Collective excitations with a sound spectrum in superconductors

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It is shown theoretically that weakly-damped oscillations of the longitudinal electric field and of the phase of the order parameter, with spectrum $\omega = kV$, can exist in superconductors with low impurity concentrations. These excitations are associated with oscillations of the difference in the populations of the branches of the quasiparticle energy spectrum.

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It has been shown recently in a number of papers that a longitudinal electric field can exist in a superconductor. In a zero-gap superconductor the characteristic length scale of the variation of the field is the correlation length [1,2], while in an ordinary superconductor with a gap, where the presence of the electric field is associated with the difference in the populations of the branches with $\xi > 0$ and $\xi < 0$ ($\xi = p^2/2m - \epsilon_F$) in the quasi-particle energy spectrum, it is the length over which equilibrium between the branches is established [3,4]. Below we shall show that, near T_c in a superconductor with a gap ($\Delta \neq 0$), weakly-damped oscillations of the longitudinal electric field and of the phase of the order parameter can exist. In this case too, the appearance of the field is connected with the imbalance in the populations of the branches of the quasi-particle spectrum.

To find the characteristic oscillations of the system we shall make use of the kinetic equation obtained by Aronov and Gurevich [5], which is valid for perturbations with wave vector k and frequency ω such that ω, kv, $\nu \ll \Delta$, where v is the Fermi velocity and ν is the collision frequency:

$$\frac{\partial n_p}{\partial t}+\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{p}}\frac{\partial n_p}{\partial \mathbf{r}}-\frac{\partial \tilde{\varepsilon}}{\partial \mathbf{r}}\frac{\partial n_p}{\partial \mathbf{p}}+I(n_p)=0, \tag{1}$$

where

$$\tilde{\varepsilon}=[(\xi+\Phi)^2+\Delta^2]^{1/2}+\mathbf{p}\mathbf{v}_s,\quad \mathbf{v}_s=\frac{1}{2m}\nabla\chi,\quad \Phi=\frac{1}{2}\frac{\partial\chi}{\partial t}+e\varphi,$$

v_S is the superfluid velocity, χ is the phase of the order parameter and Φ and φ are the gauge-invariant and electric potentials. We assume that the potential **A** is equal to zero, since perturbations of the current, or, consequently, of the magnetic field, do not occur in the oscillations under consideration.

Assuming that the quantities Φ and v_S arising during the oscillations are proportional to $\exp(-i\omega t + ikx)$, we shall seek the solution of Eq. (1) in the form $n_p = n_0(\tilde{\epsilon}) + n_+ + n_-$, where n_+ and n_- are the corrections to the Fermi distribution function $n_0(\tilde{\epsilon})$ that are symmetric and antisymmetric in the momentum p_x. We shall linearize (1) with respect to Φ and v_S. We shall assume the frequency ω to be large compared with the frequency of the energy relaxation, and, therefore, in the linearized collision integral we shall retain only the expression νn_- corresponding to the elastic scattering, where we assume that ν is independent of ϵ. For frequencies $\omega \ll \nu$ we have

$$\begin{aligned} n_+&=(\xi/\varepsilon)\,\omega n_0'\left[i\nu\Phi+\frac{kmv^2}{3}v_s\right]D^{-1}\\ n_-&=v\mu\omega n_0'\left[k\left(\frac{\xi}{\varepsilon}\right)^2\Phi+m\omega v_s\right]D^{-1}, \end{aligned} \tag{2}$$

where

$$D=(k^2v_r^2/3)-i\omega\nu,\quad n_0'=-[4T\,\mathrm{ch}^2(\varepsilon/2T)]^{-1},$$
$$\mu=p_x/p,\quad v_r=v(\xi/\varepsilon).$$

Since n_+ is odd in ξ, the appearance of the potential and superfluid velocity in the superconductor leads to redistribution of the quasi-particles over the branches of the spectrum and, to within terms of order T/ϵ_F, does not lead to a change of the total number of quasi-particles or, therefore, to a change of the energy gap Δ. This means that the oscillations of the gap occur independently. In the equilibrium case they are damped.

We shall obtain the dispersion equation for the free oscillations from the Poisson equation and the equation of continuity (this, as pointed out in a number of papers, does not follow from the kinetic equation). In the case of good conductors these equations reduce, to within terms of order $(k/k_F)^2$ (k_F^{-1} is the Thomas-Fermi screening length), to the equations

$$\delta N=0,\quad \mathrm{div}\, j=0. \tag{3}$$

The current density and particle-number fluctuation calculated by means of (2) have the form

$$j=ev_s\left[N_s+\frac{3}{2}\omega^2N\langle n_0'\mu^2/D\rangle\right]+\frac{3}{2}e\Phi\frac{k\omega}{m}N\langle(\xi/\varepsilon)^2n_0'\mu^2/D\rangle, \tag{4}$$

$$\begin{aligned}\delta N&=\frac{1}{2}v_s k\omega N\left\langle\left(\frac{\xi}{\varepsilon}\right)^2 n_0'/D\right\rangle\\ &+\frac{m^2v}{2\pi^2}\Phi\left\langle\left(\frac{\xi}{\varepsilon}\right)^2n_0'-\frac{\Delta^2}{2\varepsilon^3}\mathrm{th}\frac{\varepsilon}{2T}+i\omega\nu n_0'\left(\frac{\xi}{\varepsilon}\right)^2\Big/D\right\rangle,\end{aligned} \tag{5}$$

where

$$\langle(\ldots)\rangle=\int_{-\infty}^{+\infty}d\xi\int_{-1}^{1}d\mu(\ldots),\quad N=\frac{(mv)^3}{3\pi^2},$$

and N_S is the density of superconducting electrons. Under the condition $\Delta/T \ll 1$ we obtain from (3)–(5) the following dispersion equation:

$$\begin{aligned}&\frac{N_s}{N}k^2v^2\langle n_0'\mu^2/D\rangle-\frac{N_s}{N}\left\langle\frac{\Delta^2}{2\varepsilon^3}\mathrm{th}\frac{\varepsilon}{2T}\right\rangle+\frac{3}{2}(kv\omega)^2\langle n_0'\mu^2/D\rangle\\ &\times\left\langle n_0'\mu^2\left(\frac{\Delta}{\varepsilon}\right)^4\Big/D\right\rangle-\frac{3}{2}\omega^2\langle n_0'\mu^2/D\rangle\left\langle\frac{\Delta^2}{2\varepsilon^3}\mathrm{th}\frac{\varepsilon}{2T}\right\rangle=0.\end{aligned} \tag{6}$$

In the frequency range

$$\nu(\Delta/T)^2\ll\omega\ll\nu(\Delta/T),$$

which corresponds to

$$\nu(1-T/T_c)\ll\omega\ll\nu(1-T/T_c)^{1/2},$$

Equation (6) has a solution corresponding to weakly-damped oscillations with the spectrum

$$\omega=kV-i\nu\left[\frac{7}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{\Delta}{T}\right)^2+\frac{1}{12}\left(\frac{kv}{\nu}\right)^2\right], \tag{7}$$

where the velocity

$$V=\left[\frac{7\zeta(3)}{3\pi^3}\frac{\Delta}{T}\right]^{1/2}v\approx 0.5\left(1-\frac{T}{T_c}\right)^{1/4}v. \tag{8}$$

We note that in obtaining (7) from (6) we can neglect the terms $(kv_g)^2/3$ compared with $\omega\nu$ in the denominators of the integrands.

The result obtained is in qualitative agreement with the results of the measurements of Carlson and Goldman[6], who observed collective oscillations with a sound spectrum in a narrow range of temperatures near T_c in a superconductor with a gap. However, they used a dirty superconductor ($\nu > \Delta$). Therefore, it is not possible to make a quantitative comparison. It would be of interest to make measurements on clean superconductors.

It was shown earlier by Brieskorn et al.[7], and also by Maki and Sato[8], that in dirty zero-gap conductors there also exist oscillations of the phase, with spectrum $\omega = kV_1$. The velocity in this case differs from the one we have found:

formula $\quad V_1\sim(\Delta/T)^{1/4}(T/\nu)^{1/4}v.$

They pointed out that such oscillations are analogous to fourth sound in superfluid helium.

Baru and Sukhanov[9] have considered the stability of a non-equilibrium state of a superconductor against fluctuations of v_s for $T \ll \Delta$. At higher temperatures, in determining the stability it is also necessary to take into account the fluctuations of Φ. Thus, if the quasiparticle distribution function in the state whose stability is being investigated has the form $n = n_0(\epsilon) + \delta n(\epsilon)$, where $\delta n(\epsilon) \sim (\Delta/T)^2$, then Eq. (6) remains valid but in place of the equilibrium quantity

$$N_s=\frac{7\zeta(3)}{4\pi^2}\frac{\Delta^2}{T^2}N$$

we must take

$$N_s^*=N_s+\int d\xi\frac{\partial\delta n}{\partial\varepsilon}.$$

Then the solution of (6) for

$$N_s^*<0,\quad kv\ll\nu(\Delta/T)^{1/2}$$

has the logarithmic increment

$$\gamma=-\nu\frac{N_s^*}{2N}+\left[\left(\frac{\nu}{2}\frac{N_s^*}{N}\right)^2-\frac{4}{3\pi^2}\frac{N_s^*}{N}\frac{T}{\Delta}k^2v^2\right]^{1/2}.$$

For

$$kv\gg\nu\left|\frac{N_s}{N}\right|\frac{\Delta}{T}$$

the logarithmic increment increases linearly with k.

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Note added in proof (September 25, 1975). Papers have recently appeared in which collective oscillations in superconductors with a gap have been studied [10-11]. In [10] the spectrum of oscillations was obtained on the basis of phenomenological equations in the case of clean superconductors. However, the Landau damping, which is extremely important in the given case, was not taken into account in this paper. Schmid and Schön [11] showed that collective oscillations can exist in dirty superconductors ($\nu > \Delta$). In this case the frequency of the collective excitations differs from the one we have found.

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