Contribution to the nonlinear theory of surface electromagnetic waves in an ionizing plasma

A. G. Boev and A. V. Prokopov

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
(Submitted April 24, 1975)
Zh. Eksp. Teor. Fiz. **69**, 1208–1217 (October 1975)

A nonlinear theory of the propagation of high-frequency surface electromagnetic waves of amplitude sufficient for the ionization of the gas is developed for a plasma half-space. The dispersion characteristics of the wave and the structure of the electric field as functions of the wave amplitude and the exponent in the ionization law are analyzed on the basis of the exact solution to the nonlinear Maxwell equations that corresponds to a power-law dependence of the electron concentration on the field intensity. It is shown that as the wave amplitude increases the boundary of the transparency region for the plasma shifts toward the higher-frequency region. The propagation of the wave is then possible only when the amplitude exceeds some threshold value for the given frequency. The case of a plasma with an initial electron concentration exceeding the critical value for the wave frequency is considered in detail.

PACS numbers: 52.40.Db

When the amplitude of waves propagating in a weakly ionized plasma exceeds the plasma field E_p or the breakdown field E_i, the nonlinear effects connected with the heating of the electrons and the ionization of the plasma become important [1]. Below we consider the problem of the propagation of surface electromagnetic waves in a semi-infinite plasma that gets ionized in the wave field. Dissipative effects are neglected, which can be done when the wave frequency is high compared to the effective collision rate.

1. Let us consider a plasma with an initial electron concentration n_0, occupying the half-space $z \geq 0$, and bordering on a homogeneous dielectric; the plane $z = 0$ corresponds to the plasma boundary and the coordinates x, y denote the position of a point on it. Along the plasma boundary in the direction of the y axis propagates a surface electromagnetic wave of amplitude sufficient for the ionization of the gas.

We shall assume that the penetration depth of the wave field into the plasma is much greater than the characteristic scales of electron diffusion and electron-concentration redistribution resulting from the heating of the electrons in the inhomogeneous field and that the lifetime of the charged particles is less than their time of diffusion from the skin layer. Then the dependence of the electron concentration on the wave field can be written in the form

$$n=n_0 f(|E/E_0|^2), \quad \partial f/\partial|E|>0, \tag{1.1}$$

where E_0 is some effective ionization field, the concrete form of which is determined by the ionization mechanism.

An important particular case of the dependence (1.1) is the power ionization law:

$$n=n_0\{1+|E/E_0|^m\}, \quad m>0. \tag{1.2}$$

Such a dependence can approximate any ionization law in a finite interval of variation of $|E|$.

We shall henceforth be interested in the propagation of the wave under conditions when the change in the wave amplitude over a wavelength as a result of the ionization of the plasma is small. Seeking the solution to the Maxwell equations in the form of a running—along the y axis—wave, $e^{i(hy-\omega t)}$, we obtain for the field in the plasma the equations

$$E_\perp=\frac{\eta^{1/2}}{\varepsilon}b, \quad E_\parallel=\frac{i}{\varepsilon}\frac{\partial b}{\partial\zeta}, \quad E_x=0,$$
$$\eta^{1/2}E_\perp+i\,\partial E_\parallel/\partial\zeta=b, \tag{1.3}$$

where we have introduced the following dimensionless quantities:

$$b=H_x/E_0, \quad E_\perp=E_z/E_0, \quad E_\parallel=E_y/E_0,$$
$$\zeta=z\omega/c, \quad \eta=(hc/\omega)^2.$$

The dependence of the electron concentration on the amplitude of the wave field leads to a situation in which the permittivity of the plasma

$$\varepsilon(E)=1-\frac{\omega_\infty^2}{\omega^2}f(|E|^2), \quad \omega_\infty^2=\frac{4\pi e^2 n_0}{m_e}, \tag{1.4}$$

determining the electromagnetic properties of the plasma, also becomes dependent on the field amplitude, thereby making the Eqs. (1.3) nonlinear. Because of this, the equation for the magnetic field, the solution of which can be reduced to the solution of the problem

$$\varepsilon(b)\frac{d}{d\zeta}\left[\frac{1}{\varepsilon(b)}\frac{db}{d\zeta}\right]=[\eta-\varepsilon(b)]b, \tag{1.5}$$

$$\varepsilon(b)=1-\frac{\omega_\infty^2}{\omega^2}f\left\{\frac{1}{\varepsilon^2}\left[\eta b^2+\left(\frac{db}{d\zeta}\right)^2\right]\right\}, \tag{1.6}$$

also turns out to be nonlinear. The Eq. (1.5) admits of the following solution:

$$\frac{2\eta-\varepsilon}{\varepsilon}b^2-\varepsilon|E|^2-\int_{\varepsilon_1}^{\varepsilon}|E|^2\,d\varepsilon=C, \tag{1.7}$$

where ϵ_1 is the value of the permittivity of the plasma at the boundary $\zeta = 0$. Now solving (1.4) for $|E|^2$:

$$|E|^2=f^{-1}[(1-\varepsilon)/(1-\varepsilon_0)],$$

where f^{-1} denotes the inverse of the function f, we obtain an expression determining the magnetic field of the wave as a function of the permittivity:

$$\frac{2\eta-\varepsilon}{\varepsilon}b^2-\varepsilon f^{-1}\left(\frac{1-\varepsilon}{1-\varepsilon_0}\right)+\int_{\varepsilon_1}^{\varepsilon}f^{-1}\left(\frac{1-\varepsilon}{1-\varepsilon_0}\right)d\varepsilon=C. \tag{1.8}$$

Notice that the existence of this relation is not connected with the condition $\partial f/\partial|E| > 0$, which is characteristic of the ionization law. It is of a general character, and can be used not only for the analysis of the ionization-related nonlinearity for different ionization laws, but also for the analysis of any nonlinearity connected with the local dependence of the permittivity ϵ of the plasma on the modulus of the electric field $|E|$. For example, a particular form (of the dependence (1.8)) corresponding to the nonlinearity connected with the action on the plasma of the potential of the high-frequency forces of the surface-wave field was used in [2].

For a power-law ionization we have

$$|E|^2=[(\varepsilon_0-\varepsilon)/(1-\varepsilon_0)]^{2/m},$$

and the dependence of the magnetic field on the permittivity, (1.8), can be expressed in terms of elementary functions:

$$b=\left[\frac{\varepsilon}{2\eta-\varepsilon}\,\frac{2\varepsilon+m\varepsilon_0}{2+m}\left(\frac{\varepsilon_0-\varepsilon}{1-\varepsilon_0}\right)^{2/m}\right]^{1/2}. \tag{1.9}$$

The constant of integration in (1.9) has been determined from the condition that the field vanish deep inside the plasma (i.e., when $\epsilon = \epsilon_0$).

Using (1.9), we can easily derive from (1.6) the following expression determining the implicit dependence of the permittivity ϵ on the coordinate ζ:

$$\zeta=\int\limits_{\varepsilon}^{\varepsilon_1}\frac{db}{d\varepsilon}\left[\varepsilon^2\left(\frac{\varepsilon_0-\varepsilon}{1-\varepsilon_0}\right)^{2/m}-\eta b^2(\varepsilon)\right]^{-1/2}d\varepsilon. \tag{1.10}$$

The expressions (1.3), (1.9), and (1.10) completely determine the field of the surface wave in the plasma. This field should be matched by the continuity conditions for the tangential components of the electric and magnetic fields to the wave field in the dielectric, which field can be found from the solution to the linear equations. The condition of continuity of the magnetic field at the plasma boundary determines the permittivity ϵ_1 at the plasma boundary and, consequently, the modulus of the electric field, as functions of the field amplitude B in the dielectric and the permittivity of the unperturbed plasma:

$$\varepsilon_1=\varphi(B;\ \varepsilon_0).$$

The continuity condition for the tangential component of the electric field can be written in the form

$$\frac{1}{\varepsilon_1}\left(\frac{db}{d\zeta}\right)_{\zeta=0}=\frac{1}{\varepsilon_2}(\eta-\varepsilon_2)^{1/2}B. \tag{1.11}$$

For the field that decreases as we go into the plasma, $(db/d\zeta)_{\zeta=0} < 0$, and from (1.11) follows the inequality $\epsilon_1\epsilon_2 < 0$. The dispersion equation for the wave then has the following form:

$$\eta=\frac{\varepsilon_1^2\varepsilon_2[\varepsilon_2(m+2)-(2\varepsilon_1+m\varepsilon_0)]}{2(m+2)\varepsilon_1\varepsilon_2^2-(\varepsilon_2^2+\varepsilon_1^2)(2\varepsilon_1+m\varepsilon_0)}. \tag{1.12}$$

For a wave of relatively small amplitude, which virtually does not change the electron concentration of the plasma as compared to the initial concentration:

$$\eta=\left(\frac{c}{v_{\rm ph}}\right)^2\approx\frac{\varepsilon_0\varepsilon_2}{\varepsilon_0+\varepsilon_2}\left[1-\frac{2\varepsilon_2(1-\varepsilon_0)}{\varepsilon_0(\varepsilon_0+\varepsilon_2)(m+2)}|E_1|^2\right] \tag{1.13}$$

where $|E_1| = |E(0)|$. In the zeroth approximation in the field, we obtain the well-known—in the linear theory—dispersion equation [3].

The second term on the right-hand side of (1.13) is always negative and, consequently, the ionization of the plasma by the wave field leads to a phase velocity that is greater than the phase velocity obtained in the linear theory for the same values of ϵ_2 and ϵ_0. This was to be expected, since a change in the plasma permittivity through an increase in the plasma-electron concentration is equivalent to a decrease in the wave frequency. In the case of a strong field, for which $|\epsilon_1| \gg \epsilon_2$, ϵ_0, we have

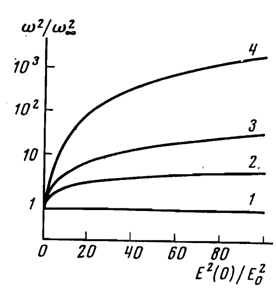

FIG. 1

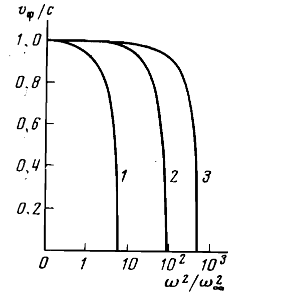

FIG. 2

FIG. 1. Dependence of the critical frequency of the wave on the electric-field intensity for different values of the exponent m in the ionization law: 1) m=0; 2) m=1; 3) m=2; 4) m=4.

FIG. 2. Dependence of the phase velocity of the wave on frequency for different electric-field intensities for m=4: 1) $|E_1|^2=5$; 2) $|E_1|^2=20$; 3) $|E_1|^2=50$.

$\eta \approx \epsilon_2$, and the phase velocity of the wave tends to the velocity of light in the dielectric.

The existence domain for the surface waves is determined by the requirement that the wave be attenuated inside the dielectric, i.e., that $\epsilon_2 < \eta(E) < \infty$. The minimum initial electron concentration $(n_0)_m$ at which the propagation of surface waves in the plasma becomes possible and, consequently, the maximum wave frequency ω_M are determined by the equation $\eta^{-1}(E) = 0$, and are functions of the wave amplitude:

$$\left[\frac{n_0}{n_c}\right]_m^{-1}=\left[\frac{\omega^2}{\omega_\infty^2}\right]_M=F(E),\quad n_c=\frac{m_e\omega^2}{4\pi e^2}. \tag{1.14}$$

For a wave of small amplitude, we obtain

$$\begin{gathered}\varepsilon_0=-\varepsilon_2+\frac{2(1+\varepsilon_2)}{m+2}|E_1|^2,\\ [\omega^2/\omega_\infty^2]_M=(1+\varepsilon_2)^{-1/2}\left[1+\frac{1}{m+2}|E_1|^m\right].\end{gathered} \tag{1.15}$$

Thus, the ionization of the plasma by the wave leads to a shift of the critical frequency $[\omega/\omega_\infty]_M$ toward the region of higher frequencies, and leads, as it were, to a "transparentization" of the plasma at higher frequencies.

In Fig. 1 we show the dependence of the square of the critical wave frequency, normalized to the plasma frequency of the unperturbed plasma ω_∞, on the electric-field intensity at the boundary for the case $\epsilon_2 = 1$ (vacuum) for different values of the exponent m in the ionization law. The curve 1 corresponds to the dependence $[\omega^2/\omega_\infty^2]_M = 1/2$, which is well-known in the linear theory.

To the existence domain for the surface waves corresponds the region

$$0<\omega^2/\omega_\infty^2<F(E)$$

below the corresponding curve. It can be seen that as the amplitude of the wave increases, the existence domain expands toward the region of higher frequencies, the extent of the expansion increasing with the exponent m. This means that, in contrast to a weak wave, an ionizing wave can propagate in a plasma with a relatively low initial electron concentration: $n_0 < 2n_c$. However, its propagation is possible only when its amplitude exceeds some threshold value, determined by the dependence (1.14), i.e., by the curves 2, 3, and 4 in Fig. 1. The threshold value of the amplitude decreases sharply as the wave frequency tends to the boundary of the usual transparency region for the plasma $\omega = \omega_\infty/\sqrt{2}$. Under these conditions all the nonlinear effects will be manifested in relatively weak fields.

Figure 2 shows the frequency dependence of the phase velocity of the wave for a number of values of the amplitude in the case when the exponent in the ionization law m = 4. Near the critical frequency the phase velocity increases very rapidly with decreasing frequency; the increase of the amplitude of a wave of fixed frequency entails the increase of the phase velocity. The wave can be effectively slowed down only when its amplitude is close to the threshold value for the given frequency. These results are in qualitative agreement with the experimental results[4,5].

Analysis of the dispersion curves for different values of the exponent m in the ionization law and comparison with the preceding results show that the influence of m on the dispersion characteristics of the wave is qualitatively equivalent to the influence of the corresponding variation of the wave amplitude at fixed m.

2. According to (1.3) and (1.9), the components of the electric field in the plasma as functions of the permittivity ϵ are determined by the following formulas:

$$E_\perp = \frac{\eta^{1/2}}{\varepsilon}\left[\frac{\varepsilon}{2\eta-\varepsilon}\,\frac{2\varepsilon+m\varepsilon_0}{m+2}\left(\frac{\varepsilon_0-\varepsilon}{1-\varepsilon}\right)^{2/m}\right]^{1/2}$$
$$E_\parallel = \frac{i}{\varepsilon}\left[\varepsilon^2\left(\frac{\varepsilon_0-\varepsilon}{1-\varepsilon_0}\right)^{2/m} - \eta b^2\right]^{1/2}. \tag{2.1}$$

The dependence of the fields on the coordinate ζ is determined through ϵ by the formula (1.10). As can be seen from these formulas, the behavior of the fields in the plasma essentially depends on the value of the permittivity ϵ_0 of the unperturbed plasma and on the exponent m. Below we shall consider the case $\epsilon_0 < 0$ in detail. The permittivity ϵ will then be negative everywhere in the plasma: $\epsilon_1 \le \epsilon \le \epsilon_0 < 0$. It is not difficult to verify that in this case $\partial b/\partial\epsilon < 0$, and the magnetic field in the plasma decreases with increasing ϵ from B at the boundary $\epsilon = \epsilon_1$ to zero at $\epsilon = \epsilon_0$.

The value of the permittivity at the plasma boundary decreases monotonically with increasing wave amplitude, i.e., $\partial\epsilon_1/\partial B < 0$, while the electron concentration at the plasma boundary monotonically increases. In the limiting case of weak fields, $\epsilon \approx \epsilon_0$, $|\epsilon - \epsilon_0| \ll |\epsilon_0|$, and from (1.9), (1.10), and (2.1) we obtain

$$\varepsilon = \varepsilon_0 - (\varepsilon_0-\varepsilon_1)\exp[-m(\eta_0-\varepsilon_0)^{1/2}\zeta], \quad \varepsilon_1 = \varepsilon_0 - (1-\varepsilon_0)\left[\frac{2\eta_0-\varepsilon}{\varepsilon_0^2}B^2\right]^{m/2},$$
$$b = B\exp[-(\eta_0-\varepsilon_0)^{1/2}\zeta],$$

where $\eta_0 = \epsilon_2|\epsilon_0|/(|\epsilon_0| - \epsilon_2)$. The penetration depth of the field into the plasma

$$\delta = \delta_0 = k^{-1}[(|\varepsilon_0| - \varepsilon_2)/|\varepsilon_0|^2]^{1/2}$$

coincides in this approximation with the penetration depth δ_0 in the linear theory. These results indicate that the field in sufficiently deep layers of the plasma for any amplitude of the wave at the boundary will always fall off exponentially.

It is worth noting that the nature of the increase of the permittivity and, consequently, the decrease of the electron concentration as we go into the plasma depend on the exponent m in the ionization law; for m > 1 the electron concentration falls off more rapidly than the field, while for m < 1 it falls off more slowly. For m = 1 the electron concentration and the field fall off at the same rate.

Allowance for the next corrections in the field leads to the following expression for the penetration depth:

$$\delta = \delta\left\{1 + \frac{[2\varepsilon_2^2 - (m+2)(\varepsilon_2+\varepsilon_0)^2](1-\varepsilon_0)B^m}{2(m+2)(|\varepsilon_0|-\varepsilon_2)\varepsilon_0^2}\left[\frac{\varepsilon_2-\varepsilon_0}{\varepsilon_0(\varepsilon_2+\varepsilon_0)}\right]^{m/2}\right\}. \tag{2.2}$$

For $|\epsilon_0| < [1 + (2/(m+2))^{1/2}]\epsilon_2$ the second term in the brackets is positive, and the ionization of the plasma by the wave field leads to an increase in the penetration depth as compared to δ_0. For $|\epsilon_0| > [1 + (2/(m+2))^{1/2}]\epsilon_2$ the sign of the second term in the brackets changes, and the wave field decreases δ. At the point

$$\overline{|\varepsilon_0|} = [1 + (2/(m+2))^{1/2}]\varepsilon_2 \tag{2.3}$$

the correction, due to the ionization of the plasma by the wave field, to the skin thickness vanishes.

Since for $m \ne 0$, $\overline{|\epsilon_0|} < 2\epsilon_2$, the value of $|\epsilon_0|$ at which the penetration depth δ_0 in the linear theory has its maximum value, the ionization of the plasma by the field will lead to a shift of the maximum of δ, as compared to the maximum of δ_0, toward the region of smaller $|\epsilon_0|$ values and to the flattening of the maximum. In this case there will be observed near the limiting value of $|\epsilon_0|$ a sharper increase in the skin thickness δ in comparison with δ_0. Beyond the maximum, the ionization of the plasma by the wave field will, as compared to what obtains in the linear theory, lead to a decrease in the skin thickness.

At sufficiently large wave amplitudes, when $|\epsilon| \gg |\epsilon_0|$, the field near the plasma surface can be assumed to be independent of the parameters of the unperturbed plasma:

$$b \sim |\varepsilon|^{(m+2)/2m}, \quad E_\perp \sim |\varepsilon|^{(2-m)/2m}, \quad E_\parallel \sim |\varepsilon|^{1/m}. \tag{2.4}$$

However, the behavior of the fields essentially depends on the value of the exponent m. At small values of m both components of the electric field behave almost identically as $|\epsilon|$ increases: $E_\perp \sim E_\parallel \sim |\epsilon|^{1/m}$; for $m \gg 2$, $E_\perp$ decreases, while $E_\parallel$ tends to some value not depending on the wave amplitude. For m = 2, $E_\perp \sim$ const and does not depend on the wave amplitude, while $E_\parallel \sim |\epsilon|^{1/2}$ and increases.

In contrast to $E_\parallel$, for which the variation of the exponent m affects only the growth rate, the transverse component of the electric field turns out to be more sensitive to changes in m: As we pass through the point m = 2, this component changes from being an increasing to being a decreasing function. However, as in the linear theory, the longitudinal component of the field always predominates over the transverse component.

The electron concentration at the plasma boundary increases with increasing amplitude of the wave (of energy W):

$$n_1 = n(0) \sim |\varepsilon_1| \sim B^{2m/(m+2)} \sim W^{m/(m+2)}. \tag{2.5}$$

Since $m/(m+2) \le 1$, this dependence is relatively weak (being a square-root dependence), and only when $m \gg 2$ does it approach a linear dependence. This seemingly unexpected result is connected with the nonlinearity of the problem and is characteristic not only of a surface wave, but also of a strictly transverse field penetrating the plasma[6]. Near the critical frequency, where $v_{ph} \to 0$ and, consequently, $\eta \to \infty$,

$$b \sim \eta^{-1/2} \to 0, \quad E_\perp \approx \varepsilon_M^{-1}\left\{\frac{\varepsilon_M(2\varepsilon_M+m\varepsilon_0)}{m+2}\left(\frac{\varepsilon_0-\varepsilon_M}{1-\varepsilon_0}\right)^{2/m}\right\}^{1/2},$$

$$E_{\|}\approx i\varepsilon_M^{-1}\left\{\left(\frac{\varepsilon_0-\varepsilon_M}{1-\varepsilon_0}\right)^{2/m}\left[\frac{\varepsilon_M[2\varepsilon_M(m+1)-m\varepsilon_0]}{2(m+2)}\right]\right\}^{1/2}, \tag{2.6}$$

where $\epsilon_M = \epsilon\ (\omega = \omega_M)$. As was to be expected, the wave field in this case degenerates into an electrostatic field.

3. To investigate the spatial structure of the wave for any wave amplitude, let us consider the effective penetration depth of the field into the plasma [7]

$$\delta=\frac{1}{kB}\int_0^\infty b(\zeta)\,d\zeta,$$

which, after going over to integration over ϵ, can be written as follows:

$$\delta=\frac{1}{kB}\int_{\varepsilon_1}^{\varepsilon_0}\frac{d(b^2)}{d\varepsilon}2^{-1}\left[\varepsilon^2\left(\frac{\varepsilon_0-\varepsilon}{1-\varepsilon_0}\right)^{2/m}-\eta b^2\right]^{-1/2}d\varepsilon. \tag{3.1}$$

In the limiting case of a weak field, from this formula follow all the preceding results pertaining to the penetration depth of the field into the plasma.

The effective penetration depth of the field into the dielectric is equal to

$$\delta_2=1/k(\eta-\varepsilon_2)^{1/2}.$$

Near the critical frequency, $\delta_2 \sim \delta \sim \eta^{-1/2} \to 0$, and the wave field shrinks toward the plasma-dielectric interface from both sides.

Let us consider the derivative of the penetration depth of the field into the plasma with respect to the wave amplitude:

$$\frac{\partial\delta}{\partial B}=-\frac{\delta}{B}+\frac{1}{kB}\int_0^\infty\frac{\partial b}{\partial\eta}\frac{\partial\eta}{\partial B}d\zeta. \tag{3.2}$$

Since

$$\frac{\partial b}{\partial\eta}=-\frac{b}{2\eta-\varepsilon}\leqslant 0, \tag{3.3}$$

and the phase velocity of the wave increases with increasing amplitude, i.e., $\partial\eta/\partial B \le 0$, the second term on the right-hand side of (3.2) is positive. As in the linear theory, the penetration depth of the field into the plasma behaves in a nonmonotonic fashion as $|\epsilon|$ and, consequently, the wave amplitude increase.

Indeed, near the threshold value of the amplitude for a given frequency, $\eta \to \infty$ and, as follows from (3.3), $\partial b/\partial\eta \approx -\eta^{-3/2}$. Since the phase velocity in this case increases rapidly with increasing wave amplitude, the absolute value of the derivative $\partial\eta/\partial B$ is large, but finite. The second term on the right-hand side of (3.2) predominates over the first, and the derivative $\partial\delta/\partial B$ tends to zero, remaining positive all the time.

For a large wave amplitude the phase velocity tends to the velocity of light in the dielectric, i.e., $\eta \to \epsilon_2$ and $\partial\eta/\partial B \approx 0$. In this case the first term on the right-hand side of (3.2) plays the dominant role, and the penetration depth of the field into the plasma decreases with increasing wave amplitude:

$$\delta\approx\text{const}/B\sim\Delta/B. \tag{3.4}$$

This result is interesting still in that it indicates the existence at high fields of a universal dependence of the penetration depth δ on the wave amplitude. On the value of the exponent m will depend only the quantity Δ and the value of the minimum amplitude starting from which this dependence is valid.

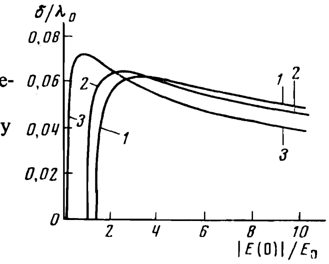

FIG. 3. Dependence of the penetration depth of the field into the plasma on the electric-field intensity for m=1: 1) γ=1; 2) γ=1.2; 3) γ=1.9.

In Fig. 3 we show the dependence of the penetration depth of the field into the plasma, divided by the wavelength λ_0 in vacuo, on the amplitude of the electric field at the plasma boundary for different values of the parameter $\gamma = \omega_\infty^2/\omega^2$. The exponent in the ionization law m = 1 and $\epsilon_2 = 1$. To a change in the parameter γ corresponds either a change in the wave frequency for a given initial electron concentration n_0, or a change in the initial electron concentration at a fixed frequency. The curves 1, 2, and 3, which correspond to different values of the parameter γ, are shifted relative to each other, owing to the difference in the threshold fields. As γ approaches the usual region of transparency of the plasma, i.e., for $\gamma \ge 2$, the threshold field, as was to be expected, decreases. The flattening of the maxima in the curves corresponding to higher threshold fields is clearly discernible. An explanation of this fact has already been given in Sec. 2.

Further, it can be seen from a comparison of the curves that as we go over to a less dense unperturbed plasma the value of the maximum decreases; beyond the maximum in this case is observed a slower decrease of δ with increasing field. To explain these results, let us note that the propagation of a wave in a less dense unperturbed plasma requires a higher threshold field and, consequently, a higher electron concentration at the plasma boundary. This leads to a decrease in the penetration depth of the field in the region of the maximum, where the permittivity ϵ_1 of the plasma at the boundary is comparable to the permittivity of the second medium, and the electron concentration in the plasma begins to determine the quantity δ. In the region beyond the maximum, for one and the same amplitude of the field at the boundary, a less dense unperturbed plasma has a lower electron concentration at the boundary, which creates more favorable conditions for the penetration of the field deep into the plasma.

The penetration depth of the field into the dielectric monotonically increases with increasing wave amplitude:

$$\frac{\partial\delta_2}{\partial B}=-\frac{1}{2(\eta-\varepsilon_2)^{3/2}}\frac{\partial\eta}{\partial B}\geqslant 0.$$

Thus, for a sufficiently large wave amplitude the wave field is expelled from the plasma and the wave propagates primarily along the dielectric. The electric field of the wave in the dielectric then becomes a transverse field. In the plasma, on the other hand, as was shown earlier, the longitudinal component of the electric field predominates.

4. The energy flux density in the plasma averaged over the oscillation period is equal to

$$S=\text{Re}\frac{c}{8\pi}[\mathbf{E}\times\mathbf{H}^*]=-\frac{c\eta^{1/2}}{4\pi}E_0^2\frac{b^2}{|\varepsilon|}. \tag{4.1}$$

As in the case of a homogeneous plasma, this quantity is negative and vanishes near the critical frequency: $S \approx \eta^{-1/2} \to 0$. The energy flux decreases together with the wave field as we go into the plasma. For a relatively small wave amplitude, or at points sufficiently far from the boundary, this decrease will be exponential.

For a sufficiently large wave amplitude

$$b^2 \sim |\varepsilon|^{(m+2)/m}, \quad \eta \to \varepsilon_2 \tag{4.2}$$

and the energy flux density is equal to

$$S \approx -(c/4\pi) E_0^2 \varepsilon_2^{1/2} |\varepsilon|^{2/m}. \tag{4.3}$$

It can be seen that, in contrast to what is obtained in the linear theory, the energy flux density in the plasma does not decrease with increasing $|\epsilon|$, but increases.

It is worth noting that as the exponent m in the ionization law increases, the energy flux density tends to some limit

$$S_0 = -(c/4\pi) E_0^2 \varepsilon_2^{1/2}, \tag{4.4}$$

not depending on the wave amplitude and determined only by the effective ionization field and the electric properties of the dielectric. Such an unusual behavior of the energy flux density is connected with the dependence of the electron concentration in the plasma on the wave field. As the amplitude of the wave in the plasma increases, the magnetic field and the electron concentration increase. The increase, however, of the electron concentration leads, in its turn, to the increase of the modulus $|\epsilon|$ and, consequently, to the decrease in the case when $m > 2$ of the transverse component of the electric field $E_\perp$. For a sufficiently large wave amplitude the growth of the magnetic field is compensated by the decrease of $E_\perp$, which explains the appearance of some wave-amplitude-independent limiting value of the energy flux density. In the $m \leq 2$ case such a compensation does not occur, and the energy flux increases with increasing $|\epsilon|$.

For a sufficiently large wave amplitude the total energy flux in the plasma

$$\bar{S} = \frac{1}{k} \int_0^\infty S \, d\zeta$$

can be estimated with the aid of (3.4) as follows:

$$\bar{S} \approx S|_{\zeta=0} \delta \approx -(c/4\pi) E_0^2 \varepsilon_2^{1/2} |\varepsilon|^{(2-m)/m} \Delta. \tag{4.5}$$

For $m > 2$ the total energy flux in the plasma decreases with increasing amplitude, while for $m < 2$ it increases. For $m = 2$ the total energy flux in the plasma does not depend on the wave amplitude:

$$\bar{S} \approx -(c/4\pi) E_0^2 \varepsilon_2^{1/2} \Delta. \tag{4.6}$$

The increase of the energy flux density in this case is exactly compensated by the decrease of the penetration depth of the field into the plasma.

In conclusion, let us estimate that interval of the plasma and wave parameters in which the assumptions made above that we can neglect the collision rate, diffusion, and the redistribution of the concentration of the plasma electrons as a result of the heating of the plasma in the inhomogeneous field are valid. These assumptions can be reduced to the following inequalities [8]:

$$\nu \ll \omega, \quad (D_a/\rho n)^{1/2} \ll \delta, \quad l(2m_e/M)^{-1/2} \ll \delta, \tag{4.7}$$

where l is the mean free path of the electron, D_a is the coefficient of ambipolar diffusion, $2m_e/M$ is twice the ratio of the electron mass to the atom mass, ρ is the coefficient of volume recombination, and δ is the penetration depth of the field into the plasma.

Let us give an example when these inequalities are not inconsistent. According to [9], the electron-neutral collision rates in the plasmas of such inert gases as Ar and Ne at an electron temperature $T_e \sim 1-3$ eV can be estimated from the formula

$$\nu\,(\text{sec}^{-1}) \approx 3 \cdot 10^9 p \ (\text{Torr}).$$

The coefficient of ambipolar diffusion, the mean free path, and the coefficient of dissociative recombination characteristic of the inert gases [10] are equal to

$$D_a(\text{cm}^2/\text{sec}) \approx 10^2/p\,(\text{Torr}), \quad l(\text{cm}) \approx 10^{-2}/p\,(\text{Torr}),$$

$$\rho \approx 10^{-6} - 10^{-8}\ \text{cm}^3/\text{sec}.$$

For a wave of frequency $\omega = 6 \times 10^{10}$ sec^{-1} ($\lambda = 3$ cm), the inequalities (4.7) are always fulfilled in the pressure range $10 \leq p\,(\text{Torr}) \leq 20$ for a plasma of electron concentration 10^{10} cm^{-3} and above, with the exception of the case when the surface wave propagates under conditions of very strong retardation.

The authors are grateful to Yu. G. Gurevich and A. P. Motornenko for a discussion and a number of valuable comments and to É. A. Kaner for his attention to the work.

[1] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plasme (Propagation of Electromagnetic Waves in a Plasma), Nauka, 1967 (Eng. Transl., Gordon and Breach, New York, 1962).

[2] Yu. R. Alanakyan, Zh. Tekh. Fiz. **37**, 817 (1967) [Sov. Phys. Tech. Phys. **12**, 587 (1967)].

[3] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz, 1959 (Eng. Transl., Pergamon, London, 1960).

[4] V. E. Golant, A. P. Zhilinskiĭ, M. V. Krivosheev, and A. I. Chernova, Zh. Tekh. Fiz. **31**, 63 (1961) [Sov. Phys. Tech. Phys. **6**, 44 (1961)].

[5] V. F. Virko and S. M. Levitskiĭ, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **9**, 691 (1966) [Sov. Radiophys. **9**, 412 (1966)].

[6] A. G. Boev and A. V. Prokopov, Zh. Tekh. Fiz. **44**, 1442 (1974) [Sov. Phys. Tech. Phys. **19**, 901 (1975)].

[7] V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (The Electromagnetic Properties of a Plasma and Plasma-like Media), Atomizdat, 1961.

[8] A. V. Gurevich and A. B. Shvartsburg, Nelineĭnaya teoriya rasprostraneniya radiovoln v ionosfere (Nonlinear Theory of the Propagation of Radio Waves in the Ionosphere), Nauka, 1973.

[9] V. E. Golant, Sverkhvysokochastotnye metody issledovaniya plazmy (Ultrahigh-frequency Methods of Investigating Plasmas), Nauka, 1968.

[10] E. W. McDaniel, Collision Phenomena in Ionized Gases, Wiley, New York, 1964 (Russ. Transl., Mir, 1967).

Translated by A. K. Agyei
133