Resonance ionization of atoms

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The probabilities for direct and resonance ionizations of atoms are found and the time dependence of these processes is investigated. It is shown that a constant (time-independent) probability for resonance ionization per unit time does not, in general, exist, but arises in only one of the two limiting cases: $\Gamma_f \gg \Gamma_i$ and $\Gamma_f \ll \Gamma_i$, where Γ_f and Γ_i are the field and ionization widths of the resonance level. The width of the resonance curve is then determined by Γ_f or Γ_i.

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1. INTRODUCTION

One of the most important problems in the theory of resonance ionization of atoms is the determination of the resonance width. In the case of weak fields, this quantity coincides with the intrinsic (radiative) width Γ_r of the resonance level. However, in experiments on the many-photon ionization of atoms the electromagnetic-wave intensity is, as a rule, so high that the radiative width Γ_r can be neglected.[1] Under these conditions the resonance width may be determined either by the probability for the ionization of the resonance state[2] (the ionization width Γ_i), or by the shift in the resonance field of the quasi-energy levels, which is proportional to the composite resonance-transition matrix element[3] (the field width Γ_f).

Kotova and Terent'ev[3] have proposed for the probability of resonance ionization of an atom per unit time a general, Breit–Wigner resonance type of formula with a total width $\Gamma = \Gamma_r + \Gamma_i + \Gamma_f$. In the case when $\Gamma_i \sim \Gamma_f$ this result is not quite convincing for the simple reason that the quantities Γ_i and Γ_f are of essentially different nature: Γ_i is the natural width and Γ_f is due to the level shifts. In Krainov's paper[4] the quantity $\bar{\Gamma} = [\Gamma_r^2 + \Gamma_i^2 + \Gamma_f^2]^{1/2}$ stands in place of the total width in the resonance formulas. This ambiguity is connected with the fact that in[2–4] the ionization width Γ_i is not a result of a consistent theory, but is extraneously introduced at one stage or another.

The correct description of the resonance ionization process requires a more exact allowance for transitions between the discrete levels and the continuous-spectrum states than is afforded by ordinary perturbation theory. As will be shown below, this question is intimately connected with the problem of the description of the time dependence of the ionization process. In the present paper these problems are solved with the aid of the well-known Fano method[5] of describing discrete levels in a continuum background. In Sec. 2, as the simplest example illustrating the potentialities of this method, we consider the process of direct (nonresonance) ionization of an atom. The obtained formulas generalize the perturbation-theory results to the region of the nonlinear (in time) regime, and describe the electron distribution over energy at the limit of 100% ionization of the atoms during the pulse time. Sections 3 and 4 are devoted to the consideration of the resonance-ionization problem. The results of the investigation show that simple resonance formulas for the ionization probability per unit time are obtainable only in the limiting cases when $\Gamma_i \ll \Gamma_f$ or $\Gamma_i \gg \Gamma_f$. In the general case the resonance-ionization picture can be considerably more complex and, in particular, the concept of a constant ionization probability per unit time is not always correct.

2. THE PROCESS OF DIRECT IONIZATION OF AN ATOM

The possibility of the application of the Fano method[5] to the processes of ionization of atoms is connected with the well-known circumstance[6] that in an external periodic field (of frequency ω) each level of an atomic system is converted into a system of quasi-energy levels with a spacing of $k\omega$ (k is a whole number). This evidently leads to the appearance of discrete quasi-energy levels on a continuum background.

Let us first assume that the frequency ω of the field is higher than the ionization potential of the atom. Let us take into account multiple transitions from the ground state ψ_0 into the continuum and back. We shall assume that the ground-state energy E_0 contains corrections determined by the nonresonance part of the dynamical Stark effect. The resonance part of the interaction needs to be allowed for more exactly, since it is precisely this part that determines the ionization probability (i.e., leads to the width Γ_i). We shall neglect all the other nonresonance transitions, as well as all transitions between states of the continuous spectrum. These simplifications are justified at field intensities $F \ll F_0$, where $F_0 = 5 \times 10^9$ V/cm is the intratomic field intensity. The validity of this assertion with respect to transitions between states of the continuous spectrum is, perhaps, not quite obvious and requires some elucidation. Although, formally, the resonance conditions can be fulfilled in virtual transitions between states of the continuum, in reality, these resonances are eliminated upon integration over the entire continuous spectrum. It can be asserted that in the case of resonance in the continuum the effective width of the resonance turns out to be very large (of the order of one Ry). Therefore, the corrections to the fundamental transition from the ψ_0 into the continuum that are due to transitions between states of the continuous spectrum are small ($\sim F/F_0$) when $F \ll F_0$.

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Our approximation, which takes only multiple transitions from the ground state ψ_0 into the continuum and back into account, is equivalent to an allowance for the dominant secular terms leading to the renormalization of the energy. The solution of the corresponding equations is equivalent to the summation of the reducible self-energy diagrams in the lowest order in the parameter $(F/F_0)^2$.

In the framework of the adopted approximation we can use the following expansion of the atomic wave function $\Psi(t)$ in terms of the unperturbed atomic functions:

$$\Psi(t)=c_0(t)e^{-iE_0t}\psi_0-\int dE\,c_E(t)e^{-iEt}\psi_E. \tag{1}$$

Here ψ_E is the wave function of the continuous spectrum (with energy E) dipole-coupled to the state ψ_0 by the interaction operator $\mathbf{df}/2$, where $\mathbf{d}$ is the dipole moment of the atom and $\mathbf{f}$ is the amplitude of the electric-field intensity in the wave $\mathbf{F}(t)=\mathrm{Re}(\mathbf{f}e^{i\omega t})$.[1)] We use a system of units in which $\hbar=1$.

The substitution of the expansion (1) into the Schrödinger equation leads to a system of equations for the coefficients $c_0(t)$ and $c_E(t)$. With the aid of the Fourier transformation

$$c_0(t)=\int dE\,a(E+E_0+\omega)e^{-iEt},\quad c_{E'}(t)=\int dE\,b_{E'}(E-E')e^{-iEt} \tag{2}$$

this system can be reduced to a system of algebraic equations for the functions $a(E)$ and $b_{E'}(E)$:

$$(E-E_0-\omega)a(E)=\int dE'W_{E'}^{*}b_{E'}(E),\quad (E-E')b_{E'}(E)=W_{E'}a(E), \tag{3}$$

where

$$W_E=-{}^1/_2\langle\psi_E|\mathbf{df}|\psi_0\rangle. \tag{4}$$

This system of equations describes the interaction of a discrete level with energy $E_0+\omega$ with the continuum through the stationary potential $\mathbf{df}/2$. The solution to such a problem is well known,[5] so that we can write down at once the normalized (to the δ-function) quasienergy wave functions $\Psi_E(t)$:

$$\Psi_E(t)=\exp[-i(E+E_0)t]\Phi_E(t),$$

$$\Phi_E(t)=\left(\frac{2\pi}{\Gamma(E)[z^2(E)+\pi^2]}\right)^{1/2}\left\{\psi_0 +e^{-i\omega t}\int dE'\left[\frac{P}{E+E_0+\omega-E'}+z(E)\delta(E+E_0+\omega-E')\right]W_{E'}\psi_{E'}\right\}, \tag{5}$$

where the symbol P denotes the principal value,

$$z(E)=2\pi[E-\Delta(E)]/\Gamma(E),$$

$$\Delta(E)+\frac{i}{2}\Gamma(E)=\int dE'\frac{|W_{E'}|^2}{E+E_0+\omega-E'+i\delta},\quad \delta\to+0. \tag{6}$$

To solve the problem of the ionization of an atom, it is necessary to specify the nature of the interaction switching-on process. We shall assume that the field is switched on instantly at $t=0$ and that the atom is in the ground state ψ_0 at the moment the interaction is switched on. The solution of the initial-value problem with the aid of (5)–(6) entails an expansion of the function ψ_0 in terms of the system of functions Φ_E, which is possible if this system is a complete system. The question of the completeness of the system of solutions to the Eqs. (3) has not, as far as we know, been investigated in the literature. This question can easily be analyzed if we neglect the slow dependence of the functions $\Delta(E)$ and $\Gamma(E)$ on the energy E and assume that $\Delta(E)\approx\Delta(0)\equiv\Delta$ and $\Gamma(E)\approx\Gamma(0)\equiv\Gamma_i$, which we shall henceforth assume. In this case the system of functions Φ_E is complete in the same sense in which the system of functions $\{\psi_0,\psi_E\}$ is complete:

$$\int dE|\Phi_E\rangle\langle\Phi_E|=|\psi_0\rangle\langle\psi_0|+\int dE|\psi_E\rangle\langle\psi_E|. \tag{7}$$

Notice that the dependence of the functions $\Delta(E)$ and $\Gamma(E)$ on E is indeed a slow dependence, since the characteristic interval ($\sim$Ry) in which $\Delta(E)$ and $\Gamma(E)$ vary is wide in comparison with their magnitudes ($\Delta,\Gamma_i\ll$Ry). Therefore, allowance for the dependences $\Delta(E)$ and $\Gamma(E)$ could introduce corrections $\sim(\Delta/\mathrm{Ry})^2\sim(\Gamma_i/\mathrm{Ry})^2\sim(F/F_0)^4$, which, according to the adopted approximation, is not justified. Consequently, we should increase the accuracy of the calculations if we exceeded the limits of the conditions $\Delta(E)=\mathrm{const}$ and $\Gamma(E)=\mathrm{const}$.

With allowance for the remarks made above, the amplitude $A(E,t)$ of the probability for the removal of an electron of energy E by the time t can be represented in the form

$$A(E,t)=\int dE'\exp[i(E-E'-E_0)t]\langle\psi_E|\Phi_{E'}(t)\rangle\langle\Phi_{E'}(0)|\psi_0\rangle, \tag{8}$$

which, after the evaluation of the integral, yields

$$A(E,t)=W_E\frac{\exp[i(E-E_0-\Delta-\omega+i\Gamma_i/2)t]-1}{E_0+\Delta+\omega-E-i\Gamma_i/2}. \tag{9}$$

It can be seen from this expression that the quantities Δ and Γ_i can be interpreted as the ground-level shift and width arising as a result of the ionization of the atom.

The total probability $W(t)$ for the ionization of the atom by the time t has the form

$$W(t)=\int dE|A(E,t)|^2=1-e^{-\Gamma_i t}. \tag{10}$$

The quantity Γ_i^{-1} determines the characteristic decay time. The assumption used above that the interaction is switched on instantaneously imposes a limitation on the time Δt it should take to switch the field on: $\Gamma_i\Delta_t\ll1$.

The formulas (8)–(10) for small times $\Delta t\ll t\ll\Gamma_i^{-1}$ coincide with the perturbation-theory results, while for $\Gamma_i t\gg1$ they describe the transition to 100% ionization of the atom, in accord with the general theoretical ideas.[7] The asymptotic electron distribution over energy in the limit of large $t\gg\Gamma_i^{-1}$ is determined by the simple resonance formula

$$\left.\frac{dW(E,t)}{dE}\right|_{t\to\infty}=\frac{\Gamma_i/2\pi}{(E_0+\omega+\Delta-E)^2+\Gamma_i^2/4}. \tag{11}$$

In the case of an n-photon ionization of the atom the Schrödinger equation should be iterated right up to the n-th order, after which the above-formulated approxi-

mations can be used (in particular, transitions between states of the continuous spectrum can be neglected at this stage). As a result, the matrix element W_E in the formulas (3), (5), (6), and (9) is replaced by an n-th order composite matrix element and the frequency ω is replaced by $n\omega$. Then the quantity $\Gamma_i \sim \mathrm{Ry}(F/F_0)^{2n}$, and by the energy E_0 should be understood the ground-state energy of the atom with allowance for right up to $2n$-th order corrections determined by the nonresonance part of the dynamical Stark effect.

3. THE RESONANCE IONIZATION

Proceeding to the description of resonance ionization, we shall first consider the most simplified case. We shall assume that resonance between the ground, ψ_0, and some excited, ψ_1, states is realized as a result of the absorption of one quantum ω' of the electromagnetic wave of electric-field intensity $\mathbf{F}' = \mathrm{Re}(\mathbf{f}' e^{i\omega' t})$. Further, let the interval from the resonance level E_1 to the continuum be larger than ω', and let the ionization of the resonance state be effected by another wave of intensity $\mathbf{F} = \mathrm{Re}(\mathbf{f} e^{i\omega t})$, the magnitude of the quantum ω being less than the ionization potential of the atom. We shall, finally, assume that the two levels E_0 and E_1 are nondegenerate.[2)]

In problems of the behavior of quantum systems in intense resonance fields,[10,11] it is convenient to take the resonance interaction into account exactly even in the initial basis, i.e., to use the quasi-energy wave functions[12]

$$\Psi_{0,1}(t) = \exp(-i\bar{E}_{0,1}t)\,\Phi_{0,1}(t),$$

$$\Phi_{0,1}(t) = \frac{1}{\sqrt{2}}\left[\left(1 \pm \frac{\varepsilon}{\Omega}\right)^{1/2} e^{i\omega' t}\psi_0 \mp \eta\left(1 \mp \frac{\varepsilon}{\Omega}\right)^{1/2}\psi_1\right]. \qquad (12)$$

corresponding to the quasi-energy values

$$\bar{E}_{0,1} = {}^1/_2(E_0 + E_1 + \omega' \mp \Omega). \qquad (13)$$

Here $\varepsilon = E_1 - E_0 - \omega'$ is the resonance detuning, $\Omega = (\varepsilon^2 + \Gamma_f^2/4)^{1/2}$, $\Gamma_f = 4|W'|$, $\eta = W'/|W'|$, and the matrix element

$$W' = -{}^1/_2\langle\psi_1|\mathbf{df}'|\psi_0\rangle. \qquad (14)$$

Let us now proceed to the consideration of the interaction with the field $\mathbf{F}(t)$. We shall take the multiple transitions between the states ψ_0, ψ_1 and the continuum into account, and shall assume that E_0 and E_1 include corrections due to the nonresonance part of the dynamical Stark effect. All the remaining nonresonance transitions, as well as the transitions between the states of the continuum, can be neglected. The applicability of these approximations can be justified by the same arguments used in the case of direct ionization.[3)] Let us again separate out in the continuous spectrum a unique linear combination, ψ_E, of wave functions (with energy E) that is dipole-coupled to the resonance level E_1 by the interaction $-\frac{1}{2}\mathbf{df}$. The wave function of the atom can be represented in the form of the expansion

$$\Psi(t) = c_0(t)\exp(-i\bar{E}_0 t)\,\Phi_0(t) + c_1(t)\exp(-i\bar{E}_1 t)\,\Phi_1(t) + \int dE\, c_E(t)\exp(-iEt)\,\psi_E. \qquad (15)$$

The equations for the coefficients $c_{0,1}$ and c_E lead, as a result of the Fourier transformation

$$c_{0,1}(t) = \int dE\, a_{0,1}(E + \bar{E}_{0,1} + \omega)\, e^{-iEt},$$
$$c_{E'}(t) = \int dE\, b_{E'}(E + E')\, e^{-iEt}, \qquad (16)$$

to a system of algebraic equations for $a_{0,1}(E)$ and $b_{E'}(E)$:

$$(E - \bar{E}_{0,1} - \omega)\, a_{0,1}(E) = \mp\frac{\eta}{\sqrt{2}}\left(1 \mp \frac{\varepsilon}{\Omega}\right)^{1/2}\int W_{E'} b_{E'}(E)\, dE'. \qquad (17)$$

$$(E - E')\, b_{E'}(E) = \frac{\eta}{\sqrt{2}} W_{E'}\left[-\left(1 - \frac{\varepsilon}{\Omega}\right)^{1/2} a_0(E) + \left(1 + \frac{\varepsilon}{\Omega}\right)^{1/2} a_1(E)\right] \qquad (18)$$

where the matrix element W_E is determined by the formula (4) with ψ_0 replaced by ψ_1.

Thus, the problem reduces to the problem, considered by Fano,[5] of two discrete levels $\bar{E}_0 + \omega$ and $\bar{E}_1 + \omega$ on a continuum background. Using the results of[5] and the formulas (15)–(18), we can easily verify that the final expression for the quasi-energy wave functions $\Psi_E(t)$, normalized to the δ-function, has the form

$$\Psi_E(t) = \exp[-i(E - \bar{E})t]\,\Phi_E(t),$$

$$\Phi_E(t) = \sqrt{\frac{\pi}{\Gamma(E)}}\frac{1}{[z^2(E) + \pi^2]}\left\{\frac{E - \Omega/2}{E - \varepsilon/2}\left(1 - \frac{\varepsilon}{\Omega}\right)^{1/2}\Phi_0(t) - \frac{E + \Omega/2}{E + \varepsilon/2}\left(1 + \frac{\varepsilon}{\Omega}\right)^{1/2}\Phi_1(t) - \sqrt{2}\,\eta e^{-i\omega' t}\int dE'\left[\frac{\mathrm{P}}{E + \bar{E} + \omega - E'} + z(E)\,\delta(E + \bar{E} + \omega - E')\right] W_{E'}\psi_{E'}\right\}. \qquad (19)$$

where

$$z(E) = \frac{2\pi}{\Gamma(E)}\left[\frac{E^2 - \Omega^2/4}{E + \varepsilon/2} - \Delta(E)\right]. \qquad (20)$$

The functions $\Delta(E)$ and $\Gamma(E)$ are determined, as before, from the formula (6), in which we only need to replace E_0 by $\bar{E} = (\bar{E}_0 + \bar{E}_1)/2$. Under the condition that $\Delta(E) \approx \Delta(0) \equiv \Delta$ and $\Gamma(E) \approx \Gamma(0) \equiv \Gamma_i$ (which is fulfilled when $F \ll F_0$), the system of functions (19) in the basis $\{\psi_0, \psi_1, \psi_E\}$ is a complete system.

We shall assume, as before, that the interaction is switched on instantaneously at the moment of time $t = 0$ and that the initial state of the atom is ψ_0. In the present case this means that the switching-on time $\Delta t \ll \min\{\Gamma_i^{-1}, \Gamma_f^{-1}\}$. From the formulas (8), (12), and (19) can be determined the amplitude $A(E, t)$ of the probability for the removal of an electron of energy E by the moment of time t:

$$A(E, t) = \frac{W' W_E}{E_a - E_b}\left(\frac{\exp[-i(E_a - E)t] - 1}{E_a - E} - \frac{\exp[-i(E_b - E)t] - 1}{E_b - E}\right). \qquad (21)$$

The poles E_a and E_b of the amplitude $A(E, t)$ lie in the lower half-plane of the complex variable E, and are determined by the expressions[4)]

$$E_{a,b} = \bar{E} + \omega + {}^1/_2\Delta - {}^1/_2 i\Gamma_i \mp {}^1/_2[(\Delta + \varepsilon - {}^1/_2 i\Gamma_i)^2 + {}^1/_4\Gamma_f^2]^{1/2}. \qquad (22)$$

The total probability of ionization of an atom by the

moment of time t has the form

$$W(t)=\int dE|A(E,t)|^2=\frac{\Gamma_i\Gamma_f^2}{32|E_a-E_b|^2}\left(\frac{\exp(2\operatorname{Im}E_at)-1}{\operatorname{Im}E_a}\right.$$
$$\left.+\frac{\exp(2\operatorname{Im}E_bt)-1}{\operatorname{Im}E_b}-4\operatorname{Im}\frac{\exp[i(E_a^*-E_b)t]-1}{E_a^*-E_b}\right). \quad (23)$$

In the case of multiphoton resonance ionization in a monochromatic field $\mathbf{F}(t)$, the matrix elements W_E and W', in terms of which the ionization Γ_i and field Γ_f widths are expressed, are replaced by composite matrix elements corresponding to the minimum number of quanta n and n' necessary for transitions respectively from ψ_1 into the continuum and from ψ_0 to ψ_1, while the frequencies ω and ω' are replaced by $n\omega$ and $n'\omega'$. In this case $\Gamma_i\sim\mathrm{Ry}(F/F_0)^{2n}$ and $\Gamma_f\sim\mathrm{Ry}(F/F_0)^{n'}$, and we should take into account in the energies E_0 and E_1 the corrections determined by the nonresonance part of the dynamical Stark effect right up to the order given by the $\min\{2[n'/2], 2n\}$, where $[X]$ denotes the integral part of the number X. The condition of applicability of the resonance approximation in the general case has the form $\omega>\mathrm{Ry}(F/F_0)^2$.

Thus, the resonance ionization of an atom is determined by three constants, characterizing the damping ($\mathrm{Im}E_a$ and $\mathrm{Im}E_b$) and the oscillation ($\mathrm{Re}(E_a-E_b)$) of the probability. The characteristic decay time of the atom is determined by the quantity

$$\tau\sim\max\{|\mathrm{Im}E_a|^{-1}, |\mathrm{Im}E_b|^{-1}\}.$$

If by chance one of the damping constants or the oscillation frequency is anomalously high, then a linear (in time) ionization regime is possible in the time interval

$$\tau'\sim\min\{|\mathrm{Im}E_a|^{-1}, |\mathrm{Im}E_b|^{-1}, |\mathrm{Re}(E_a-E_b)|^{-1}\}\ll t\ll\tau.$$

In the case when $|\mathrm{Im}E_a|\sim|\mathrm{Im}E_b|\sim|\mathrm{Re}(E_a-E_b)|$, however, the time τ' required by the system to get into the linear (in time) regime is comparable to the total-ionization time τ for the atom, and the linear regime does not occur.[5)] It is clear that in this case the question of the shape of the resonance curve for the ionization probability per unit time loses its meaning. The use of a Breit–Wigner type of formula with one or another resonance width to describe the linear (in time) ionization regime is, in the general case, impossible.

4. THE LINEAR REGIME AND OTHER PARTICULAR CASES

The general expressions for the probability amplitude $A(E, t)$ and the total ionization probability $W(t)$ get simplified when certain relations exist between the parameters ε, Δ, Γ_i, and Γ_f.

1) Narrow ionization widths. Under conditions when $\Gamma_i\sim|\Delta|\ll\Omega$ we obtain from (22) that

$$E_{a,b}\approx\bar{E}+\omega\mp\Omega/2=\bar{E}_{0,1}+\omega.$$

It is not difficult to see that at times $\Omega^{-1}\ll t\ll\Gamma_i^{-1}$ it follows from (21) that

$$\frac{dW(E,t)}{dE}=|A(E,t)|^2=\frac{\Gamma_i\Gamma_f^2t}{16\Omega^2}[\delta(E-\bar{E}_0-\omega)+\delta(E-\bar{E}_1-\omega)], \quad (24)$$

i.e., that the linear (in time) ionization regime is realizable. In this case the resonance width is determined by the field width Γ_f, and the conservation laws correspond to independent ionization from each of the $\bar{E}_0$ and $\bar{E}_1$ levels.

2) Small field widths. For $\Gamma_f\ll|\Delta+\varepsilon-\frac{1}{2}i\Gamma_i|$

$$E_a\approx\bar{E}+\omega-\frac{\varepsilon}{2},\quad E_b\approx\bar{E}+\Delta+\omega+\frac{\varepsilon}{2}-\frac{i}{2}\Gamma_i.$$

At the relatively small times $|\varepsilon|^{-1}\ll t\ll\Gamma_i^{-1}$ the electron distribution over energy is given by the formula

$$\frac{dW(E,t)}{dE}=\frac{\Gamma_i\Gamma_f^2t}{16\varepsilon^2}[\delta(E_0+\omega+\omega'-E)+\delta(E_1+\omega-E)]. \quad (25)$$

This result is similar to the corresponding formula obtained in[2] if we extrapolate the latter result to the case of a small number of absorbable quanta. Notice that the formula (25) is valid in the region of sufficiently large detunings $|\varepsilon|\gg\Gamma_i$, and, therefore, allowance for the ionization width Γ_i in the resonance denominator in (25) is not warranted. The appearance of the second δ-function in (25) is a well-known consequence of the fact that the interaction is switched on instantaneously (see, for example,[14]).

At larger times $t\gg\Gamma_i^{-1}$ the second term in the square brackets in (21) becomes nonresonant, and can be dropped. The linear regime is realizable in the time range $\Gamma_i/\Gamma_f^2\gg t\gg\Gamma_i^{-1}$:

$$\frac{dW(E,t)}{dE}=\frac{\Gamma_i\Gamma_f^2t}{16[(\Delta+\varepsilon)^2+\Gamma_i^2/4]}\delta(E_0+\omega+\omega'-E). \quad (26)$$

The resonance width in this case is, as was to be expected, determined by the ionization width Γ_i. It is interesting to note that the total-ionization time (the decay time) is in this case $\tau\sim\Gamma_i/\Gamma_f^2$.

3) For a sufficiently long interaction duration $t\gg|\mathrm{Im}E_{a,b}|^{-1}$, the ionization probability reaches saturation. It can be directly verified that in this case it follows from (23) that $W(t)\to1$. The asymptotic (in time) form of the electron distribution over energy is determined by the expression

$$\left.\frac{dW(E,t)}{dE}\right|_{t\to\infty}=\frac{\Gamma_i\Gamma_f^2}{32\pi}\left\{\left[(E-E_0-\omega-\omega')^2-(\Delta+\varepsilon)(E-E_0-\omega-\omega')\right.\right.$$
$$\left.\left.-\frac{\Gamma_f^2}{16}\right]^2+\frac{\Gamma_i^2}{4}(E-E_0-\omega-\omega')^2\right\}^{-1}. \quad (27)$$

4) For small times ($t\ll\tau'$), the series expansion of the probability $W(t)$ in powers of t begins with t^3:

$$W(t)\approx\frac{\Gamma_i\Gamma_f^2}{48}t^3. \quad (28)$$

5) The formula (23) for the probability $W(t)$ assumes a simple form for specially chosen relations between the detuning ε, the level shift Δ, and the ionization Γ_i and field Γ_f widths: $\Delta+\varepsilon=0$ and $\Gamma_i=\Gamma_f$. In this case

$$E_a=E_b=\bar{E}+\omega-{}^1/_2\varepsilon-{}^1/_4 i\Gamma_i$$

and, expanding the indeterminate form in (23), we obtain

$$W(t)=1-e^{-\Gamma_i t/2}[1+{}^1/_2\Gamma_i t+{}^1/_8(\Gamma_i t)^2]. \quad (29)$$

Thus, the linear (in time) ionization regime is indeed possible only under the conditions of a relatively narrow field or ionization width.

5. CONCLUSION

Let us briefly summarize the main results and the conditions under which they were obtained. In the paper we have found the probability amplitudes and the total probability of direct and resonance ionization of an atom. The obtained formulas determine the time dependence of the ionization process. It turns out that the notion of a constant (time independent) ionization probability per unit time is not always justified. Under conditions when the field and ionization widths are quantities of the same order of magnitude, no region exists in which the ionization regime is linear in time. The conditions of applicability of the obtained results are determined by the basic assumptions, and amount to the following: 1) the field intensity F in the wave is significantly less than the intratomic field intensity F_0; 2) the discrete levels E_0 and E_1 are not degenerate; 3) the interaction with the field is switched on "instantaneously." If the first condition is fundamentally necessary for the construction of some closed theory of the interaction between a real atom and radiation, then the last two conditions are used to simplify the problem. Therefore, the construction of a theory free from these limitations and, in particular, the investigation of the dependence of the results on the way the interaction is switched on are of unquestionable interest. These questions, however, fall outside the limits of the present paper, and we hope to return to them in another place.

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[1)] For any state ψ we can always construct a unique linear combination, ψ_E, of the wave functions of the continuous spectrum with energy E such that $\langle\psi|\mathbf{df}|\psi_E\rangle\neq 0$, while the corresponding matrix elements with all other functions with the same energy E will be equal to zero.

[2)] This assumption is common to all the papers on resonance ionization.[2-4] It should, of course, be borne in mind that atomic levels may be degenerate with respect to a component of the total angular momentum. However, this degeneracy is removed by a sufficiently strong magnetic field, and it generally turns out to be unimportant in the case when the fields $\mathbf{F}$ and $\mathbf{F}'$ have the same polarization. The model of nondegenerate E_0, E_1 levels can also be used to describe resonance multiphoton ionization under the action of a monochromatic field $\mathbf{F}(t)$. For example, in atoms of the noble gases a two-photon resonance between the nondegenerate S-states is realizable. If, on the other hand, the resonance arises as a result of the absorption of three or more photons, then of vital importance will be the nonresonance dynamical Stark,[8,9] which may lead to the splitting of each of the degenerate levels into levels with a spacing significantly exceeding the widths Γ_i and Γ_f.

[3)] A formulation of the problem of resonance ionization, similar to the one adopted in the present paper, is given in[13]. In it, however, the author replaces the entire continuous spectrum of atomic states by a single discrete level, and considers transitions in a system that is essentially a three-level system. Such a replacement does not allow for the important distinctive features of the continuum, and is not adequate.

[4)] The constants E_a and E_b coincide with the "quasi-energy" values $\tilde{E}_{0,1}$, (13), for the two-level system if we introduce into it a width Γ_i and a shift Δ for the E_1 level, i.e., if we make the substitution $E_1\to E_1+\Delta-i\Gamma_i/2$ (and shift the reference energy point by $\omega-\varepsilon/2$). As applied to resonance ionization, the two-level system with damping has been considered in[9].

[5)] Notice that the characteristic times τ and τ' are not, generally speaking, anomalously small. For example, in the case of the resonance ionization of helium atoms under the action of rubidium laser radiation,[1] $n=1$, $n'=2$, and $\Gamma_1\sim\Gamma_f\sim \mathrm{Ry}(F/F_0)^2$, which, for $|\varepsilon|\lesssim\Gamma_i$ and $F\sim 5\times 10^5$ V/cm yields $\Gamma_i^{-1}\sim\Gamma_f^{-1}\sim 10^{-8}$ sec.

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