-x}\right)^{m_1}\left(\beta\frac{v^{-1}-x}{u-x}\right)^{m_2}(u-x)^{-2l-2} \times{}_2F_1\left(-m_1,-m_2,2l+2,\frac{u_1x}{(v-x)(v^{-1}-x)}\right), \tag{11}$$

where we have introduced the following notation:

$$\alpha=\frac{\nu+n_1}{\nu-n_1},\quad \beta=\frac{n_2+\nu}{n_2-\nu},\quad v=-\frac{\alpha}{\beta},\quad u=-\alpha\beta,$$
$$u_1=u+u^{-1}-v-v^{-1},\quad m_1=n_1-l-p-1,\quad m_2=n_2-l-q-1.$$

The hypergeometric function in the integrand of (11) is essentially a polynomial, since either $-n_1+l+p$ or $-n_2+l+q$ is a negative integer. For integral $\nu \geq l$ the function $M(l,p,q)$ has, as was to be expected, simple poles. If one of the states (for definiteness, the $|2\rangle$ state) lies in the continuous spectrum, then the integrand has a bifurcation, and we should choose that branch that remains finite for $n_2=i/k\to i\infty$.

3. COHERENT AND RAMAN LIGHT SCATTERING CROSS SECTIONS

In the case of bound–bound transitions the light-scattering cross section, summed over the final, and averaged over the initial, magnetic quantum numbers of the electron, has the following form:

$$d\sigma=r_0^2\frac{\omega_2}{\omega_1}\left\{\delta_{n_1n_2}\delta_{l_1l_2}(\mathbf{e}_1\mathbf{e}_2)^2\left[1-\sum_{l=l_1\pm1}\frac{2\,\mathrm{Re}\,A_{0l}}{(6l_1+3)^{1/2}}\right] +\frac{1}{2l_1+1}\sum_{l,l'=l_1\pm1}\sum_{j=0}^{2}A_{jl}A_{jl'}^{*}C_j(\mathbf{e}_1\mathbf{e}_2)\right\}d^2\Omega_{k_2}, \tag{12}$$

where

$$A_{jl}=\begin{Bmatrix} l_2 & l_1 & j\\ 1 & 1 & l\end{Bmatrix}[Q_1(l_2,l,l_1)+(-1)^jQ_2(l_2,l,l_1)],$$
$$C_0=1/3(\mathbf{e}_1,\mathbf{e}_2)^2,\quad C_1=1/6[1-(\mathbf{e}_1,\mathbf{e}_2)^2],\quad C_2=1/30[3+(\mathbf{e}_1,\mathbf{e}_2)^2],$$

r_0 is the classical electron radius. It follows from (12) that if $l_2=l_1\pm2$, then the angular distribution of the outgoing photon does not depend on its frequency. This is connected with the fact that in the intermediate state the electron may have only one, totally defined angular momentum $l=l_1\pm1$, and then under the summation sign in (12) will remain only the term with $j=2$.

A characteristic feature of the behavior of the cross section (12) is the presence of poles at photon energies corresponding to the poles of the electron Green function. These poles have a point of accumulation at $\omega=\omega_0$, where ω_0 is the photoelectric threshold frequency for the initial state. At certain frequency values lying between the poles and lower than ω_0, the scattering cross section vanishes. If the initial state is not the ground state, then the cross section may have poles also at $\omega>\omega_0$, but in this case it will no longer vanish between the poles. The vanishing of the cross section is connected with the reality of the amplitude for $\omega<\omega_0$, and will not occur if we take the level widths into account. In this case the poles of the cross section will also disappear. In the case of coherent scattering the cross section tends to the Thomson limit as $\omega\to\infty$. If, on the other hand, the initial and final states of the electron do not coincide, then the cross section falls off like ω^{-4}. To illustrate the behavior of the cross section, we present the plot (Fig. 1) of the cross section for coherent scattering of light from the 3s level as a function of the incident-photon energy. In con-

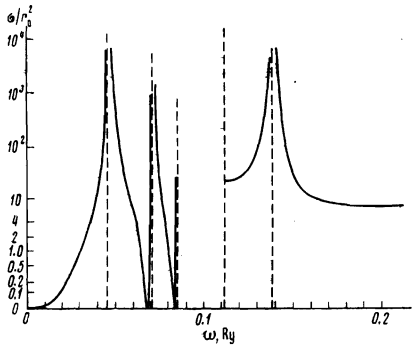

FIG. 1. Coherent scattering of light from the 3s level. The photoelectric threshold frequency ω_0 in this case is equal to $\frac{1}{9}$ Ry.

trast to the previously published calculations for the $1s \to 1s$ and $1s \to 2s$ transitions,[12] in the present case the cross section exhibits a pole at $\omega > \omega_0$.

Let us further consider the asymptotic forms of the amplitude and the cross section for $n_2 \to \infty$. Let $n_1 \sim |\nu| \ll n_2$. Then we can go over in (11) to the asymptotic form directly under the integration sign. Then, since the dominant term in the asymptotic form of $M(l,p,q)$ does not depend on q (and is therefore partially canceled out in the expression for Q), we should take into account terms of higher order in q/n_2, whose contributions to (7) will be of the same order of magnitude. Thus, for large n_2 we should substitute for $M(l,p,q)$ in (7) the following quantity:

$$\bar{M}(l,p,q)=\frac{2^{4l+2}}{(2l+1)!}\nu^{2l+3}\frac{1}{n_2^{l+2}}\frac{n_1^{l}}{(\nu-n_1)^{2l+2}}$$
$$\times\int_0^\infty dx\, x^{l-\nu}\frac{(1-\alpha x)^{m_1}}{(\alpha+x)^{n_1+l-p+1}}\exp\left(\frac{2\nu x}{\alpha+x}\right) \qquad (13)$$
$$\times\left[\left(1+\frac{q}{n_2}\frac{2\nu x}{\alpha+x}\right){}_1F_1(-m_1,2l+2,z)+\frac{q}{n_2}{}_1F_1(1-m_1,2l+3,z)\right],$$

where

$$z=4\nu(\alpha-\alpha^{-1})x/(\alpha+x)(\alpha^{-1}+x).$$

Substituting (13) into (7), we see that the cross section

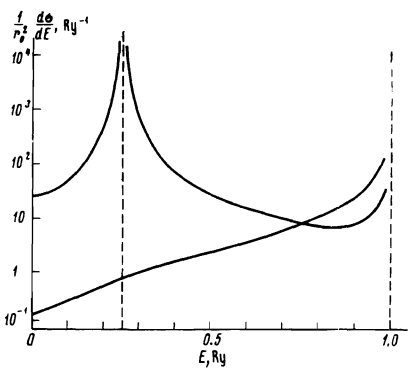

FIG. 2. Cross section for light scattering with electron ionization from the 2s and 2p states. The energy of the incident photon is equal to 1.25 Ry. The upper curve corresponds to the detachment of an electron from the 2p level; the lower curve, to electron detachment from the 2s level.

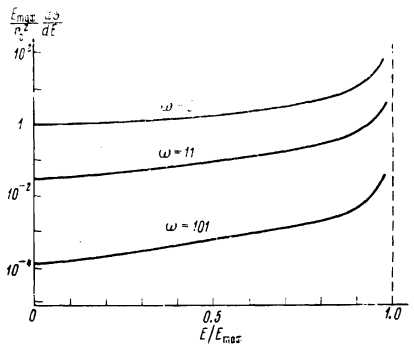

FIG. 3. Cross section for light scattering with electron ionization from the ground state for three values of the incident-photon frequency $\omega = 2$, 11, and 101 Ry, to which correspond the following values of the electron energy in the photoelectric effect: $E_{max} = 1$, 10, and 100 Ry.

is proportional to n_2^{-3}, which is in agreement with Polikanov's result.[14]

4. THE CROSS SECTION FOR LIGHT SCATTERING WITH IONIZATION OF THE HYDROGEN ATOM

When the final state of the electron belongs to the continuous spectrum, we can, as before, use the formula (7) for the quantity Q if we make the substitution $n_2 \to i/k$ and introduce the additional factor

$$k^{-1}[2\pi i/(1-e^{-2\pi/k})]^{1/2}.$$

Here we assume that the radial wave function is real and normalized "according to the scale $k/2\pi$." Then, averaging over the initial magnetic quantum numbers, we can represent the cross section for light scattering with ionization in the following form:

$$d\sigma=\frac{\omega_2}{\omega_1}\frac{r_0^2}{2l_1+1}\frac{d^2\Omega_{\mathbf{k}}\,d^3\mathbf{k}}{\sqrt{\pi}\,k^2}\sum_{l_2,l_2'}\sum_{l,l'}\sum_{j,j'}\sum_{J}\sum_{j_2j_3j_3'}(-1)^{l_1}(2j+1)(2j'+1)(2J+1) \qquad (14)$$
$$\times\begin{Bmatrix} j_3 & j & j' \\ 1 & 1 & J\end{Bmatrix}\begin{Bmatrix} j_3 & j_2 & j_1 \\ 1 & 1 & J\end{Bmatrix}\begin{Bmatrix} j_3 & j & j' \\ l_1 & l_2 & l_2'\end{Bmatrix}B_{l_2lj}B^{*}_{l_2'l'j'}P^{l_2l_2'}_{j_2j_3j_3'}(\mathbf{e}_1,\mathbf{e}_2,\mathbf{n}).$$

Here

$$B_{l_2lj}=\exp(i\delta_{l_2})\begin{Bmatrix} l_2 & l_1 & j \\ 1 & 1 & l\end{Bmatrix}[Q_1(l_2,l,l_1)+(-1)^jQ_2(l_2,l,l_1)],$$

$$P^{l_2l_2'}_{j_1j_2j_3}(\mathbf{e}_1,\mathbf{e}_2,\mathbf{n})=i^{j_1+j_2+j_3}\begin{pmatrix} l_2 & l_2' & j_3 \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & 1 & j_2 \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & 1 & j_1 \\ 0 & 0 & 0\end{pmatrix}$$
$$\times[(2l_2+1)(2l_2'+1)(2j_1+1)(2j_2+1)(2j_3+1)]^{1/2}\cdot$$
$$\times\sum_{m_1m_2m_3}\begin{pmatrix} j_3 & j_2 & j_1 \\ m_3 & m_2 & m_1\end{pmatrix}Y_{j_1m_1}(\mathbf{e}_1)Y_{j_2m_2}(\mathbf{e}_2)Y_{j_3m_3}(\mathbf{n}),$$

$\mathbf{n}=\mathbf{k}/k$, and $\delta_l=\arg\Gamma(l+1-i/k)$ is the Coulomb phase of the scattering.

For a fixed ω_1 the cross section as a function of the energy of the outgoing electron has a pole at $E=E_{max}$, where E_{max} is the energy that the electron would have in a photoelectric-effect process with the same initial conditions. Such a behavior of the cross section is a direct consequence of the infrared catastrophe, and does not by itself make sense. If the initial electron state is an excited state, so that dipole transitions to less excited levels are allowed, then the cross section will

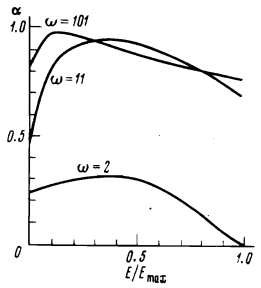

FIG. 4. The angular distribution of the outgoing photon for the same processes to which the curves in Fig. 3 correspond. Along the ordinate axis is plotted the quantity α, which is determined from the expression $d\sigma/dEd^2\Omega_{k_2} \sim 1-\alpha(\mathbf{e}_1\mathbf{e}_2)^2$ (E is the electron energy).

have poles at electron energies corresponding to the photoelectric effect from these less excited levels. These poles will naturally disappear if we take the level widths into account.

Figure 2 shows the plots of the dependence of the cross section for light scattering with electron ionization from the $2s$ and $2p$ states on the energy of the outgoing electron. As can be seen from this figure, the cross section has a pole only when the ionization is from the $2p$ level.

Figures 3 and 4 show the cross sections for electron emission and the angular distributions of the outgoing photon for three values of the incident-photon frequency; initially, the electron is in the ground state. At constant ω_2 the cross section monotonically decreases with increasing ω_1. If, on the other hand, we fix the outgoing-electron energy E, then the monotonic decrease of the cross section may not occur, since E may approach a pole singularity of the cross section as the incident-photon frequency increases.

The asymptotic form, (13), of the amplitude is also applicable in the case when $n_2 \to \infty$. From it we easily obtain that $d\sigma/dE \to$ const as $E \to 0$.

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Passage of low-energy particles through a nonstationary potential barrier and the quasi-energy spectrum

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The passage of a particle through a narrow potential barrier with a periodically varying depth is investigated. A set of wave functions with a definite quasi-energy are constructed and the concept of scattering eigenphases and eigenamplitudes is used. It is shown that the quasi-energy spectrum is a continuous spectrum with an infinite degree of degeneracy. Examples are presented of cases when there exist a discrete (nondecaying) quasi-energy state superimposed on the continuous-spectrum background. The effect of total reflection of particles from a nonstationary potential barrier is discovered and found to be of a resonance nature.

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1. INTRODUCTION. FORMULATION OF THE PROBLEM

The interest in the problem of the passage of a quantum particle through a potential barrier with a periodically varying depth is explained by its connection with the theory of the interaction of laser radiation with matter—in particular, with the theory of many-photon ionization (see, for example, the monograph by Baz', Zel'dovich, and Perelmov[1] and the references