# Magnetic ordering in nonequilibrium systems

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The possibility is studied of magnetic ordering caused by coexistence of singlet and triplet electron-hole pairings in systems with a nonequilibrium concentration of excitations, for a model of an exciton insulator: that is, for a semiconductor with a small forbidden band $E_g$ and for a semiconductor with a large value of $E_g$. In the first case, when the (diamagnetic or antiferromagnetic) equilibrium state of the semiconductor is itself connected with the existence of either singlet or triplet electron-hole pairing, nonequilibrium excitations insure the coexistence of singlet and triplet pairings; that is, ferromagnetic ordering occurs. In the case of a semiconductor with a large value of $E_g$, nonequilibrium excitations undergo electron-hole pairing because of Coulomb interaction and of a strong electromagnetic pumping field. In this case the doped semiconductor goes over to a ferromagnetic state.

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## 1. INTRODUCTION

It is known that semimetals whose electron ($\epsilon_1(\mathbf{p})$) and hole ($\epsilon_2(\mathbf{p})$) bands satisfy the condition $\epsilon_1(\mathbf{p}) = -\epsilon_2 \times (\mathbf{p} + \mathbf{q})$ are unstable with respect to electron-hole pairing. A similar property is possessed by single-band metals with a spectrum $\epsilon(\mathbf{p}) = -\epsilon(\mathbf{p} + \mathbf{q})$ and by semiconductors in which the width $E_g$ of the forbidden band does not exceed the binding energy $E_{exc}$ of an exciton. In all these cases, depending on the spin structure of the electron-hole pair, there occurs either a charge-density wave (CDW), for singlet structure of the pair, or a spin-density wave (SDW), for triplet structure of the pair. That state is realized that corresponds to the larger coupling constant. The CDW state is accompanied by a rearrangement of the crystal lattice.

Examples of states with CDW are certain semiconductors of the group $A_4B_6$, a number of layered and quasiunidimensional systems, and certain compounds of the transition metals (see, for example, [1]). The antiferromagnetism of chromium is explained by the occurrence of SDW. The semiconductor state that occurs as a result of CDW or SDW is customarily called an exciton dielectric. It was shown earlier[2] that SDW and CDW can coexist, which leads to a ferromagnetic state.

Semiconductors with $E_g > E_{exc}$, in which electron-hole pairing does not occur under equilibrium conditions, behave in a similar manner under pumping of the electrons and holes by an external source; that is, a nonequilibrium CDW or SDW state is possible[3]. Electron-hole pairing with CDW may be accompanied by Bose condensation of photons[4], which transforms the system to the laser mode. A strong electromagnetic wave induces, in the Fermi quasilevels of the electrons and holes, a field gap[5], which makes the CDW state more favorable than the SDW state[6].

In semiconductors with an indirect gap $E_g$, the electrons and holes have a tendency toward condensation in ordinary but not in momentum space; that is, they have a tendency toward formation of electron-hole droplets[7], within which Bose condensation in momentum space may still occur. A peculiarity of the nonequilibrium CDW or SDW state is that the phase of the corresponding order parameter is not fixed, both because of the absence of distortion of the crystal when CDW occurs, and because of the nonexistence of interelectron interaction terms connected with interband transitions of the particles[8]. Therefore a phase transition in the CDW or SDW state is a transition of the second kind, and superfluidity is possible in the system[4]. Furthermore, in such a system, in contrast to an equilibrium system, there is possible a phenomenon[9] analogous to the nonstationary Josephson effect. If the pumping occurs in a doped semiconductor, then when there is effective interelectron attraction, there is a possibility of simultaneous existence of SDW or CDW and of superconductivity[6]. Then the temperature of the superconducting transition may be significantly higher than when CDW and SDW are absent.

We shall suppose below that the effective interelectron interaction is repulsive. In this case also there is in general a possibility of a superconducting state, if the width $E_g$ of the forbidden band is less than a certain critical $E_{cr}$[10]. We shall not consider this case ($E_g < E_{cr}$); that is, we shall assume that superconductivity, with or without pumping, is impossible. In Sec. 3 it will be shown that under pumping of a doped semiconductor, coexistence of CDW and SDW is possible; that is, a nonequilibrium ferromagnetic state is established. In Sec. 2, the possibility is investigated of a magnetically ordered state of an undoped exciton dielectric under the influence of pumping. Without pumping, a ferromagnetic state of an exciton dielectric is possible only with doping.

The question arises: in an exciton dielectric, which itself is a dielectric because of equilibrium electron-hole pairing, cannot electrons and holes excited by an external source become bound into pairs, as in the case of an ordinary semiconductor with $E_g > E_{exc}$? As a result, in the electron and hole Fermi quasilevels there would be formed a new dielectric gap $\Delta_b$, also of collective nature, like the basic gap $\Delta_0$ in the equilibrium state of an exciton dielectric. An analogous question about the possibility of pairing of electron and hole excitations exists in superconductors with pumping.

Analysis of the system of equations for $\Delta_0$ and $\Delta_b$ shows that this sytem is incompatible[11]; that is, electron-hole pairing of excitations in an exciton dielectric is impossible. Apparently a similar situation occurs also in superconductors with pumping. Therefore what is investigated in Sec. 2 is the influence of noninter-

acting excitations of the electronic and hole types on the possibility of coexistence of CDW and SDW. Here excitations of the concentration n play the same role as does the doping in the equilibrium case[2], although the form of the solutions for the magnetic phase and the bounds on its existence differ significantly from the results of the previous work[2].

During pumping of an exciton insulator with gap $2\Delta$ larger than the energy $\hbar\omega_D$ of a Debye phonon, the distribution function of the excitations may be of Fermi type[10]. In the absence of magnetic ordering, the more advantageous solution for $\Delta(\mathbf{r})$ and $n(\mathbf{r})$ is a nonuniform one, of the type of the solutions for a superconductor in an internal magnetic field[11]. Here regions in space with concentrations of excitations above and below the mean alternate. Such a state is like the state of electron-hole drops in a semiconductor with $E_g > E_{exc}$[7], when inside a drop there occurs Bose condensation of electron-hole pairs.

In the present paper (Sec. 2), we shall investigate the spatially uniform magnetically ordered state of an exciton dielectric with pumping.

## 2. SPIN ORDERING IN AN EXCITON DIELECTRIC WITH PUMPING

Under the influence of an external source, for example an electromagnetic field with frequency $\hbar\omega > 2\Delta$ or a tunnel current, let there be produced an excitation of quasiparticles and quasiholes with concentration n in an exciton dielectric with gap $\Delta$. In contrast to the analogous problem in superconductors[12], the distribution function $f(\epsilon)$ of excitations in an exciton dielectric may be almost of Fermi form; that is, for T = 0 it may have form of a step, equal to unity up to a certain level $\mu$ and to zero for $\epsilon > \mu$. For this it is necessary that the dielectric gap $\Delta$ be larger than half the Debye phonon energy, $\hbar\omega/2$[13]. Since the value of $\Delta$ in an exciton dielectric is determined by the Fermi energy $\epsilon_F$ instead of $\hbar\omega_D$ as in a superconductor, the condition

$$\Delta > \hbar\omega_D/2 \tag{1}$$

can be easily satisfied. Because of the contrary situation in superconductors ($\Delta < \hbar\omega_D/2$), the distribution function $f(\epsilon)$ is always less than $1/2$[14]; that is, it is significantly non-Fermi, in contradiction to the assumption of [12b].

On the basis of the diagram technique of Keldysh[16], for nonequilibrium processes it can be shown[10] that in an exciton dielectric under the condition (1), the distribution function $f(\epsilon)$ with pumping can have the form

$$f(\varepsilon) = [e^{(\varepsilon-\mu)/T}+1]^{-1}, \tag{2}$$

while the normal and anomalous Green functions $G_{11}(\mathbf{p}, \omega)$ and $G_{21}(\mathbf{p}, \omega)$ can be expressed in the form (T = 0)

$$G_{11}(\mathbf{p}, \omega) = \frac{u^2}{\omega - E + i\delta} + \frac{v^2}{\omega + E - i\delta} + 2\pi i n_p [u^2\delta(\omega - E) - v^2\delta(\omega + E)], \tag{3}$$

$$G_{21}(\mathbf{p}, \omega) = \frac{-\Delta}{(\omega - E + i\delta)(\omega + E - i\delta)} + \frac{\pi\Delta i}{E} n_p [\delta(\omega - E) + \delta(\omega + E)]. \tag{4}$$

Here

$$G_{11} = -i\langle T(a_1 a_1^+)\rangle,\ G_{21} = -i\langle T(a_2 a_1^+)\rangle,$$
$$E = (\xi^2 + \Delta^2)^{1/2},\quad u^2 = {}^1/_2(1 + \xi/E),\quad v^2 = {}^1/_2(1 - \xi/E),$$

$$n_p = \begin{cases} 1 & E \leqslant \mu \\ 0 & E > \mu \end{cases},\qquad \Delta = \frac{-ig}{(2\pi)^4}\int G_{21}(\mathbf{p}, \omega)\, d\omega\, d\mathbf{p}. \tag{5}$$

The value of $\mu$ is determined from the condition that the total number N of excitations (quasiparticles) is given:

$$(\mu^2 - \Delta^2)^{1/2} = n. \tag{6}$$

Here n is the concentration, expressed in energy units: $n = N/4VN(0)$, where N(0) is the density of states at the Fermi level and V is the volume of the system. On substituting $G_{21}$ from (4) in equation (5) and using condition (6), we get

$$n = {}^1/_2(\Delta/\Delta_0)^{1/2}(\Delta_0 - \Delta), \tag{7}$$

where $\Delta_0 = 2\tilde{\omega}\, e^{-1/gN(0)}$. The dependence of $\Delta$ on n, determined by equation (7), is shown in Fig. 1 (Curve 1). The value of $\Delta$ falls with increase of n and vanishes discontinuously from a value $\Delta_{cr} = 1/3\Delta_0$ at $n_{cr} = 0.192\Delta_0$. There is a second branch of the solution $\Delta(n)$, going out from the origin, which corresponds to an unstable state[17].

We shall compare these results with the case of a doped exciton dielectric, in which, depending on the type of doping, there are at T = 0 either electrons or holes, but not both together, as for pumping. In the latter case there are two Fermi quasilevels: $+\mu$ for electrons and $-\mu$ for holes. For doping in the equilibrium state, of course, there is a single Fermi level $\mu$. Because of this difference, the gap $\Delta$ in the case of doping is a single-valued monotonic function of n and approaches zero continuously at $n_{cr} = 1/2\Delta_0$.

As was shown earlier[2], in doping of an exciton dielectric there is a possibility of coexistence of singlet and triplet electron-hole pairings, which leads to spin splitting of the bands and to ferromagnetic ordering. Without doping, coexistence is impossible in the equilibrium state. We shall consider the possibility of such a state in an undoped exciton dielectric with pumping. Formally, the system of equations for the Green functions has the same form as in the equilibrium case[2].

The nonequilibrium of the system can be taken into account by a change of the rules for bypassing poles in the Green function:

$$G_{11}^{\alpha\alpha}(\mathbf{p}, \omega) = \frac{u_\alpha^2}{\omega - E_\pm + i\delta} + \frac{v_\alpha^2}{\omega + E_\pm - i\delta} + 2\pi i n_p (u_\alpha^2\delta(\omega - E_\pm) - v_\alpha^2\delta(\omega + E_\pm)], \tag{8}$$

$$G_{21}^{\alpha\alpha}(\mathbf{p}, \omega) = \frac{-\Delta_\pm}{(\omega - E_\pm + i\delta)(\omega + E_\pm - i\delta)} + \frac{\pi\Delta_\pm i}{E_\pm} n_p [\delta(\omega - E_\pm) + \delta(\omega + E_\pm)], \tag{9}$$

where $\alpha$ is the spin index, equal to $\pm 1$;

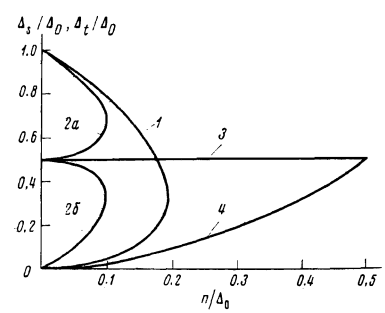

FIG. 1

$$\Delta_\pm=\Delta_s\pm\Delta_t, \quad E_\pm=(\xi^2+\Delta_\pm^2)^{1/2}$$

(in formulas (8) and (9), $\Delta_+$ and $E_+$ are taken for $\alpha = +1$, $\Delta_-$ and $E_-$ for $\alpha = -1$);

$$\Delta_s=-\frac{ig_s}{2(2\pi)^4}\int d\omega\, d\mathbf{p}\{G_{21}^{11}(\mathbf{p},\omega+i0)+G_{21}^{-1,-1}(\mathbf{p},\omega+i0)\},$$
$$\Delta_t=-\frac{ig_t}{2(2\pi)^4}\int d\omega\, d\mathbf{p}\{G_{21}^{11}(\mathbf{p},\omega+i0)-G_{21}^{-1,-1}(\mathbf{p},\omega+i0)\}, \quad (10)$$

and $g_S$ and $g_t$ are, respectively, the singlet and triplet coupling constants[1]. The values of $n_p(E_\pm)$, $u_\alpha^2(E_\pm)$, and $v_\alpha^2(E_\pm)$ are given by formulas (5).

On substituting, in the conditions for compatibility of (10), the expression for $G_{21}^{\alpha\alpha}$ from (9), and on carrying out the integration, we get the equations that determine $\Delta_S$ and $\Delta_t$. In the case $\mu > |\Delta_+|$, $|\Delta_-|$ they have the form

$$\Delta_s \ln \Delta_{s0}=1/2[\Delta_+L_++\Delta_-L_-], \quad (11)$$
$$\Delta_t \ln \Delta_{t0}=1/2[\Delta_+L_+-\Delta_-L_-]. \quad (12)$$

In the case $|\Delta_-| < \mu < |\Delta_+|$,

$$\Delta_s \ln \Delta_{s0}=1/2[\Delta_+ \ln |\Delta_+|+\Delta_-L_-], \quad (13)$$
$$\Delta_t \ln \Delta_{t0}=1/2[\Delta_+ \ln |\Delta_+|-\Delta_-L_-], \quad (14)$$

$$L_\pm=\ln\frac{[(\mu^2-\Delta_\pm^2)^{1/2}+\mu]^2}{|\Delta_\pm|};$$
$$\Delta_{s0}=2\tilde{\omega}\exp\{-1/g_sN(0)\}, \quad \Delta_{t0}=2\tilde{\omega}\exp\{-1/g_tN(0)\},$$

where $\tilde{\omega}$ is a characteristic cutoff energy.

We note that, although in pumping as in doping $\mu \neq 0$ occurs, equations (11), (12) and (13), (14) for $\Delta_S$ and $\Delta_t$ differ significantly from those in the equilibrium state with doping[2] because of the presence, with pumping, of electron and hole Fermi quasilevels simultaneously. Actually a similar difference leads to the possibility of superconductivity when there is repulsion in the nonequilibrium case[10].

Since in this problem the concentration n of nonequilibrium carriers is considered given, but not the position of the Fermi quasilevels of the electrons and holes, it is necessary to append to equations (11), (12) and (13), (14) a relation connecting $\mu$ and n. In the case $\mu > |\Delta_+|$, $|\Delta_-|$ we have

$$(\mu^2-\Delta_-^2)^{1/2}+(\mu^2-\Delta_+^2)^{1/2}=2n, \quad (15)$$

and in the case $|\Delta_-| < \mu < |\Delta_+|$

$$(\mu^2-\Delta_-^2)^{1/2}=2n. \quad (16)$$

We shall consider first the equations for $\Delta_S$ and $\Delta_t$ when $\Delta_{S0} = \Delta_{t0} = \Delta_0$; that is, on a diagonal in the $(\Delta_{S0}, \Delta_{t0})$ plane.

For $\mu > |\Delta_+|$, $|\Delta_-|$ we get from equations (11) and (12)

$$\Delta_\pm(L_\pm-\ln\Delta_0)=0. \quad (17)$$

Since the equations for $\Delta_+$ and $\Delta_-$ are identical, they have solutions $\Delta_+ = \Delta_- = \Delta_S$ ($\Delta_t = 0$), and the value of $\Delta_S$ is determined from an equation analogous to (7). Besides these solutions, the system of equations (17) admits a solution

$$\Delta_-=0, \quad \Delta_+=4n^2/\Delta_0, \quad n<1/2\Delta_0 \text{ or } \Delta_s=\Delta_t=2n^2/\Delta_0. \quad (18)$$

It can be shown that in the case $\Delta_-$, $\Delta_+ > 0$ the system of equations (17) has no other solutions.

In the case $|\Delta_-| < \mu < |\Delta_+|$ and $\Delta_{S0} = \Delta_{t0}$, we get from equations (13) and (14), with use of the condition (16),

$$\Delta_+ \ln(|\Delta_+|/\Delta_0)=0, \quad (19)$$
$$\Delta_- \ln\frac{[2n+(4n^2+\Delta_-^2)^{1/2}]^2}{\Delta_0|\Delta_-|}=0. \quad (20)$$

The system of equations (19), (20) has the following solutions:

a) $\Delta_+=\Delta_0$, $\Delta_-=0$, $\Delta_s=\Delta_t=\Delta_0/2$, $n<\Delta_0/2$; (21)

b) $\Delta_+=\Delta_0$, $n=1/4(\Delta_-/\Delta_0)^{1/2}(\Delta_0-\Delta_-)$. (22)

The dependences of $\Delta_S$ and $\Delta_t$ on n in the case $\Delta_{S0} = \Delta_{t0}$ are shown in Fig. 1 (curve 1, the nonmagnetic solution (7); 2a and 2b, $\Delta_S$ and $\Delta_t$, respectively, corresponding to formulas (22); 3, the line $\Delta_S = \Delta_t = \Delta_0/2$; 4, $\Delta_S = \Delta_t = 2n^2/\Delta_0$). We found the solutions on the assumption that $\Delta_- > 0$; that is, $\Delta_S > \Delta_t$. When $\Delta_- < 0$ ($\Delta_S < \Delta_t$), the solutions of the system (11), (12), and (13) are obtained from the above by the substitution $\Delta_S \rightleftharpoons \Delta_t$.

Thus for $\Delta_{S0} = \Delta_{t0}$ we obtain the whole set of solutions for the order parameters $\Delta_S$ and $\Delta_t$. The presence of singlet and triplet pairings leads to a broadening of the concentration range within which a dielectric state of the system is possible, in comparison with the case of a single type of pairing ($n_{cr} = 3^{-3/2}\Delta_0 = 0.192\Delta_0$ in the case of a single type of pairing, $n_{cr} = 0.5\Delta_0$ for coexistence of two types of pairing).

The question arises, which of the solutions obtained for $\Delta_{S0} = \Delta_{t0}$ (Fig. 1) will be realized. Starting from a qualitative analysis of the dependence of the energy E of the system on $\Delta$, we find that some of these solutions correspond to a maximum on the $E(\Delta)$ curve; that is, they correspond to an unstable state. For example, the lower branch of Curve 1 (the nonmagnetic solution $\Delta_S \neq 0$, $\Delta_t = 0$) is unstable[17]. If there were no magnetic solutions ($\Delta_S \neq 0$, $\Delta_t \neq 0$), then with increase of the concentration n of nonequilibrium excitations, the gap $\Delta$ would decrease and would become zero discontinuously at $n = n_{cr} = 0.192\Delta_0$. For $n > 0.192\Delta_0$ there are two types of magnetic solutions; in both, $\Delta_S = \Delta_t$.

One solution (Curve 4 in Fig. 1) corresponds to $\mu > |\Delta_+|$ ("weak" ferromagnetism); here $\Delta_S$ and $\Delta_t$ have a nonphysical behavior, since they increase with increase of pumping. This branch is unstable. Therefore in the range $0.192 < n/\Delta_0 < 0.5$ solution (21) is realized ("strong" ferromagnetism, $|\Delta_-| < \mu < |\Delta_+|$, Curve 3 in Fig. 1). For $0.096 < n/\Delta_0 < 0.192$ the solution (21) is energetically more advantageous than the nonmagnetic solution (7), and therefore it is also realized. In the range $n < 0.096\Delta_0$ the energetically advantageous solution is (22) ("strong" ferromagnetism), corresponding to the upper branch of Curve 2a for $\Delta_S$ and the lower branch of Curve 2b for $\Delta_t$. The lower branch of Curve 2a for $\Delta_S$ and the upper part of Curve 2b for $\Delta_t$ correspond to an unstable state.

Thus for $\Delta_{S0} = \Delta_{t0}$ and for $n < 0.096\Delta_0$, the parameters $\Delta_S$ and $\Delta_t$ vary according to the stable sections of Curve 2 of Fig. 1 ($\Delta_S$ decreases with increase of n, while $\Delta_t$ increases). There was an analogous behavior also in the case of a doped exciton insulator[2]. True, in the latter case $\Delta_S$ and $\Delta_t$ varied smoothly up to the value $\Delta_0/2$, and on further increase of n remained equal to the value $\Delta_0/2$ until $n = 0.5\Delta_0$, where both became zero discontinuously. In the present case, at the point $n = 0.096\Delta_0$ the value of $\Delta_S$ drops discontinuously to

the value $\Delta_0$ and $\Delta_t$ rises discontinuously to this same value; at $n = 0.5\Delta_0$ both become zero discontinuously, as in the case of doping.

A difference in principle between the "magnetic" solutions with pumping and the doping case consists in the fact that a net magnetic moment does not occur in pumping, since the moment of the electrons above the gap is compensated by the moment of the hole states below the gap. But in the doping case[2] there are, depending on the type of doping, either only "excess" electrons above the gap, or only hole states below the gap. Thus a net moment can exist in pumping only in a doped material, in which magnetization existed also before the pumping. Here the influence of the pumping reduces to a change of character of the magnetization.

In the case $\Delta_{s0} \neq \Delta_{t0}$, the system of equations (11), (12), (15) has a solution $\Delta_t = 0$, $\Delta_s = \Delta_s^0$ ($\Delta_s^0$ is determined by formula (7)) and a solution $\Delta_s = 0$, $\Delta_t = \Delta_t^0$ ($\Delta_t^0$ is determined by formula (7)). Both of these solutions are nonmagnetic. In addition to these solutions, the system (11), (12), (15) may, for certain values of the parameters $\Delta_{s0}$ and $\Delta_{t0}$, have ferromagnetic solutions. We shall find on the ($\Delta_{s0}$, $\Delta_{t0}$) phase diagram the line on which $\Delta_t$ first appears when $\Delta_s \neq 0$. For this purpose we shall set $\Delta_t = 0$ in equations (11) and (15) and shall expand the right side of (12) up to quadratic terms in $\Delta_t$:

$$\Delta_s \ln \Delta_{s0} = \Delta_s \ln \frac{[(\mu^2-\Delta_s^2)^{1/2}+\mu]^2}{\Delta_s} \qquad (\mu^2-\Delta_s^2)^{1/2} = n,$$

$$\Delta_t \ln \Delta_{t0} = \Delta_t \Big[ \ln \frac{[(\mu^2-\Delta_s^2)^{1/2}+\mu]^2}{\Delta_s} - \frac{2\Delta_s^2}{[(\mu^2-\Delta_s^2)^{1/2}+\mu](\mu^2-\Delta_s^2)^{1/2}} - 1 \Big] + O(\Delta_t^2). \tag{23}$$

On letting $\Delta_t$ approach zero, we obtain from (23) after simple transformations

$$\Delta_{s0}\Delta_s = [n+(n^2+\Delta_s^2)^{1/2}]^2, \quad \ln \frac{\Delta_{t0}}{\Delta_{s0}} + \frac{2\Delta_s^2}{n[n+(n^2+\Delta_s^2)^{1/2}]} + 1 = 0. \tag{24}$$

Equations (24) determine the line on the ($\Delta_{s0}$, $\Delta_{t0}$) plane on which $\Delta_t$ first appears. This is a line of phase transition of the second kind into a ferromagnetic state. We find similarly the line on which $\Delta_s$ first appears when $\Delta_t \neq 0$) (the line $\Delta_s = 0$). In Fig. 2 the line $\Delta_t = 0$ is represented by Curve 1, the line $\Delta_s = 0$ by Curve 1′. It is easy to show that Curve 1 has the asymptotes $\Delta_{t0} = 0$ and $\Delta_{t0} = \Delta_{s0}/e$, Curve 1′ the asymptotes $\Delta_{s0} = 0$ and $\Delta_{t0} = e\Delta_{s0}$.

The system of equations (11), (12) (15) corresponds to weak ferromagnetism ($\mu > |\Delta_+|$, $|\Delta_-|$) and has

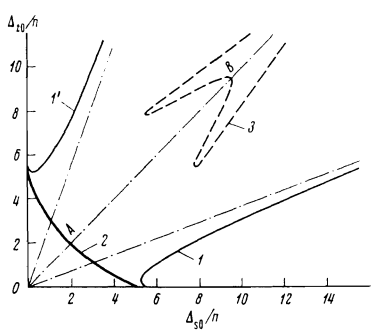


FIG. 2

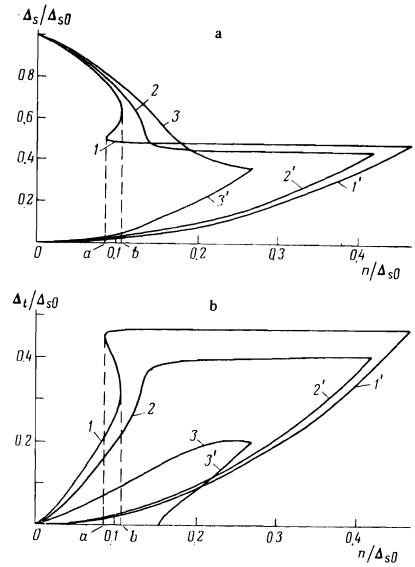


FIG. 3

magnetic solutions only outside the region bounded by Curves 1 and 1′ in Fig. 2. We note that in the doping case the curve $\Delta_s = 0$ underwent a smooth transition into the curve $\Delta_t = 0$. In the pumping case these curves as it were repel each other.

From consideration of the case $\Delta_{s0} = \Delta_{t0}$ it is clear that in addition to the lines of phase transition of the second kind already found, the ($\Delta_{s0}$, $\Delta_{t0}$) phase diagram should contain lines of transition of the first kind. In order to find these lines, the systems of equations (11), (12), (15) for $\mu > |\Delta_+|$ and (13), (14), (16) for $|\Delta_-| < \mu < |\Delta_+|$ were solved numerically.

Figure 3a shows the dependence of $\Delta_s$ on the concentration n of nonequilibrium excitations, Fig. 3b the dependence of $\Delta_t$ on n, for various values of $\Delta_{t0}/\Delta_{s0} < 1$ (for 1 and 1′, $\Delta_{t0}/\Delta_{s0} = 0.9$; for 2 and 2′, $\Delta_{t0}/\Delta_{s0} = 0.7$; for 3 and 3′, $\Delta_{t0}/\Delta_{s0} = 0.2$). For $\Delta_{t0}/\Delta_{s0} > 1$, the solutions for $\Delta_s$ and $\Delta_t$ are obtained from those given above by the substitution $\Delta_s \rightleftharpoons \Delta_t$, $\Delta_{t0} \rightleftharpoons \Delta_{s0}$. From the figure it is evident that in the case of coexistence of singlet and triplet pairings, the range of existence of a dielectric state is broadened.

With increase of n, at a certain $n_{cr}$ the singlet and triplet gaps simultaneously become zero discontinuously. This is a point of phase transition of the first kind in the metallic state. In the ($\Delta_{s0}$, $\Delta_{t0}$) plane (Fig. 2), the line of transitions of this type is represented by curve 2. Beginning with $\Delta_{t0}/\Delta_{s0} \approx 0.7$ and up to $\Delta_{t0}/\Delta_{s0} \approx 1.4$, on the $\Delta_s$ vs n and $\Delta_t$ vs n curves for $|\Delta_-| < \mu < |\Delta_+|$ there are regions of multivaluedness (for example, the region (a, b) on curve 1 of Fig. 3). Points of the type a and b may be points of first-order phase transition within the region of strong ferromagnetism. Lines of such transitions are shown in Fig. 2 (curve 3). On the diagonal $\Delta_{s0} = \Delta_{t0}$, point B of curve 3 (Fig. 2) corresponds to a discontinuous transition from line 3 to line 2 in Fig. 1; that is, on section AB (Fig. 2) the state (21) is realized.

As is evident from Fig. 3, for a given value of n there exist up to two solutions both for $\Delta_s$ and for $\Delta_t$, of which one corresponds to $|\Delta_-| < \mu < |\Delta_+|$ (curves 1, 2, 3) and the other to $\mu > |\Delta_+|$ (curves 1′, 2′, 3′). The latter solution is apparently unstable, since on the

branches corresponding to it both parameters $\Delta_S$ and $\Delta_t$ have a nonphysical behavior, increasing with increase of the pumping intensity. Therefore curves 1 and 1′ in Fig. 2 also correspond to a nonphysical state. Thus everywhere above and to the right of curve 2 in Fig. 2 a state of strong ferromagnetism arises discontinuously; that is, $|\Delta_-| < \mu < |\Delta_+|$, $\Delta_S \neq 0$, $\Delta_t \neq 0$. Below curve 2 the system is a metal.

Everywhere above, we supposed that the concentration n of excitations was given, and the position of the Fermi quasilevels $+\mu$ for quasiparticles and $-\mu$ for quasi-holes was determined by the pumping n and the parameters $\Delta_S$ and $\Delta_t$ (see formulas (15), (16)). An analogous situation is realized in an exciton dielectric in equilibrium, with doping, when the concentration of the surplus carriers (either electrons or holes) is set by the impurity concentration. In the equilibrium case one can imagine a situation in which there is an additional band at the Fermi level with an infinite density of states. Then the position of the Fermi level $\mu$ will be fixed by this band. A state with a prescribed position of the Fermi quasilevel ($+\mu$ for quasiparticles and $-\mu$ for quasi-holes) can be formally realized by prescription of the frequency $\Omega$ of a powerful monochromatic source in the saturation mode[5], the energy of a quantum being larger than the width of the forbidden band. In this case equations (11)–(14) for $\Delta_S$ and $\Delta_t$ must be solved with $\mu$ = const; that is, without the additional conditions (15) and (16).

Results of a numerical solution of these equations for $\Delta_S$ and $\Delta_t$ as functions of $\mu$ are shown in Fig. 4. Curves 1, 2, 3 correspond to the parameter $\Delta_S$, curves 1′, 2′, 3′ to the parameter $\Delta_t$, for the case $|\Delta_-| < \mu < |\Delta_+|$ and $\Delta_{t0}/\Delta_{S0}$ equal to 0.9, 0.5, 0.2. Curves 4, 5, 6 correspond to $\Delta_S$, 4′, 5′, 6′ to $\Delta_t$, for $\mu > |\Delta_+|$ and the same respective values of the parameter $\Delta_{t0}/\Delta_{S0}$. Curve 7 corresponds to the nonmagnetic solution $\Delta_S = \Delta_{S0}$, $\Delta_t = 0$, $\mu < \Delta_S$, curve 8 to the nonmagnetic solution $\Delta_S \neq 0$, $\Delta_t = 0$, $\mu > \Delta_S$. As is evident from Fig. 4, the most favorable solution is $\Delta_S = \Delta_{S0}$, $\mu < \Delta_S$. The saturation mode, analogously to that considered by Galitskiĭ and others[5], would correspond to the case $\mu > \Delta_S$, which is not realized, since for $\mu > \Delta_{S0}$ the solution $\Delta_S = \Delta_{S0}$ (curve 7) becomes zero discontinuously. The reason for the impossibility of this state is as follows: As soon as, with increase of the field frequency, $\Omega$ exceeds the value $2\Delta_0$ by ever so little, the gap $\Delta_S$ must decrease because of the quasiparticles that appear. Then because of the fixedness of the frequency $\Omega$ of the position of the Fermi quasilevels $\pm\mu$, the concentration of quasiparticles will increase, and so on. There occurs an avalanche-type disappearance of the gap $\Delta_S$.

In addition to the nonmagnetic solutions $\Delta_S = \Delta_{S0}$ and $\Delta_S \neq 0$, $\Delta_t = 0$, $\mu > \Delta_S$ (curve 8 in Fig. 4), for $\mu$ = const

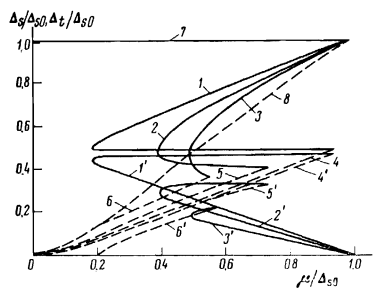


FIG. 4

there are magnetic solutions with $\Delta_S \neq 0$, $\Delta_t \neq 0$. In the equilibrium case (in the presence of a Fermi level either of electrons $+\mu$ or of holes $-\mu$), the branch corresponding to the magnetic solution is single-valued. Here $\Delta_S$ and $\Delta_t$ nonphysically increase with increase of $\mu$, which is a sign that the state is unstable; that is, a magnetic state for $\mu$ = const is impossible[2]. But in our case the magnetic-solution branch is three-valued; that is, there appears a stable-state section, on which $\Delta_S$ drops with increase of $\mu$. On this section $\mu > \Delta_-$ ($\mu = \hbar\Omega/2$); that is, a saturation mode can occur for the magnetic state (strong ferromagnetism). As $\Delta_{t0}/\Delta_{S0}$ approaches unity, the $\mu$ range (and consequently also the range of frequency $\Omega$ of the external source) broadens, as is evident from Fig. 4.

We emphasize once more that the saturation mode in the nonmagnetic state of an exciton dielectric is impossible.

## 3. MAGNETIC STATE OF A SEMICONDUCTOR IN THE FIELD OF A STRONG ELECTROMAGNETIC WAVE

We shall consider a semiconductor in which the extrema of the conduction and valence bands coincide in momentum space, in the field of a coherent light source of frequency $\Omega$, where $\hbar\Omega > E_g$ ($E_g$ is the width of the forbidden band). For simplicity we shall suppose that the effective masses are the same in absolute value in the two bands. In such a system, with sufficient source power, a saturation mode is possible[6], in which the states in the conduction band up to the level $+\epsilon_F$ (from the bottom of the band) will be filled with electrons, and the states in the valence band up to the level $-\epsilon_F$ (from the top of the band) will be filled with holes. At the corresponding Fermi quasilevels $\pm\mu$, in the absence of electron-hole interaction, there arises a dielectric gap $\lambda$, proportional to the amplitude $E_0$ of the field:

$$\lambda = (e/2\Omega)\, V_{cv} E_0$$

($V_{ve}$ is the matrix element of the interband transition for the velocity operator). Such a state is possible for a normal semiconductor, in distinction from an exciton dielectric, because the value of $E_g$ is determined principally by noncollective effects.

We shall suppose that the interaction between electrons within each band is of repulsive nature, so that the normal superconducting state considered earlier[6] is impossible. At a sufficiently large value of $E_g$, which is also assumed below, there is also no possibility of superconductivity caused by interelectron repulsion with inversion[10]. We shall therefore disregard intraband interelectron interaction.

The Hamiltonian of the system has the following form:

$$\hat{H}(t) = \sum_{p\alpha} \left\{ \left( \frac{\mathbf{p}^2}{2m} + \frac{E_g}{2} \right) (a^+_{cp\alpha} a_{cp\alpha} - a^+_{vp\alpha} a_{vp\alpha}) + \lambda a^+_{cp\alpha} a_{vp\alpha} e^{-i\Omega t} + \lambda a^+_{vp\alpha} a_{cp\alpha} e^{i\Omega t} \right\} + \sum_{\substack{qpp' \\ \alpha\beta}} g a^+_{cp\alpha} a^+_{vp'\beta} a_{v,p'-q,\beta} a_{c,p+q,\alpha}, \tag{25}$$

where $a^+_{cp\alpha}$ and $a^+_{vp\alpha}$ are the creation operators of electrons in the conduction band and in the valence band, and where $\alpha$ and $\beta$ are the spin indices. In the Hamiltonian (25), all that remains of the interband interaction is the

term with effective constant

$$g=(e/\hbar v_F)\ln(4p_F^2/\varkappa_D^2),$$

corresponding to interaction of the plane-plane type (here $v_F$, $p_F$, and $\kappa_D$ are determined by the concentration n of electrons and holes).

We shall suppose that the frequency $\Omega$ and the amplitude $E_0$ of the field are so chosen that $na_B^3 \gg 1$. Then for a large value of $E_g$ ($E_g \gg me^4/\epsilon^2\hbar^2$), interaction terms connected with conversion of particles of one band to particles of another band[2] should be omitted; this has also been taken into account in the Hamiltonian (25). In this case the constants $g_S$ of singlet and $g_t$ of triplet electron-hole pairing will be the same ($g_S = g_t = g$). The interband conversion term in (25), proportional to $\lambda$, does not become small because of a large value of $E_g$, because for $\hbar\Omega \geq E_g$ there is possible a real process of interband transition in such a field, and therefore there does not occur a large energy denominator for the transition probability.

After the unitary transformation[5]

$$U(t)=\exp\left\{-i\frac{\Omega t}{2}\sum_{p\alpha}(a_{cp\alpha}^+a_{cp\alpha}-a_{vp\alpha}^+a_{vp\alpha})\right\} \tag{26}$$

the Hamiltonian (25) reduces to the form

$$\hat{H}=\sum_{p\alpha}\{\xi_p(a_{cp\alpha}^+a_{cp\alpha}-a_{vp\alpha}^+a_{vp\alpha})+\lambda a_{cp\alpha}^+a_{vp\alpha}+\lambda^* a_{vp\alpha}^+a_{cp\alpha}\}$$
$$+\sum_{\substack{qpp'\\ \alpha\beta}} g a_{cp\alpha}^+ a_{vp'\beta}^+ a_{v,p'-q,\beta}a_{c,p+q,\alpha}; \tag{27}$$
$$\xi_p=p^2/2m-\varepsilon_F,\quad 2\varepsilon_F=\hbar\Omega-E_g.$$

For $\lambda = 0$, the expression (27) coincides exactly with the Hamiltonian for a semimetal with $g_S = g_t$[1]; that is, in this case the excited electrons and holes play the same role as in the equilibrium state of a semimetal. If the original semiconductor was undoped, then, as will be shown below, spin splitting does not occur; that is, a nonmagnetic saturation mode is realized[5]. But in the case of a doped semiconductor, in which the Fermi quasi-levels of the electrons and holes are outside the gap that is formed as a result of electron-hole pairing, spin splitting occurs; that is, a magnetic saturation mode is realized.

We introduce the Green function:

$$G_{11}^{\alpha\beta}=-i\langle T(a_{cp\alpha}a_{cp\beta}^+)\rangle,\quad G_{21}^{\alpha\beta}=-i\langle T(a_{vp\alpha}a_{cp\beta}^+)\rangle.$$

In the z-representation we obtain as in [2]

$$G_{11}^{\alpha\alpha}(\mathbf{p},\omega)=\frac{u_\alpha^2}{\omega+\mu-E_\pm+i\delta\,\mathrm{sign}(E_\pm-\mu)}+\frac{v_\alpha^2}{\omega+\mu+E_\pm-i\delta}, \tag{28}$$

$$G_{21}^{\alpha\alpha}(\mathbf{p},\omega)=\frac{-\Delta_\pm}{[\omega+\mu-E_\pm+i\delta\,\mathrm{sign}(E_\pm-\mu)](\omega+\mu+E_\pm-i\delta]}, \tag{29}$$
$$\Delta_\pm=\Sigma\pm\Delta_t,\quad \Sigma=\lambda+\Delta_s,$$

where the sign + corresponds to $\alpha = 1$, the sign − corresponds to $\alpha = -1$, $\mu$ is the displacement of the chemical potential because of doping, and

$$\Delta_s=-\frac{ig}{2(2\pi)^4}\int d\omega\,d\mathbf{p}\{G_{21}^{11}(\mathbf{p},\omega+i0)+G_{21}^{-1,-1}(\mathbf{p},\omega+i0)\},$$
$$\Delta_t=-\frac{ig}{2(2\pi)^4}\int d\omega\,d\mathbf{p}\{G_{21}^{11}(\mathbf{p},\omega+i0)-G_{21}^{-1,-1}(\mathbf{p},\omega+i0)\}. \tag{30}$$

On substituting in the condition for compatibility of (30) the functions $G_{21}^{\alpha\alpha}$ from (29), we get the equations that determine the order parameters $\Delta_S$ and $\Delta_t$:

$$\frac{\Delta_s}{gN(0)}=\frac{1}{2}\left\{\Delta_+\ln\frac{2\tilde{\omega}}{\mu+(\mu^2-\Delta_+^2)^{1/2}}+\Delta_-\ln\frac{2\tilde{\omega}}{\mu+(\mu^2-\Delta_-^2)^{1/2}}\right\},$$
$$\frac{\Delta_t}{gN(0)}=\frac{1}{2}\left\{\Delta_+\ln\frac{2\tilde{\omega}}{\mu+(\mu^2-\Delta_+^2)^{1/2}}-\Delta_-\ln\frac{2\tilde{\omega}}{\mu+(\mu^2-\Delta_-^2)^{1/2}}\right\}; \tag{31}$$

here N(0) is the density of states at the Fermi quasi-level; it is proportional to the intensity of the pumping. The system of equations (31) has been written for the case $\mu > |\Delta_+|, |\Delta_-|$. To equations (31) must be appended the condition for electrical neutrality, which in this case has the form

$$(\mu^2-\Delta_-^2)^{1/2}+(\mu^2-\Delta_+^2)^{1/2}=2n, \tag{32}$$

where $n = N/4VN(0)$ is the impurity concentration expressed in energy units.

Equations (31) can be reduced to the form

$$\Delta_\pm\ln\frac{\mu+(\mu^2-\Delta_\pm^2)^{1/2}}{\Delta_0}=\frac{\lambda}{gN(0)}, \tag{33}$$

where $\Delta_0 = 2\tilde{\omega}e^{-1/gN(0)}$.

We obtain similarly the equations for $\Delta_+$ and $\Delta_-$ in the case $|\Delta_-| < \mu < |\Delta_+|$,

$$\Delta_+\ln\frac{|\Delta_+|}{\Delta_0}=\frac{\lambda}{gN(0)}, \tag{34}$$

$$\Delta_-\ln\frac{\mu+(\mu^2-\Delta_-^2)^{1/2}}{\Delta_0}=\frac{\lambda}{gN(0)} \tag{35}$$

and the condition for electrical neutrality,

$$(\mu^2-\Delta_-^2)^{1/2}=2n. \tag{36}$$

In the absence of doping ($\mu = n = 0$), the equations for $\Delta_+$ and $\Delta_-$ are identical and independent:

$$\Delta_\pm\ln\frac{|\Delta_\pm|}{\Delta_0}=\frac{\lambda}{gN(0)}. \tag{37}$$

It is easy to show that for positive $\Delta_+$ and $\Delta_-$, when $\lambda \neq 0$, equations (37) have only nonmagnetic solutions ($\Delta_- = \Delta_+ = \Sigma$, $\Delta_t = 0$). If $\Delta_- < 0$ (that is, $\Delta_t > \Delta_S + \lambda$), then for $\lambda < \Delta_0 gN(0)/e$ the system of equations (37) permits ferromagnetic solutions (curve 1 for $\Delta_t$ and 1′ for $\Sigma$ in Fig. 5). We note that when $\lambda = 0$, there is a ferromagnetic solution of the system (37) when $\Delta_-, \Delta_+ \geq 0$ ($\Delta_- = 0$, $\Delta_+ = \Delta_0$).

When n is different from zero, the system (32), (33), in addition to the nonmagnetic solutions, has ferromagnetic solutions also when there is a certain relation between n and $\lambda$. We shall find the line in the (n, $\lambda$) plane that separates the regions of nonmagnetic and of ferromagnetic solutions (the line $\Delta_t = 0$). From equations (32) and (33) we get the system of equations that determines the dependence of $\lambda$ on n and the value of $\Sigma$ when $\Delta_t = 0$:

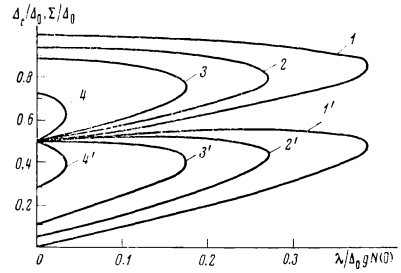


FIG. 5

$$\Sigma \ln \frac{n+(n^2+\Sigma^2)^{1/2}}{\Delta_0} = \frac{\lambda}{gN(0)}, \qquad \frac{\Sigma^3}{n[n+(n^2+\Sigma^2)^{1/2}]} = \frac{\lambda}{gN(0)}, \tag{38}$$

Curve 1 in Fig. 6 shows the dependence of n on λ obtained by numerical solution of the system of equations (38). We shall find the line in the (n, λ) plane that separates the regions of weak ($\mu > |\Delta_+|$) and strong ($|\Delta_-| < \mu < |\Delta_+|$) ferromagnetism. When n and λ lie on this line, $\Delta_+ = \mu$. Therefore on setting $\mu = \Delta_+$ in equations (32) and (33), we obtain after some transformations

$$\Delta_+ \ln \frac{|\Delta_+|}{\Delta_0} = \frac{\lambda}{gN(0)}, \qquad (\Delta_+^2+4n^2)^{1/2} \ln(\Delta_+ + 2n) = \frac{\lambda}{gN(0)}. \tag{39}$$

By numerical solution of the system of equations (39) we find the desired line (curve 2 in Fig. 6).

Thus in the (n, λ) plane (see Fig. 6) three regions can be distinguished: a) above curve 1 in Fig. 6, a region of nonmagnetic solutions; b) between curves 1 and 2, where along with nonmagnetic solutions there are solutions with $\mu > |\Delta_+|$ (a region of weak ferromagnetism); c) below curve 2, where in addition to nonmagnetic solutions there are solutions with $|\Delta_-| < \mu < |\Delta_+|$ (a region of strong ferromagnetism).

When λ = 0, Eqs. (33)–(35) go over to the corresponding equations of a previous paper[2] on the diagonal ($g_S = g_t$). In this case there are solutions corresponding only to strong ferromagnetism, which are more favorable than the nonmagnetic. In the general case with λ and n different from zero, the systems of equations (32), (33) and (34)–(36) were solved numerically. For the case $\Delta_- < 0$ (that is, $\Delta_t > \lambda + \Delta_S$), graphs of the dependence of $\Delta_t$ and Σ on λ are shown in Fig. 5 (curves 1, 2, 3, 4 correspond to the parameter $\Delta_t$, curves 1′, 2′, 3′, 4′ to the parameter Σ for $|\Delta_-| < \mu < |\Delta_+|$ and for values of $n/\Delta_0$ equal to 0, 0.1, 0.2, 0.4 respectively). It can be shown that when $\Delta_- < 0$, the system of equations (32), (33) corresponding to $\mu < |\Delta_+|$, $|\Delta_-|$ has no solutions.

Figure 7 shows the dependences of $\Delta_t$ and of Σ on n for various values of λ in the case $\Delta_- > 0$ (curves 1, 2, 3, 4 correspond to the parameter Σ, curves 1′, 2′, 3′, 4′ to the parameter $\Delta_t$ for values of $\lambda/\Delta_0 gN(0)$ equal to 0, 0.1, 0.5, and 1.0 respectively; the dotted lines correspond to nonmagnetic solutions).

We shall consider, for example, curves 2 and 2′ in Fig. 7. For $n/\Delta_0 < 0.544$ they describe a solution of the system (34)–(36); that is, they correspond to "strong" ferromagnetism. The system of equations (32), (33) has no solutions in this n interval. For $n/\Delta_0 = 0.544$ the displacement of the chemical potential $\mu = \Delta_+$, and for larger values of n the system of equations (34)–(36) has no solutions. Starting with $n/\Delta_0 = 0.544$ there appears a solution of the system (32), (33) ($\mu >$

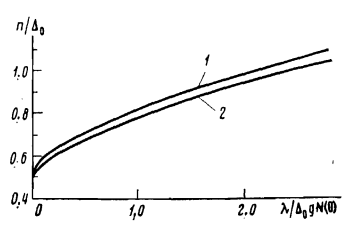


FIG. 6

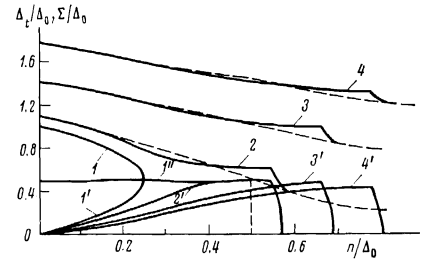


FIG. 7

$|\Delta_+|$, "weak" ferromagnetism); and when $n/\Delta_0 \approx 0.574$, $\Delta_t$ vanishes. For $n/\Delta_0 > 0.574$ there are only nonmagnetic solutions.

Curves 1, 1′, 1″ in Fig. 7 correspond to λ = 0 and are completely analogous to the case of an exciton dielectric[2]. From a comparison of these curves with curves 4, 4′; 2, 2′; 3, 3′ ($\lambda \neq 0$) it is seen that when $\lambda \neq 0$ the n region for magnetic solutions increases; instead of the discontinuous simultaneous disappearance of $\Delta_S$ and $\Delta_t$ when λ = 0, the parameter $\Delta_t$ disappears smoothly when $\lambda \neq 0$, whereas Σ meanwhile remains finite, approaching zero for $n \to \infty$. We remark that in the case of an equilibrium exciton dielectric (in the absence of pumping), the role of the parameter λ is qualitatively analogous to the effect of interband hybridization.

The expression for the change of energy δE in unit volume in our case is completely analogous to the corresponding expression for an exciton dielectric[2]. From a comparison of the energies for different solutions of the equations for $\Delta_t$ and Σ it is evident that for $n < n_1$ ($n_1 \approx 0.54\Delta_0$ for $\lambda/\Delta_0 gN(0) = 0.1$; $n_1 = 0.63\Delta_0$ for $\lambda/\Delta_0 gN(0) = 0.5$; $n_1 = 0.71\Delta_0$ for $\lambda/\Delta_0 gN(0) = 1.0$), the solutions of equations (34), (36), corresponding to $\Delta_-, \Delta_+ > 0$, are more favorable than either the nonmagnetic or the ferromagnetic solutions, corresponding to $\Delta_- < 0$ ($n_1$ is less than the concentration at which there occurs a transition from strong to weak ferromagnetism). For $n > n_1$, the most favorable of all is the nonmagnetic solution.

Thus in a doped semiconductor with pumping, states with $\Delta_S + \lambda > \Delta_t$ will be realized; when $0 < n < n_1$, the energetically favorable state is that of strong ferromagnetism, $|\Delta_-| < \mu < |\Delta_+|$, whereas when $n > n_1$ it is the nonmagnetic saturation mode[5]; that is, the state of weak ferromagnetism is not realized under any conditions, just as in the case of an exciton dielectric[2].

We note that the possibility, which we have studied in Sec. 3, of magnetic ordering (spin orientation) of nonequilibrium electrons in a semiconductor with a large gap $E_g$ differs importantly from the case, investigated experimentally[18] and theoretically[19] for semiconductors with a peculiar band structure, in which dipole transitions under the action of a circularly polarized electromagnetic wave are different in states with spin projections 1/2 and −1/2. In our model, such transitions were supposed to be equally probable, and furthermore the electromagnetic wave was supposed to be unpolarized. The spin orientation arose as a manifestation of a collective effect, absent in the single-electron (band) approximation; therefore in our case it remains as long as the pumping source is acting. But under the conditions of the papers of Zakharchenya et al.[18] and of D'yakonov and Perel'[19], the orientation exists only for

times of the order of the spin-relaxation times.

In the papers of Kokin and Popovkin[20], as distinguished from the paper of D'yakonov and Perel'[19], the electromagnetic field is supposed to be strong and is therefore taken into account exactly, just as in section 3 of the present paper. Furthermore, in [20] it was deduced qualitatively that just one singlet electron-hole pairing because of Coulomb interaction in the scheme of D'yakonov and Perel' (see [19]) leads to a diminution of the degree of orientation produced by an electromagnetic field of circular polarization.

The effects, studied in the present paper, of spin ordering of nonequilibrium excitations can be detected experimentally both by magnetic measurements and from the character of the frequency spectrum and polarization of luminescence. In the spectrum of the radiation there should be two dips in the frequency dependence of the intensity, corresponding to the parameters $\Delta_+$ and $\Delta_-$ in the electron spectrum. In distinction from the experiments of Zakharchenya et al.[18], the polarization of the radiation should take place during the entire time of action of the pump, and not only at the instant when it is turned on. Furthermore, the magnetic ordering produced by the collective effects, which we have considered, of coexistence of singlet and triplet electron-hole pairings should, with rise of temperature, disappear by phase transition.


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