Quantum spin waves in ferromagnets

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The ground state and the excitation spectrum of the electron liquid in ferromagnetic metals located in a quantizing magnetic field are considered. A material equation determining the dependence of the magnetization on the magnetic field strength is deduced for arbitrary temperatures. The quasiclassical-magnon and quantum-spin-wave spectra are obtained for arbitrary wavelengths in the case of propagation of the oscillations along the direction of magnetization. It is shown that the character of the quantum-wave spectrum depends on the location of the transparency windows in which Landau damping is negligible. If the magnon quasiclassical dispersion curve falls in the transparency windows, giant oscillations of magnon damping occur. In the opposite case the quantum dispersion curves lie near the window boundaries. The most favorable conditions for observation of quantum spin waves are realized near the quasiclassical dispersion curve.

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1. The quantization of the orbital motion of electrons in a magnetic field makes it impossible, under certain conditions, to obtain the collision-free damping of the waves due to the Cerenkov effect on the electrons (Landau damping). This effect appears in giant oscillations of the wave absorption and in the generation of new branches of excitations of the solid—quantum waves (see, for example,[1,2]). Among such new excitations, we must note quantum spin waves (QSW), the theory of which, as applied to the electron liquid of normal metals, was developed in[3,4]. However, spin waves in normal metals have been studied comparatively little. On the other hand, spin waves in ferromagnetic metals have been the object of an extraordinarily broad range of investigations. In this connection, special interest should be attached to the quantum spin waves in ferromagnets, the theory of which is developed in this paper.

Two possibilities of the appearance of quantization of orbital motion of electrons in the laws for the propagation of magnons are manifest, depending on the location of the regions of prohibition or, at finite temperatures, on the location of regions of significant suppression of the Landau damping, which can be called magnon transparency windows. To be precise, if the quasiclassical dispersion curve of the magnons (see, for example,[5] crosses the transparency window, then giant oscillations appear in the magnon damping, similar to those predicted for ultrasound.[1] On the other hand, if the quasiclassical dispersion curve does not fall in the transparency window, then the ordinary magnons cannot propagate. However, the existence of quantum spin waves turns out to be possible in this case. The dispersion curves of these waves are close to the boundaries of the transparency window.

2. To consider the properties of the quantum waves in a ferromagnetic metal, we first formulate the quantum dispersion equation. Such a problem can be solved only with the use of definite model representations, as is clear from the classical theory (cf.[5,6]). We shall assume below that the energy of the electron in the absence of a quantizing magnetic field depends quadratically on the momentum, while the exchange interaction between the electrons is characterized by a single Fermi-liquid constant (the contact interaction approximation). We then have for the energy of the electrons

$$\varepsilon_n^\sigma(p)=\varepsilon_n(p)-\sigma b,$$

where p is the projection of the momentum of the electron on the direction of the magnetic field B, σ is the spin variable (± 1), and n an integer. In this case,

$$\varepsilon_n(p)=(n+{}^1/_2)\hbar\Omega+p^2/2m,$$

where Ω is the cyclotron frequency and m is the effective mass.

The exchange energy b, in an approximation in which it is small in comparison with the Fermi energy of the electron ϵ_F at zero temperature ($T = 0$), is connected with the constant B_0 of Fermi-liquid interaction in the following way:

$$B_0=-\left[1+\frac{1}{24}\left(\frac{b}{\varepsilon_F}\right)^2-\frac{\Omega_B}{\Omega_0}\right], \tag{1}$$

where $\Omega_0 = 2b/\hbar$ is the exchange frequency, $\Omega_B = 2\mu B/\hbar$, B is the magnetic induction in the ferromagnet, μ is the magnetic moment of the electron, $B_0 = \psi p_F^2/\pi^2\hbar^3 v_F$,[2] and $p_F = mv_F = (2m\epsilon_F)^{1/2}$ is the Fermi momentum of the electrons.

That term in the right side of Eq. (1) which depends on the magnetic induction B plays an important role in ferromagnets even in the absence of an external magnetic field, because of the presence of a strong internal field[7,8] $B = 4\pi M$ due to spontaneous magnetization.

At finite temperatures, the equation connecting the exchange energy $b(T)$ with the constant B_0 differs from (1) in the presence on the right hand side of the term $-(\pi^2/24)(T/\epsilon_F)^2$ (cf[9]). Accordingly, it is not difficult to obtain the following equation for the magnetization per unit volume $M = -\mu b/\psi$

$$\alpha M+\beta M^3=H, \tag{2}$$

where

$$\alpha=-\frac{\psi}{\mu^2}\left[1+B_0+\frac{4\pi\mu^2}{\psi}+\frac{\pi^2}{24}\left(\frac{T}{\varepsilon_F}\right)^2\right],\quad \beta=-\psi^3/24\mu^4\varepsilon_F^2, \tag{3}$$

and $H = B - 4\pi M$ is the magnetic field strength in the ferromagnet. This equation for the magnetization determines its temperature and magnetic field strength (paraprocess) dependence and corresponds to the thermodynamic theory of ferromagnetic transition (see, for example,[10]).

We shall be interested in transverse-polarization spin waves propagating in a degenerate ferromagnetic electron liquid along the direction of constant magnetization. In the approximation $\omega \ll kv_F(\hbar k/c/\epsilon_F)^2$, the spectrum of such waves is determined by the singularities of the magnetic susceptibility $\chi^\pm$ (see, for example,[11]). Using the general relations obtained in[3], we can write down the following expression for the nonequilibrium magnetization ($\sim e^{-i\omega t+ikz}$):

$$m^\pm=-\frac{\mu^2S^\pm(\omega,k)}{1-\psi S^\pm(\omega,k)}b^\pm,$$

where $m^\pm = m_x \pm im_y$, $b^\pm = b_x \pm ib_y$ are the transverse components of the nonequilibrium magnetization and of the magnetic field,

$$S^\pm(\omega,k)=\frac{2eB}{c}\sum_n\int\frac{dp}{(2\pi\hbar)^2}\frac{n[\varepsilon_n^+(p\mp\hbar k)]-n[\varepsilon_n^-(p)]}{\pm\hbar\omega+\varepsilon_n^+(p\mp\hbar k)-\varepsilon_n^-(p)\pm i0}$$

Here $n(\epsilon)$ is the equilibrium distribution function of Fermi particles.

From this it is now easy to obtain an expression for the magnetic susceptibility

$$\chi^\pm(\omega,k)=-\mu^2S^\pm(\omega,k)\{1-[\psi-4\pi\mu^2]S^\pm(\omega,k)\}^{-1},$$

which allows us to write down the spin-waves dispersion equation that connects the frequency ω and the wave vector $\mathbf{k}$:

$$(\psi-4\pi\mu^2)S^\pm(\omega,k)=1.$$

Taking into account the fact that $-4\pi\mu^2/\psi = 4\pi\mu M/b \equiv \Omega_M/\Omega_0$, we can write down the dispersion equation of spin waves of left polarization at zero temperature in the following explicit form:

$$D(\omega,k)=0,$$

$$D(\omega,k)=\frac{\Omega}{2kv_F}\sum_{\sigma=\pm1}\sum_{n=0}^{N^\sigma}\sigma\left[\ln\left|\frac{\omega-\Omega_0+kv^\sigma(n)-\sigma\hbar k^2/2m}{\omega-\Omega_0-kv^\sigma(n)-\sigma\hbar k^2/2m}\right|-i\pi\int\limits_{-kv^\sigma(n)}^{kv^\sigma(n)}\delta(\omega-\Omega_0-\sigma\hbar k^2/2m-x)\,dx\right]-B_0^{-1}\left(1-\frac{\Omega_M}{\Omega_0}\right). \quad (4)$$

Here N^σ is the number of levels in the magnetic field occupied by particles with spin σ, $v^\sigma(n) = \{(2/m)[\epsilon_F + \sigma b - (n + \frac{1}{2})\hbar\Omega]\}^{1/2}$ is the longitudinal velocity of particles with spin σ on the Fermi surface. The dispersion relation for waves of right polarization differs from (4) in the sign of ω.

For the problem of interest to us, the decisive role is played by those regions on the (ω, k) plane for which the imaginary part of Eq. (4) vanishes. We first observe that the imaginary part of the dispersion equation is equal to zero in the region

$$\omega<\Omega_0-kv^+(0)+\hbar k^2/2m, \quad (5)$$

which corresponds to the result of quasiclassical theory that there is no collision-free damping of the magnons. However, along with this, the imaginary part of the dispersion equation also vanishes in regions determined by the inequalities

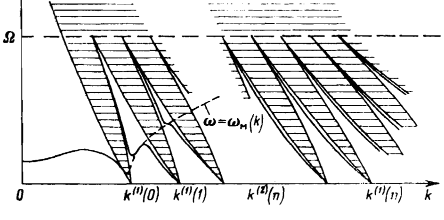

FIG. 1

$$\Omega_0-kv^-(n-1)-\hbar k^2/2m<\omega<\Omega_0-kv^+(n)+\hbar k^2/2m \quad (n=1,2,\dots N^-). \quad (6)$$

We assume that $\Omega_0 \gg \Omega$ and, consequently, $v^+(n) > v^-(n-1)$. In the (ω, k) plane, the boundaries of the regions (6), for which we use the name transparency window, begin with the axis $\omega = 0$, where they are bounded by the points $k^{(1)}(n) = m[v^+(n) - v^-(n)]/\hbar$, and end with $\omega = \Omega$ at the points $k^{(2)}(n) = m[v^+(n) - v^-(n-1)]/\hbar$, which determine the minimum possible value of the wave vector in the transparency window. In Fig. 1 the regions of absence of Landau damping are not shaded. The transparency window on the extreme right of Fig. 1 is determined by the inequalities

$$\omega>\Omega_0-kv^-(N^-)-\hbar k^2/2m,\quad \omega<\Omega_0+kv^-(N^-)-\hbar k^2/2m,$$
$$\omega<\Omega_0-kv^+(N^-+1)+\hbar k^2/2m, \quad (7)$$

from which it follows that the maximum value of k in the transparency region corresponds to $\omega = 0$ and is equal to $k_m = m[v^+(N^-) + v^-(N^-)]/\hbar$. The left and right boundaries for $\omega = \Omega$ correspond respectively to the values $k^{(2)}(N^- + 1)$ and $\tilde{k} = m[v^+(N^- + 1) + v^-(N^-)]/\hbar$. The curve that forms the upper boundary of the transparency window (5) has a minimum at the point $k = mv^+(N^- + 1)/\hbar$, equal to $\Omega - (\epsilon_F/\hbar)[v^-(N^-)/v_F]^2$. The value of the wave vector in the transparency windows changes from the value $k \approx (\mathbf{k}/\epsilon_F)p_F/\hbar$ in windows with small $n \ll N^-$ to $k \approx (2b/\epsilon_F)^{1/2}p_F/\hbar$ in windows with $n \sim N^-$.

Outside the transparency regions (5)—(7), the imaginary part of the dispersion equation turns out to be comparable with the real part, which corresponds to solutions of the dispersion equation corresponding to strongly damped waves. In this connection, we shall concentrate our attention below on determining the consequences that follow from the dispersion equation (4) in the regions (5)—(7).

3. The possibility of the existence in the transparency windows (6) and (7) of solutions of the dispersion equation (4) corresponding to undamped waves follows from the fact that, inside the transparency window, the left side of Eq. (4) changes from $-\infty$ on the right boundary to $+\infty$ on the left. This property of the dispersion equation, along with general confirmation of the existence of quantum waves in ferromagnets, reveals a way for estimating the approximate solutions in the transparency windows.

We separate out those components in Eqs. (4) which become infinite on the boundaries of the n-th transparency window ($D_{qc}(\omega, k)$) (qc = quasi classical). We calculate the sum of the remaining components approximately, replacing the summation by integration ($D_{cl}(\omega, k)$). As a result, in the region of absence of collision-free damping, the dispersion equation takes the form

$$D_{qc}(\omega,k)+D_{cl}(\omega,k)=0, \quad (8)$$

where

$$D_{qc}(\omega,k)=\frac{\Omega}{2kv_F}\left\{\ln\left|\frac{[\omega-\Omega_0+kv^+(n)-\hbar k^2/2m][\omega-\Omega_0-kv^-(n)+\hbar k^2/2m]}{[\omega-\Omega_0-kv^+(n)-\hbar k^2/2m][\omega-\Omega_0+kv^-(n)+\hbar k^2/2m]}\right|+\ln\left|\frac{[\omega-\Omega_0+kv^+(n-1)-\hbar k^2/2m][\omega-\Omega_0-kv^-(n-1)+\hbar k^2/2m]}{[\omega-\Omega_0-kv^+(n-1)-\hbar k^2/2m][\omega-\Omega_0+kv^-(n-1)+\hbar k^2/2m]}\right|\right\}, \quad (9)$$

$$D_{cl}(\omega,k)=\frac{p_F}{4\hbar k}\left\{\left[\left(\frac{v^+}{v_F}\right)^2-\left(\frac{\omega-\Omega_0-\hbar k^2/2m}{kv_F}\right)^2\right]\times\ln\left|\frac{\omega-\Omega_0+kv^+-\hbar k^2/2m}{\omega-\Omega_0-kv^+-\hbar k^2/2m}\right|-\left[\left(\frac{v^-}{v_F}\right)^2-\left(\frac{\omega-\Omega_0+\hbar k^2/2m}{kv_F}\right)^2\right]\right.$$

$$\times \ln \left| \frac{\omega-\Omega_0+kv^-+\hbar k^2/2m}{\omega-\Omega_0-kv^-+\hbar k^2/2m} \right| +2\frac{v^+}{v_F}\frac{\omega-\Omega_0-\hbar k^2/2m}{kv_F} - 2\frac{v^-}{v_F}\frac{\omega-\Omega_0+\hbar k^2/2m}{kv_F} \Big\} - B_0^{-1}\left(1-\frac{\Omega_M}{\Omega_0}\right). \quad (10)$$

Here $v^{\pm} = v_F(1 \pm b/\epsilon_F)^{1/2}$. The error connected with the approximate calculation of the sum in the dispersion equation (4) does not exceed $(\hbar\Omega/\epsilon_F)(kv_F/\Omega_0)^2$.

In the classical region (5), the effect of $D_{qc}(\omega, k)$ on the spectrum of spin waves is unimportant and the dispersion equation can be written approximately in the form

$$D_{cl}(\omega, k)=0. \quad (11)$$

We shall say that Eq. (11) determines the quasiclassical spectrum of the magnons ($\omega_M(k)$). Not too far from the point $k = k_0$, which corresponds to the intersection of the boundaries of region (5) with the straight line $\omega = 0$, in the broad range of wavelengths

$$\frac{|k_0-k|}{2k_0}\left[\ln\left|\frac{2k_0}{k_0-k}\right|\right]^2 \gg \frac{\Omega_H}{\Omega_0}+\frac{1}{12}\left(\frac{b}{\varepsilon_F}\right)^2 \quad (12)$$

the following comparatively simple expression for the magnon frequency can be obtained:

$$\omega_M(k) = \left[\Omega_H+\frac{\Omega_0}{12}\left(\frac{\hbar k}{p_F}\right)^2\right]\frac{2k}{k_0}\Big/\ln\left|\frac{k_0+k}{k_0-k}\right|, \quad (13)$$

where $\Omega_H = 2\mu H/\hbar$.

Using the condition (5), it is not difficult to obtain the following expression for k_0:

$$k_0 = m(v^+-v^-)/\hbar = \frac{\Omega_0}{v_F}\left[1+\frac{1}{8}\left(\frac{b}{\varepsilon_F}\right)^2\right]. \quad (14)$$

The second term in the right side of (14) is due to quantum effects of the finite character of the momentum $\hbar k$ and turns out to be important near the boundaries of the region (5):

In the longwave limit $(kv_F)^2 \ll \Omega_0^2$ the well known expression for the low-frequency magnon spectrum follows from (13) (see, for example,[5,6]):

$$\omega_M(k) = \Omega_H + {}^1/_{12}\Omega_0(\hbar k/p_F)^2. \quad (15)$$

We note here that the approach based on the use in[5] of the kinetic equation does not take into account the quantum (in the sense described above) effects and enables us to consider only the longwave excitation of the type (15). On the other hand, Eqs. (11) and (13) evidently take the quantum effects into account[3] (see also[6]).

In correspondence with Fig. 1, the quasiclassical magnon spectrum, as is seen from Eq. (13), at first (with increase in k) corresponds to an increase in the frequency, the frequency reaching the maximum value

$$\omega_M \approx \frac{4}{3}\left[\Omega_H+\frac{\Omega_0}{12}\left(\frac{b}{\varepsilon_F}\right)^2\right]\Big/\ln\left(\frac{1}{a}\right) \quad (16)$$

at $k \approx [1-(4/3)a/\ln(1/a)]\Omega_0/v_F$. Here $a = [\Omega_H + (1/12)\Omega_0(b/\epsilon_F)^2]/2[\Omega_H + 1/4\,\Omega_0(b/\epsilon_F)^2] \approx 1/6$ to $1/4$—is a parameter that generally depends on the magnetic field strength. In addition, as k approaches the value $k = k_0$, the frequency of the magnons decreases according to the law

$$\omega_M(k) = 2\left[\Omega_H+\frac{\Omega_0}{12}\left(\frac{b}{\varepsilon_F}\right)^2\right]\Big/\ln\left|\frac{2k_0}{k_0-k}\right|. \quad (17)$$

The dispersion curve of the magnons crosses the boundary of the classical region (5), in the absence of Landau damping, at the point $k = k^+$, which is determined by the equation

$$\frac{k_0-k^+}{2k_0}\left[\frac{v^-}{v_F}\ln\left|\frac{2\varepsilon_F}{b}\frac{k_0}{k_0-k^+}\right|+1\right] = \frac{\Omega_H}{\Omega_0}+\frac{1}{12}\left(\frac{b}{\varepsilon_F}\right)^2. \quad (18)$$

For the frequency of the magnons at this point, the following equation holds:

$$\frac{\omega}{2\Omega_0}\left[\ln\left|\frac{2\varepsilon_F}{b}\frac{\Omega_0}{\omega}\right|+1\right] = \frac{\Omega_H}{\Omega_0}+\frac{1}{12}\left(\frac{b}{\varepsilon_F}\right)^2. \quad (19)$$

The dispersion equation (8) reduces to the quasiclassical equation (11) not only in the region (5), but also relatively far (see below for further detail) from the boundary of the transparency window, when the component $D_{qc}(\omega, k)$ in Eq. (8) turns out to be negligibly small. Therefore, it is convenient to follow the path of the quasiclassical dispersion curve even in the classical region.

After crossing the boundary of the classical region (5), the quasiclassical dispersion curve passes through a minimum located in the region of strong damping, and then crosses the left boundary of the first transparency window. The value $k = k^-$ corresponding to the point of such an interaction is determined by Eq. (8), in which we must replace v^- by v^+. The frequency of the magnons at this point is given by Eq. (19), the approximate solution of which can be written in the form

$$\omega_M(k^{\pm}) \approx 2\left[\Omega_H+\frac{\Omega_0}{12}\left(\frac{b}{\varepsilon_F}\right)^2\right]\Big/\left[\ln\left|\frac{2\varepsilon_F}{b}\frac{\Omega_H}{\Omega_0}+\frac{1}{6}\frac{b}{\varepsilon_F}\right|+1\right]. \quad (20)$$

With increase in k, when the condition (12) is satisfied, the behavior of the quasiclassical frequency of the magnons is determined by Eq. (13). As follows from (13) and (19), the frequency of the magnons remains approximately equal to the value (20) in the region of wavelengths

$$\left|\frac{k-k_0}{k_0}\right| \leqslant \frac{1}{8}\left(\frac{b}{\varepsilon_F}\right)^2 \quad (21)$$

corresponding to transparency windows with $n \lesssim (b/\epsilon_F)^2 N^-/8$.

It must be emphasized that in reality the quasiclassical dispersion curve crosses the transparency windows only in the case in which

$$\omega_M(k) < \Omega. \quad (22)$$

This corresponds to the requirement of smallness of the right side of Eq. (20) in comparison with the cyclotron frequency Ω of the electrons. Such a requirement can be satisfied also in the absence of an external magnetic field, since $4\pi M \sim 2 \times 10^4$ Ga[8] and $b/\epsilon_F \sim 10^{-1} - 10^{-2}$ for typical ferromagnets.

We note here that even more favorable conditions can exist for satisfaction of the inequality (22) in metals with anisotropic Fermi surfaces, when the cyclotron mass of the electrons turns out to be less than the effective mass characterizing the motion of the electron along the direction of magnetization. Such a situation is possible for metals of the iron group.[12] Here the range of values of k for which the quasiclassical curve lies in the transparency window is broader. The limitation of the possibility of such a position at large values of k follows from Eq. (13) which, for $(kv_F)^2 \gg \Omega_0^2$ gives

$$\omega_M(k) = \left[\Omega_H+\frac{\Omega_0}{12}\left(\frac{\hbar k}{p_F}\right)^2\right]\left(\frac{kv_F}{\Omega_0}\right)^2. \quad (23)$$

For example, inasmuch as, in windows with $n \sim N^-$, $(kv_F/\Omega_0)^2 \approx 2\epsilon_F/b$, it turns out that here $\omega_M \approx \Omega_0/3$. Under these conditions, the inequality (22) is violated; it is then clear that for a window with a large number n, crossings with the quasiclassical dispersion curve do not occur. The maximum value of the wave number for which there is a possibility of such a crossing is determined by equating the expression (13) to the cyclotron frequency.

In correspondence with established terminology (see, for example,[1,13]), the crossing of the quasiclassical curve $\omega_M(k)$ with the transparency window can be accompanied by giant quantum oscillations of the absorption of the magnons. We shall associate the new branches of spin waves, which arise in the transparency windows, with quantum waves.

4. We now discuss the form of the dispersion curves determined by the quantum equation (8) inside the transparency window. We begin such a discussion with the case of comparatively small n, when the inequality (22) can be satisfied. Then, far from the boundaries of the transparency window, the spectrum of quantum waves turns out to be close to the quasiclassical $\omega_M(k)$, and upon approach to the boundaries of the window, the dispersion curve of the quantum spin waves turns out to be very close to the corresponding boundary curve. Upon increase in k, the dispersion curve, beginning at the points $\omega = \Omega$ and $k = k^{(2)}(n)$, corresponds to a falling off of the frequency according to the law

$$\omega=\omega^-(n)+\frac{\hbar k}{m}\frac{[k-k^{(2)}(n)][k^{(1)}(n)-k]}{k^{(1)}(n)-k^{(2)}(n)}\left[\frac{v^-(n-1)}{v^+(n)}\right]^2\times\exp\left\{\frac{2kv_F}{\Omega}D_{\rm cl}[\omega^-(n),k]\right\}, \qquad (24)$$

where $\omega^-(n) = \Omega_0 - kv^-(n-1) - \hbar k^2/2m$ determines the left boundary of the transparency window (6).

Upon approach of the curve (24) to the quasiclassical dispersion curve, the condition of applicability of formula (24) breaks down:

$$|D_{\rm cl}(\omega,k)|\gg\hbar\Omega/\varepsilon_F. \qquad (25)$$

Therefore, the falling off of the frequency is slowed and the quantum dispersion curve merges with the quasiclassical one. The dispersion curve then approaches the right boundary of the transparency window with increase in the wave vector according to the law

$$\omega=\omega^+(n)-\frac{\hbar k}{m}\cdot\frac{[k^{(1)}(n)-k][k-k^{(2)}(n)]}{k^{(1)}(n-1)-k^{(2)}(n-1)}\left[\frac{v^+(n)}{v^-(n-1)}\right]^2\times\exp\left\{-\frac{2kv_F}{\Omega}D_{\rm cl}[\omega^+(n),k]\right\}, \qquad (26)$$

where $\omega^+(n) = \Omega_0 - kv^+(n) + \hbar k^2/2m$ determines the right boundary of the window. The condition of applicability of formula (26) is given by the inequality (25).

Under the conditions (12), $D_{\rm cl}(\omega, k)$ takes the form

$$D_{\rm cl}(\omega,k)=\frac{\omega_M(k)-\omega}{2kv_F}\ln\left|\frac{k+k_0}{k-k_0}\right|, \qquad (27)$$

where $\omega_M(k)$ is given by Eq. (13). The inequality (25) here can be represented in the form

$$\frac{|\omega_M(k)-\omega^\pm(n)|}{\Omega}\gg\frac{4\hbar k}{p_F}\Big/\ln\left|\frac{k+k_0}{k-k_0}\right|.$$

The behavior of the dispersion curves of the quantum waves inside the transparency window, in their interaction with the quasiclassical dispersion curve, is illustrated in Fig. 1.

We now proceed to a discussion of the situation in which the quasiclassical dispersion curve does not fall in the transparency window. In this case, waves with frequency close to $\omega_M(k)$ cannot be propagated. However, the dispersion equation (8) in this case also has undamped solutions in the transparency windows, corresponding to some other type of quantum waves. To be precise, the dispersion curve of such waves in the transparency windows (6) lies near the right side of the window. The quantum spin wave spectrum is given by Eq. (26) everywhere in the window. Finally, on the edge of the right side of the window (7), the quantum dispersion curve diffuses to the upper $\omega^+(N^-+1) = \Omega_0 = kv^+(N^-+1) + \hbar k^2/2m$ and the right $\tilde\omega = \Omega_0 + kv^-(N^-) - \hbar k^2/2m$ boundaries. Near the upper boundary, the following expression can be obtained for the frequency of the quantum spin wave:

$$\omega=\frac{1}{2}[\omega^+(N^-+1)+\tilde\omega]-\left[\frac{1}{4}[\omega^+(N^-+1)-\tilde\omega]^2+16\Omega_0^2\frac{k-k^{(2)}(N^-+1)}{k^{(1)}(N^-)-k^{(2)}(N^-+1)}\exp\left\{-\frac{2\Omega_0}{\Omega}\frac{\omega_M(k)}{kv_F}\right\}\right]^{1/2}. \qquad (28)$$

The dependence of the frequency on k not too close to the upper boundary is given by the expression

$$\omega=\frac{1}{2}[\omega^+(N^-)+\tilde\omega]+\left[\frac{1}{4}[\omega^+(N^-)-\tilde\omega]^2-16\Omega_0^2\frac{\tilde k-k^{(2)}(N^-+1)}{k-\tilde k}\exp\left\{-\frac{2\Omega_0}{\Omega}\frac{\omega_M(k)}{kv_F}\right\}\right]^{1/2}, \qquad (29)$$

where $\omega^+(N^-) = \Omega_0 - kv^+(N^-) + \hbar k^2/2m$, and the frequency $\omega_M(k)$ is determined by the expression (23). The regions of applicability of Eqs. (28) and (29) overlap. The behavior of the quantum dispersion curves under conditions when the inequality (22) is not satisfied is shown schematically in Fig. 2.

5. Up to now, we have not taken into account the effects associated with the finiteness of the temperature and the scattering of quasiparticles. These effects limit the region of existence of the quantum waves, "diffusing" the boundaries of the transparency window. The spreading out of the boundaries Δ because of scattering has the order of magnitude

$$\Delta\sim\tau^{-1},$$

where τ is the relaxation time of the momentum of the quasiparticles. The temperature effects lead to the following value of the spreading out of the boundaries

$$\Delta\sim kT/mv^+(n).$$

The condition for the existence of quantum spin waves, the frequencies of which lie close to the quasiclassical dispersion curve, will be the smallness of the spreading out of the boundaries in comparison with the width of the transparency window $\sim k[v^+(n-1) - v^+(n)]$. This condition thus takes the form of the following two inequalities:

$$(\Omega\tau)^{-1}<2b/\varepsilon_F,\quad T<\hbar\Omega. \qquad (30)$$

For the existence of quantum waves with spectra (24), (26) and (28), (29), the dispersion curves of which spread out to the boundaries of the transparency windows, it is necessary to require the satisfaction of more rigorous conditions. Such conditions are the smallness of spreading out of the boundaries in comparison with the quantities $|\omega - \omega^\pm(n)|$, $|\omega - \tilde\omega|$:

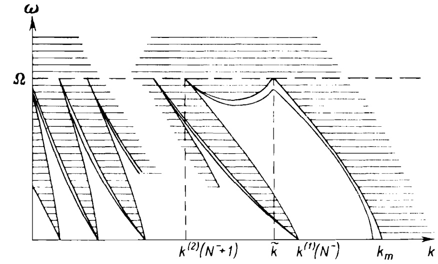

FIG. 2

$$(\Omega\tau)^{-1} \ll (b/\varepsilon_F)^2, \quad T \ll (b/\varepsilon_F)\hbar\Omega \qquad (31)$$

in transparency windows with numbers $n \ll N^-$, and

$$(\Omega\tau)^{-1} \ll \Omega/\Omega_0, \quad T \ll (\Omega/\Omega_0)\hbar\Omega \qquad (32)$$

in windows with $n \sim N^-$.

Thus the most favorable conditions for observation of quantum spin waves in ferromagnets exist near the quasiclassical dispersion curve of the magnons.

In conclusion, we note that in real ferromagnets, the condition (30) can be satisfied in sufficiently pure samples, where the content of impurities does not exceed 0.01–0.001% ($\tau \sim 10^{-9}$–10^{-10} sec)[7,8] at liquid helium temperatures ($T \lesssim 4.2$°K) and at magnetic fields $B \gtrsim 4\pi M \sim 2 \times 10^4$ Ga.

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