Experimental study of the evolution of the nonlinear wave excited by a modulated electron beam in a plasma

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The spatial evolution of the anharmonic profile of the electric-field wave in a beam-plasma system is investigated experimentally and the dynamics of the electron bunches is discussed on the basis of the experimental results. The variability of the structure of the bunches, and in particular, the breaking up of the bunches, which was previously predicted theoretically, is observed. It is found that the distortion of the initial profile of the wave becomes more severe as the ratio of the modulation frequency to the plasma frequency decreases. Direct experimental data are obtained on the high-frequency radial fields in the system.

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The dynamics of an electron beam in the field of the initially monochromatic plasma wave that it generates has been the subject of a considerable number of theoretical papers[1-6]. According to these studies, the beam electrons, during the nonlinear stage of their interaction with the plasma, group themselves into dense bunches whose motion gives rise to periodic changes in the amplitude of the oscillations. That such bunching occurs has also been established experimentally over a wide range of conditions[7-10]. Recent experimental data[10] on the behavior of the resulting electron bunches, on the other hand, are limited only to cases of very low beam density n_b ($(n_b/n_0)^{1/3} \ll 1$, where n_0 is the plasma density). Analogous experiments with the no less prevalent beam-plasma systems, for which $n_b/n_0 \ll 1$, have not yet been performed.

In[11] an auxiliary electron beam was used to probe such a beam-plasma system and to measure the profile of the longitudinal electric-field wave in the nonlinear stage of the interaction, and it was argued that the observed anharmonic wave profile is due to strong bunching of the beam. In the work reported here we have made a direct experimental study of the spatial evolution of the nonlinear profile of the electric-field wave in a beam-plasma system and have analyzed the dynamics of the electron bunches on the basis of the measured wave profiles. We also investigated the effects of the principal parameters of the system on the wave profile.

The experiments were performed in the absence of external fields. The electron-beam energy U_0 and current I_b were ~100 eV and ~10 mA, respectively. The beam passed along the axis of a metallic plasma chamber whose characteristic transverse dimension was 6 cm. The diameter of the beam varied along its length; the beam diameter in the region where the measurements were made usually fell within the range 0.4–2 cm. The beam catcher could be moved longitudinally so as to vary the length of the system. In most of the experiments the beam was ~12 cm long. The plasma was produced in argon by the electron beam itself, and the plasma density n_0 could either be close to the beam density n_b or could exceed the latter by an order of magnitude, depending on the gas pressure. The absolute plasma concentration within the beam could be varied in the range 10^9-10^{10} cm^{-3}. A spatially growing monochromatic wave was excited by velocity-modulating the beam at a frequency ω somewhat lower than the plasma frequency.

We used an electron probe beam directed perpendicular to the main beam to determine the temporal profile of the wave. After being deflected first by the field of the wave and then by a synchronous harmonic sweep field, the probe beam fell onto a fluorescent screen, where it traced a curve from which the time dependence of the strength of the investigated electric field could be determined. The technique used is described in more detail in[11]. Our present technique differs from that described in[11] in that the probe beam can now be moved parallel to itself in two directions: parallel and perpendicular to the principal beam. We accordingly determined the longitudinal-wave profile at various distances and also measured the strength of the high-frequency radial electric field that arises because of the finite radius of the beam.

Figure 1 shows a typical series of curves traced on the fluorescent screen by the probe beam when the latter is set at different distances from the point where the beam enters the plasma. For each such curve the figure also shows both the corresponding temporal profile of the longitudinal electric-field wave xxx $\bar{E}_z$, averaged over the radius of the system,

$$\bar{E}_z \sim \int_0^R E_z(r)\,dr,$$

and the corresponding time dependence of the alternating longitudinal convection current $\bar{j}_c$ in the plasma, also averaged over the cross section of the system. The $\bar{E}_z(t)$ curves were constructed directly from the probe traces, and the $\bar{j}_c(t)$ curves were obtained from the derivatives of the corresponding $\bar{E}_z(t)$ curves on the basis of the following considerations:

Let us write Maxwell's equation connecting the electric field E with the convection current density j in the form

$$\operatorname{div}\left(j-\frac{1}{4\pi}\frac{\partial E}{\partial t}\right)=0 \qquad (1)$$

and integrate it over the cross section of a cylinder of radius R whose axis coincides with the axis of the beam. We choose R so large that $E_r(R)$ and $j_r(R)$ can be neglected. (Our measurements of the radial distribution of the amplitude of the oscillations detected by an ordinary probe showed that these oscillations are localized virtually entirely within the region occupied by the beam [this is in agreement with results of other experiments—see, e.g.,[12]], so that R may be chosen close to the radius of the beam.) The integration yields

$$\frac{\partial}{\partial z}\left(\int_0^R j_z r\,dr-\frac{1}{4\pi}\frac{\partial}{\partial t}\int_0^R E_z r\,dr\right)=0. \qquad (2)$$

Since there are no external fields in the system (the

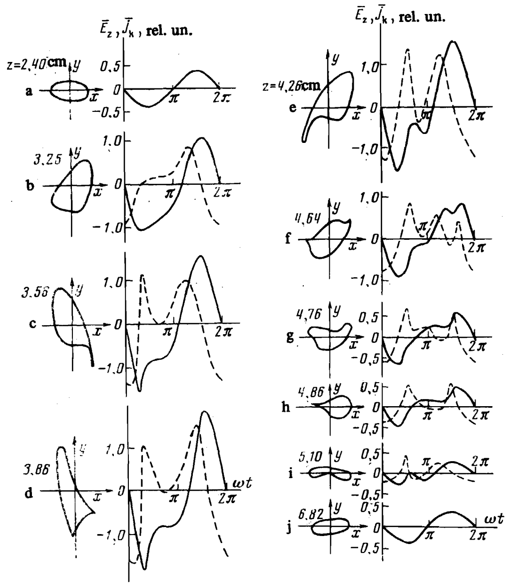

FIG. 1. Spatial evolution of the temporal profiles of the electric-field wave $\bar{E}_z$ (full curves) and the convection current density $\bar{j}_c$ in the plasma (dashed curves). Plasma frequency $f_p = \Omega/2\pi \approx 500$ MHz, modulation frequency $f = \omega/2\pi = 325$ MHz, $x \sim \sin \omega t$, $y \sim \bar{E}_z$, $I_b = 10$ mA, $U_0 = 137$ V, $p = 1.8 \times 10^{-3}$ mm Hg; the probe beam intersects the axis of the system.

measured t dependence of $\bar{E}_z$ did not change when the beam catcher was moved considerable distances along the beam, indicating that there was no appreciable alternating current in the external circuit), it follows from Eq. (2) that

$$\int_0^R j_z r\,dr - \frac{1}{4\pi}\frac{\partial}{\partial t}\int_0^R E_z r\,dr = \frac{1}{2\pi} I_b, \tag{3}$$

where I_b is the dc component of the beam current.

Substituting $\bar{E}_z$ for E_z in Eq. (3), we obtain the approximation relation

$$\bar{j}_c \sim \partial \bar{E}_z/\partial t,$$

which determines the time dependence of the quantity

$$\bar{j}_c \sim \int_0^R j_z r\,dr - \frac{I_b}{2\pi}.$$

Turning to the discussion of the experimental results, we first consider the structure of the waves on the basis of the simplest possible theoretical model, consisting of a one-dimensional plasma with cold electrons and stationary ions through which passes an electron beam whose current density $j_b(z, t)$ is periodic in time. From Eq. (1), the equation of continuity for the plasma, and the equation of motion for the electrons, we can derive the following equation for the current density j_p due to the plasma electrons in the approximation in which the plasma oscillations are assumed to be linear:

$$\partial^2 j_p/\partial t^2 + \Omega^2 j_p = -\Omega^2 j_b, \tag{4}$$

where Ω is the plasma frequency. Expanding the periodic function $j_b(z, t)$ in a Fourier series,

$$j_b(z, t) = \sum_{n=1}^{\infty} B_n(z)\cos[n\omega t - \varphi_n(z)], \tag{5}$$

we find the steady-state solution of Eq. (4):

$$j_p = -\sum_{n=1}^{\infty} \frac{1}{1-\alpha^2 n^2} B_n(z)\cos[n\omega t - \varphi_n(z)], \tag{6}$$

where $\alpha = \omega/\Omega$.

Thus, the time dependence of the total current $j_c(z, t) = j_b + j_p$ in the plasma can be expressed in terms of the beam current density:

$$j_c = -\sum_{n=1}^{\infty} \frac{1}{1-\alpha^2 n^2} B_n(z)\cos[n\omega t - \varphi_n(z)] + j_b(z, t). \tag{7}$$

On substituting the values of the coefficients B_n and φ_n and the function $j_b(t)$ calculated with the formulas of [3] into Eqs. (6) and (7), we obtain the relations shown graphically in Fig. 2 for the stage in which the beam bunching is appreciable. A characteristic feature of the calculated $j_c(t)$ curves is the presence in the positive half cycle of a prominent dip, which is due to strong bunching of the beam. We note that the change in the fine structure of the bunches is associated mainly with changes in the amplitudes B_n of the higher harmonics. It accordingly follows from (6) that if $\alpha \sim 1$ the time dependence of j_p will remain virtually unchanged, and in that case the shape of the j_c dip will be determined by the profile of the beam-electron bunches.

Let us turn again to Fig. 1 and compare it with Fig. 2. It will be seen that the experimental $\bar{j}_c(t)$ curves are very similar to the calculated $j_c(t)$ curve, and if they are to be interpreted in the same way, the observed spatial development of the dip essentially illustrates the dynamics of a beam-electron bunch, in which the following most significant points can be distinguished:

1. Following the linear stage of exponential growth of the wave amplitude, the beam electrons group themselves into bunches and the initially harmonic profile of the $\bar{j}_c(t)$ wave becomes distorted (Fig. 1, a–e). Since the bunching of the beam electrons as a result of their interaction with the plasma has been reliably established in earlier studies, the evolution of the wave profile in this region can serve as additional confirmation of the correctness of its interpretation and of the subsequent conclusions.

2. The electron bunches that are formed do not remain intact in their subsequent motion: they increase in length, with a consequent decrease in the mean electron density within them. Being stretched, the electron bunches break up into two parts–there are two dips on the $\bar{j}_c(t)$ curve (Fig. 1, f, g). After that the bunches merge again. In the following stage the beam becomes so thoroughly debunched that the wave profile again becomes virtually sinusoidal (Fig. 1, j).

3. The electron bunches decay considerably faster than their phase changes. In fact, it is evident from Fig. 1 that, both in the bunch forming stage and in the region of maximum bunching, the center of a bunch is always displaced somewhat into the decelerating half cycle of the field and no significant motion of the bunch in the direction of the accelerating half period can be observed. Only when the bunches break into two can one of the parts enter the accelerating phase of the wave (Fig. 1, f, g).

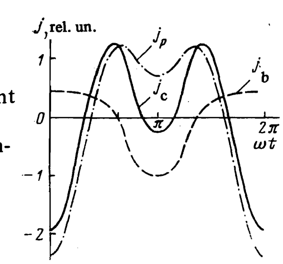

FIG. 2. Theoretical alternating current density curves: $j_p(t)$–plasma electrons, $j_b(t)$–beam, $j_c(t)$–total. Bunching parameter $X = 0.7$, $\alpha = 0.75$.

The debunching of the beam, established in the present work, is in accordance with the results of numerical calculations[4, 6]. However, this debunching was not observed in any previous experiments—not even in those of Gentle and Lohr[10], mentioned in the introduction, which were specially designed to investigate the dynamics of the bunches in the case $(n_b/n_0)^{1/3} \gg 1$, although the theory predicts that the bunches will break up even in this case[1, 2].

The fact that the higher harmonics of the field become more and more important as the ratio ω/Ω decreases is also in accordance with the theory, and in particular, with Eq. (7). Figure 3 shows a series of curves recorded with different pressures, and accordingly different plasma densities, in the chamber. In recording each curve, the plasma was probed at the point where the oscillations reached their maximum intensity. It will be seen that the wave profile deviates more and more from the harmonic form as the plasma density increases. As was predicted theoretically[3, 13], the second harmonic dominates the electric-field oscillations when the plasma density is high enough (Fig. 3, d). At still higher plasma densities the electric field oscillates with the frequency 3ω (Fig. 3, e).

Summing up this part of the work, we may conclude that under the conditions of our experiments the behavior of the wave is determined primarily by the deformation of the electron bunches, and not by their oscillation as a whole in the potential well of the wave.

Along with the above mentioned agreement between theory and experiment we can also easily perceive an important discrepancy concerning the space dependence of the wave amplitude. According to the theory, the debunching of the beam and its subsequent rebunching should lead to corresponding spatial oscillations of the amplitude[4, 6]. Experimentally, however, repeated maxima usually do not occur, or, if they do occur, they exhibit no clear connection with the evolution of the longitudinal-wave profile. This can apparently be explained by the fact that the real system differs from the one-dimensional theoretical model in that the beam undergoes strong angular spreading in the nonlinear stage of the interaction, and this leads to a change in the beam density with distance.

It is natural to suppose that the angular spreading of the beam is due to a high-frequency radial field. In the present experiments we also made measurements, using the probe beam, that enable us to estimate the strength of this field. Figure 4 shows a series of curves obtained by probing the system at different distances r from the axis in the plane z = const at which the amplitude of the longitudinal wave reaches its maximum. These probe traces were obtained without external sweeping of the probe beam along the x axis, the displacement of the beam in this direction being effected by the electric field of the plasma oscillations. It is evident that the radial field is of the same order as the longitudinal one. It reaches a strength of ~100 V/cm and is quite strong enough to account for the large angular spreading observed visually[7].

As we mentioned before, the experimental data presented above were obtained under such conditions that $n_b/n_0 \ll 1$. By increasing the beam current density and at the same time slightly reducing the gas pressure one can bring about conditions in which n_b is comparable with n_0. A corresponding set of curves recorded at different distances along the axis is shown in Fig. 5. There is no nonlinear theory of beam-plasma interaction for this case. We note (see Fig. 5) that here, too, along with the spatial growth of the wave, the wave profile becomes anharmonic; moreover, the higher harmonics contribute much more strongly here than in the case of a lower-density beam.

Thus, it has been shown in the present study that the initially monochromatic electric-field wave excited by an electron beam in a plasma under such conditions that $n_b/n_0 \ll 1$ assumes a specific nonsinusoidal profile in the nonlinear stage of the interaction, owing to bunching of the beam. The data obtained on the spatial evolution of the wave profile indicate that the structure of the electron bunches changes continuously; this is in qualitative agreement with a numerical analysis made by other authors. It has also been shown that the relative contribution of the higher harmonics to the electric-field wave increases as the beam density increases and the ratio ω/Ω decreases. The presence in the system of a

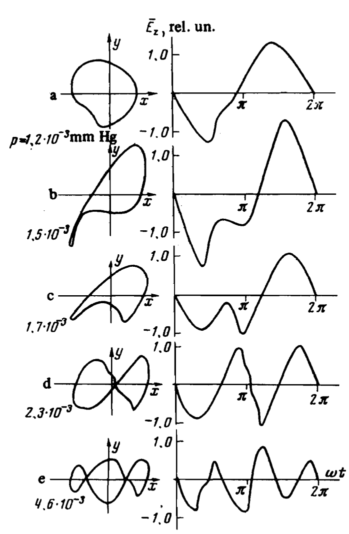

FIG. 3. Probe beam traces and the corresponding electric-field wave profiles for different pressures in the plasma chamber. I_b = 6 mA, U_0 = 165 V, f = 270 MHz, f_p = 300 MHz (plot a), 350 (b), 400 (c), 600 (d), and 900 MHz (e).

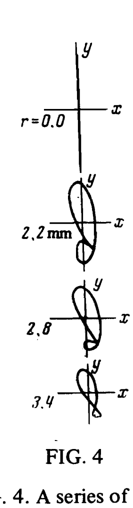

FIG. 4

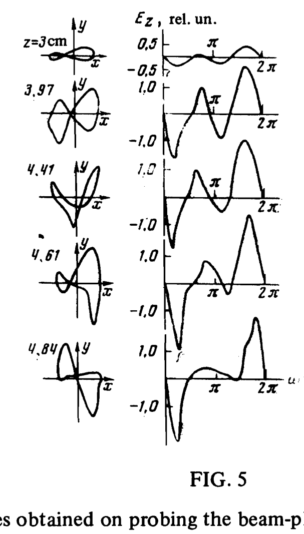

FIG. 5

FIG. 4. A series of traces obtained on probing the beam-plasma system at different distances r from the beam axis; I_b = 10 mA, U_0 = 125 V, f = 330 MHz, p = 1.6 X 10^{-3} mm Hg, $f_p \approx$ 550 MHz, beam diameter 1 cm.

FIG. 5. Spatial evolution of the temporal profile of the electric-field wave in the case $n_b \sim n_0$; I_b = 21 mA, U_0 = 167 V, f = 272 MHz, p = 1.3 X 10^{-3} mm Hg, f_p = 650 MHz.

high-frequency radial field close in strength to the field of the longitudinal oscillations has been established.

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