The pomeron as a Goldstone boson

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This paper developes the concept that the Pomeranchuk pole (pomeron) is the result of a spontaneous breaking of a continuous symmetry of the system of Regge poles of the vacuum channel. An interpretation is proposed for the appearance of the spontaneous symmetry breaking for reggeons. The choice of symmetry group naturally introduces a second vacuum pole P' shifted from $j = 1$ and having an α' different from that of the pomeron. The possible interactions of the P and P' are determined up to the squares of the reggeon momenta $\mathbf{k}_i$. They lead to a quasistable pomeron in the same manner as in the model of weak pomeron coupling.

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INTRODUCTION

In Regge pole theory the asymptotic behavior of the strong interactions is described by the Pomeranchuk pole (the pomeron P). According to the traditional concepts this singularity is a pole of the t-channel partial-wave amplitude $f_j(t)$ having the quantum numbers of the vacuum ($t = -\mathbf{k}^2$ is the momentum transfer, $\mathbf{k}$ is the transverse momentum). The trajectory of the pole in the complex angular momentum plane j passes through the point $j = 1$ for $t = 0$. A pole exhibiting these properties quarantees the constancy of the total cross sections at high energies.

The interaction of the pomerons with each other (and with other poles of the t-channel partial-wave amplitudes) leads to the appearance of branch points in the j-plane; the contributions of these branch points to the asymptotic behavior determine corrections to the simple Regge formulas. The corresponding calculations are described by Gribov's reggeon diagram technique[1]. For small momenta $\mathbf{k}_i$ of the reggeons the diagrams can be obtained from a nonrelativistic nonhermitean Lagrangian with imaginary time[2–5]

$$L=-\frac{1}{2}\left(\psi^+\frac{\partial\psi}{\partial\xi}-\frac{\partial\psi^+}{\partial\xi}\psi\right)-\alpha'\nabla\psi^+\nabla\psi-\Delta\psi^+\psi+\frac{r}{2}(\psi^+\psi^2-\psi\psi^{+2})$$
$$+\frac{\lambda_1}{3!}(\psi^+\psi^3+\psi^{+3}\psi)+\lambda_2\psi^{+2}\psi^2+(\text{higher powers of } \psi, \psi^+, \nabla\psi \text{ and } \nabla\psi^+). \tag{1}$$

In Eq. (1) the reggeon wave fields ψ and ψ^+ are defined in the three-dimensional space of imaginary time ξ and of the two spatial coordinates $\boldsymbol{\rho}$:

$$\psi(\xi,\boldsymbol{\rho})=\sum_{\mathbf{k},\omega=-\alpha(\mathbf{k}^2)} a(\mathbf{k},\omega)e^{i\mathbf{k}\boldsymbol{\rho}+\omega\xi},$$
$$\psi^+(\xi,\boldsymbol{\rho})=\sum_{\mathbf{k},\omega=-\alpha(\mathbf{k}^2)} a^+(\mathbf{k},\omega)e^{-i\mathbf{k}\boldsymbol{\rho}-\omega\xi}. \tag{2}$$

Here $\omega = j - 1$ plays the role of the reggeon energy, $\alpha(\mathbf{k}^2)$ is the bare reggeon trajectory (its energy spectrum ($E = -\omega = -\alpha(\mathbf{k}^2)$) before the interaction is switched on). In Eq. (1) which corresponds to small values of $\mathbf{k}$, we have

$$\alpha(\mathbf{k}^2)=\Delta+\alpha'\mathbf{k}^2. \tag{3}$$

The "time" ξ is canonically conjugate to the angular momentum $\omega = j - 1$. It is equal to the logarithm of the energy per reggeon, $\xi = \ln(s/s_0)$. The spatial coordinates $\boldsymbol{\rho}$ form the plane of impact parameters and are canonically conjugate to the momenta $\mathbf{k}$.

The quantity Δ determines the shift of the position of the pole from the point $\omega = 0$ for $\mathbf{k}^2 = 0$ (the "energy gap"). For the pomeron $\Delta = 0$. Therefore the problem of summing all pomeron contributions is the same as the problem of the interaction of two-dimensional nonrelativistic excitations without an energy gap. In general the interactions lead to a shift of the pomeron position from $\omega = 0$. In order to keep the bare pomeron pole at $\omega = 0$ ($\Delta_P = 0$) it is necessary to attribute to it an initial shift which compensates exactly the renormalization effects.[4] This assumption does not follow from anything, but without it a massless excitation does not appear in the theory.

The problem of a massless nonrelativistic excitation inevitably leads to a pomeron which is unstable for $\mathbf{k} \neq 0$, since the energy-momentum conservation laws allow a pomeron $\omega = -\alpha'\mathbf{k}^2$ to decay into two-pomeron, three-pomeron, etc. states having for the same momentum $\mathbf{k}$ the thresholds $\omega = -\alpha'\mathbf{k}^2/2$, $\omega = -\alpha'\mathbf{k}^2/3$, etc. (Figs. 1 and 2). For nonvanishing r and λ in (1) this instability strongly affects the trajectory and the character of the pomeron singularity (strong coupling)[4,5]: the main resulting singularity is a branch point; for $\mathbf{k} = 0$ the pole disappears. For this reason the hypothesis of a bare pomeron as a pole in the strong coupling variant appears to be rather artificial.

Another variant of solution to the problem (weak coupling) starts from the desire to have a stable pomeron pole at $\mathbf{k} \neq 0$. This imposes on the pomeron interaction vertices in (1) a number of interdiction rules.[2] Thus, e.g., in the weak coupling variant the vertices r and λ (Fig. 2) must be proportional to a power of the momentum $\mathbf{k}$ ($r = \lambda_i = 0$ for $\mathbf{k} = 0$), i.e., the Lagrangian (1) can contain only interaction terms with $\nabla\psi$ and $\nabla\psi^+$. Another number of restrictions appears when one tries to make the weak coupling agree with the s-channel unitarity condition[6].

Without a dynamical principle to be put at the basis of such a chain of restrictions the weak coupling variant also seems very artificial. At the same time, in a theory with a massless particle the existence of restrictions on its interactions is quite natural. For an arbitrary chosen interaction the infrared stability of the

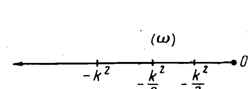

FIG. 1

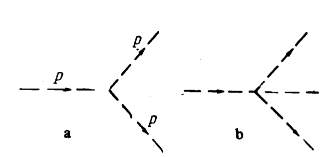

FIG. 2

particle may lead to its disappearance, which in fact did occur in the variant of pomeron strong coupling. Therefore, if the total cross section is constant as $s \to \infty$ the existence of a dynamical principle guaranteeing the stability of the pomeron problem is quite likely.

In the present paper we discuss the possibility and consequences of the hypothesis that the reason for the appearance of restrictions and constraints on the quantities in the pomeron problem is the existence of a continuous group of symmetries of the reggeon system in the vacuum channel. A spontaneous symmetry breaking, if possible for this system, leads to the appearance of a massless excitation[7,8]. Thus, the pomeron appears in the theory as a Goldstone boson, and its interactions will necessarily be subject to conditions which guarantee the infrared stability.

In field theory spontaneous symmetry breaking occurs in systems for which the potential energy has a specific shape (with a minimum for a $\Phi \neq 0$ value of the field operator). In such systems the vacuum turns out to be unstable and the stable vacuum corresponds to the value of Φ at the minimum. In reggeon theory the concepts of vacuum and potential energy are not obvious. Therefore the problem of the meaning of spontaneous symmetry breakdown and instability of the vacuum for the reggeon system must be cleared up. A possible interpretation of these concepts in reggeon language is discussed in Sec. 1 and in the Appendix. The continuous symmetry group and the interactions for the Goldstone pomeron are investigated in Secs. 2—4. The properties of the solutions and their characteristic experimental manifestations will be considered in another paper.

1. THE INSTABILITY OF THE VACUUM OF A REGGEON SYSTEM

Spontaneous symmetry breakdown is possible in physical systems for which the vacuum is unstable with respect to small shifts in the field operators Φ. Such a shift $\delta\Phi$ leads to the appearance in the Lagrangian of the system of terms describing transitions of excitations into the vacuum and vice versa. Consequently a Lagrangian with $\delta\Phi \neq 0$ yields diagrams of the tree type (Fig. 3). Instability of the vacuum means that vacuum terms in the Lagrangian and tree diagrams disappear for some finite value $\delta\Phi = \Phi_0 \neq 0$. For this value of Φ the system has minimal energy, a new vacuum is defined, as well as new excitations existing in the system.

It is easy to think of a similar situation in reggeon theory. The diagrams describing reggeon-vacuum transitions can be given the same intuitive interpretation as for the reggeons themselves[9,10]. As is well known the reggeon is described by a process of multiperipheral parton (or virtual particle[9]) production from the state of the incident particle. Such a "parton comb" interacts with the target, and then again coalesces into the scattered particle (Fig. 4,a). This picture reproduces exactly the properties of reggeon contributions to hadronic amplitudes at high energies.

For the vacuum poles the formation and coalescence of the multiperipheral parton fluctuation can be multiply ramified (Fig. 4,b). One of the ramifications interacts with the target, and the others must coalesce, simulating transitions into the vacuum. To the processes represented by Fig. 4,b correspond the reggeon dia-

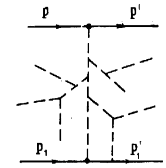

FIG. 3

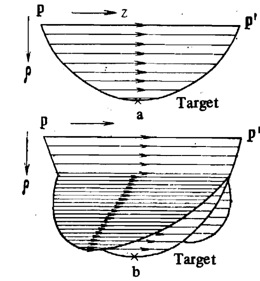

FIG. 4

grams of Fig. 3. We note that the diagrams of Fig. 3 cannot be obtained by investigating the asymptotic behavior of Feynman diagrams[1] and require a partonic description of the reggeons.

Thus the "parton vacuum reggeons" (PR) and their diagrams may correspond to an unstable vacuum and spontaneous symmetry breaking of the system. The Lagrangian (1) contains vacuum terms in the PR operators. For "hadronic reggeons" (HR), corresponding to definite terms in the asymptotic behavior of Feynman amplitudes, the Lagrangian must not contain either tree diagram terms or terms corresponding to transitions into the vacuum. The HR are obtained from the PR by summing all possible diagrams involving transitions into the vacuum (Fig. 3). For appropriate values of the parameters such a procedure leads to the appearance of a finite shift of the reggeon operators, in the same manner as in ordinary field theory. The shift corresponds to a transition from the vacuum of the PR to the vacuum of the HR and the vacuum terms in the Lagrangian disappear as a result. In order to explain the summation procedure of the reggeon-vacuum transitions we consider in the Appendix a simple model with the help of which are defined the quantities entering the reggeon problem as well as the regions in which the parameters of the problem lead to spontaneous symmetry breakdown. The results of this consideration are, of course, completely analogous to the corresponding problems in field theory. Therefore in the following section we shall consider the problem of spontaneous symmetry breakdown of the reggeon system directly for the reggeon Lagrangian.

2. THE INTERACTIONS OF A GOLDSTONE POMERON. THE SIMPLEST RENORMALIZABLE LAGRANGIAN

The channel with the quantum numbers of the vacuum contains two reggeon poles: the pomeron P and the second vacuum pole P′. The existence of two states is a strong support for the hypothesis of the Goldstone boson, since for the spontaneous breaking of a continuous symmetry there must appear several states with different mass values.

We shall denote the two fields which after spontaneous symmetry breaking lead to the P and P′ by ψ_i ($\psi_i^\dagger$), $i = 1, 2$. The symmetry group which transforms the subscript i must be a real group, since according to the arguments of Sec. 1 before the spontaneous symmetry breaking the invariant Lagrangian may contain the reggeon-vacuum transition terms ψ_i^{+2}, ψ_i^2, etc. Therefore, the group which will be considered here is the group of rotations around an axis O(2).

In place of the complex operators ψ_i and $\psi_i^\dagger$ (for imaginary time ξ this complex character is a matter of convention) it is convenient[1)] to introduce their real combinations

$$\varphi_i^- = {}^1/_2(\psi_i^+ - \psi_i), \quad \varphi_i^+ = {}^1/_2(\psi_i^+ + \psi_i). \tag{4}$$

It is easy to rewrite the free Lagrangian in terms of the operators (4):

$$L_0 = -\left[\varphi_i^- \frac{\partial \varphi_i^+}{\partial \xi} - \frac{\partial \varphi_i^-}{\partial \xi}\varphi_i^+\right] + \alpha'(\nabla\varphi_i^-)^2 - \alpha'(\nabla\varphi_i^+)^2. \tag{5}$$

The kinetic part of (1) which contains the operators $\nabla\varphi^-$ and $\nabla\varphi^+$ appears unchanged in (5). In Sec. 4, where the interactions are discussed, the terms involving α' are included in the general procedure outlined below. As potential energy we write any invariant form made up of the operators φ_i^- and φ_i^+ effect in it the shift which is responsible for the spontaneous symmetry breaking. The broken-symmetry Lagrangian describes the interaction of the real states P and P′ and should not contain transitions into the vacuum. This program necessarily leads to a Goldstone boson and determines the conditions imposed on it by the interactions.

In this section we consider the simplest form for the potential energy, including only terms up to φ^4. The modifications introduced by interactions involving higher powers (which are nonrenormalizable) will be discussed in the sequel.

The most general invariant form of fourth order in φ_i^+ and φ_i^- is

$$\Delta_1\varphi_i^{-2} + \Delta_2\varphi_i^{+2} + \frac{2^4}{4!}[\lambda_1(\varphi_i^{-2})^2 + \lambda_2(\varphi_i^{+2})^2 + \lambda_3\varphi_i^{-2}\varphi_k^{+2} + \lambda_4(\varphi_i^-\varphi_i^+)^2] + \tilde{\Delta}(\varphi_i^-\varphi_i^+)\varepsilon_{kl}\varphi_k^-\varphi_l^+. \tag{6}$$

Here ϵ_{kl} is the second rank antisymmetric unit tensor. On first thought one might add to (6) the invariant expression

$$(\varphi_i^-\varphi_i^+)[\Delta' + \lambda_1'\varphi_i^{-2} + \lambda_2'\varphi_i^{+2}] + \varepsilon_{ik}\varphi_i^-\varphi_k^+ \cdot [\Delta'' + \lambda_1''\varphi_i^{-2} + \lambda_2''\varphi_i^{+2}]. \tag{7}$$

However, there is a fundamental difference between the expressions (6) and (7). The expression (6) does not change sign under the substitution $\varphi_i^- \to -\varphi_i^-$, $\varphi_i^+ \to \varphi_i^+$ (or $\varphi_i^- \to \varphi_i^-$, $\varphi_i^+ \to -\varphi_i^+$) whereas (7) does change sign under this substitution. Such a substitution realizes a conjugation of the "temporal" parity, since the free Lagrangian (5) is invariant under a simultaneous change of signs of ξ and one of the $\varphi^\pm$. This difference in the properties of the expressions (6) and (7) leads to the result that the reggeon interactions to which they give rise exhibit opposite hermiticity properties.

According to the analysis carried out by Gribov[1], the interactions involving an even number of reggeons of positive signature must be hermitean, and the interactions involving an odd number of such reggeons must be anti-Hermitian. This rule refers both to the pomeron and to the other poles of positive signature if their shift Δ from j = 1 is not too large ($\Delta < {}^1/_2$). It is easy to verify that only the interactions which are obtained from (6) satisfy this rule. Consequently the expression (7) cannot be included in the expression for the potential energy of positive signature poles. The interactions of PR with positive signature conserve the "temporal" parity.

The parity rule also leads to a unique selection of the operator in which the constant shift appears through spontaneous symmetry breakdown. This shift leads to the appearance in (6) of vertices with an odd number of reggeons. Since such vertices must be anti-Hermitian the constant shift affects the operator φ^-:

$$\varphi_i^- \to \varphi_i^- + {}^1/_2\varphi\delta_{i,1}. \tag{8}$$

Substituting (4) and (7) into (6) and setting the coefficients of terms which contain only the operators ψ_i^+ or ψ_i equal to zero, we obtain a system of equations for the coefficients of the form (6)

$$\begin{gathered}\tilde{\Delta}=0, \quad \lambda_1+\lambda_2+\lambda_3+\lambda_4=0,\\ 2\lambda_1+\lambda_3+\lambda_4=0, \quad 4\lambda_1+\lambda_4=0,\\ {}^1/_4\Delta_1 + {}^1/_4\Delta_2 + {}^1/_{12}\lambda_1\varphi^2 + \lambda_3\varphi^2/4! = 0,\\ \varphi(\Delta_1 + {}^1/_3\lambda_1\varphi^2) = 0.\end{gathered} \tag{9}$$

The solution of (9) is

$$\lambda_1=\lambda_2, \quad \lambda_3=2\lambda_1, \quad \lambda_4=-4\lambda_1. \tag{10}$$

Calculating the coefficients of all the interaction terms in (6) with the help of (10) one can obtain the following simple Lagrangian for the system of P′ (i = 1) and P (i = 2):

$$L = -\frac{1}{2}\left[\psi_i^+ \frac{\partial\psi_i}{\partial\xi} - \frac{\partial\psi_i^+}{\partial\xi}\psi_i\right] - \alpha'\nabla\psi_i^+\nabla\psi_i - \Delta\psi_1^+\psi_1 + (\Delta\lambda)^{1/2}[\psi_1^+\psi_i^2 - \psi_1\psi_i^{+2}] + \lambda\psi_i^{+2}\psi_k^2. \tag{11}$$

In (11) we have changed the notation for the coefficients:

$$\Delta = {}^2/_3\lambda_1\varphi^2, \quad \lambda = {}^2/_3\lambda_1.$$

The vacuum conditions (9) or (17) (cf. Sec. 3) lead to interesting and important consequences which are valid in nonrelativistic theory (for Hermitian and non-Hermitian interactions) independently of the invariant form chosen for the potential energy of interaction—the simple form (6) or the more complicated expressions (13), (14), (15) which include higher powers of $\varphi_i^\pm$ (or even terms of the opposite "temporal" parity, (7)):

1. The pomeron mass, being the mass of a Goldstone boson, vanishes. This is, of course, trivial, since we expect exactly this. However, we stress the fact that in a nonrelativistic theory the vanishing of the mass is a direct consequence of the vacuum conditions.

2. All decay constants of the pomeron vanish identically. This is also a direct consequence of (6) or (17). For the Lagrangian (11) the interaction vertices are represented in Fig. 5. Among them there are not the decay vertices of Fig. 2 and Fig. 6. Any pomeron decays are also forbidden for the more complicated interactions of Sec. 4. The vacuum conditions preserve the absolute stability of the pomeron as a Goldstone particle. This property is characteristic only for a nonrelativistic problem. The symmetry group preserves only those pomeron interactions which cannot lead to a shift from its position at $\omega = 0$. In relativistic models[7,8] such an absolute interdiction does not exist. This is related to the fact that the Goldstone particle is situated at the same point $k_\mu^2 = 0$ as the invariant thresholds of its possible decay reactions. Therefore in the relativistic case the problem of stability of the position of the Goldstone particle is solved together with the infrared problem.

3. For the infrared stability of the theory in models with spontaneous breakdown of a continuous symmetry there appear relations between the coupling constants of various interactions. In a nonrelativistic theory these relations are again consequences of the vacuum conditions (9) or (17). Such a relation is explicitly written in (11). Owing to its existence the scattering of

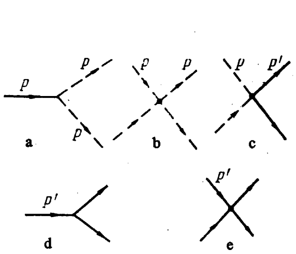

FIG. 5

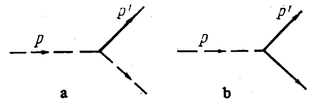

FIG. 6

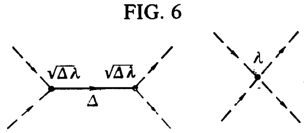

FIG. 7

pomerons on pomerons (Fig. 7) vanishes for $\omega = 0$ and $\mathbf{k} = 0$. Indeed, the sum of the two diagrams of Fig. 7 equals

$$-\frac{\Delta\lambda}{\omega+\Delta+\mathbf{k}^2}+\lambda=\lambda\frac{\omega+\mathbf{k}^2}{\omega+\Delta+\mathbf{k}^2}, \tag{12}$$

Eq. (12) signifies that the infrared singularities will be inessential in all the diagrams that have only pomerons as external lines. In the diagrams with P′ as external particles[2)] the infrared singularities may give a large contribution, but the conditions (12) let them survive only in the lowest-order diagrams. For the invariant form (6) these consequences are derived very easily. However, it is not at all trivial that they are also confirmed for the forms discussed in the following section.

Finally, it will be shown in Sec. 4 that the inclusion of terms with $\nabla\psi$ and $\nabla\psi^+$ into the interaction leads, in general, to a violation of the consequence 2.

3. MORE COMPLICATED FORMS FOR THE POTENTIAL ENERGY OF REGGEON INTERACTION

The main problem of this section is to show that the conclusions reached in Sec. 2 remain unchanged when the simple invariant form (6) is replaced by a potential energy of arbitrary form, containing higher powers of $\varphi_i^{\pm}$. Terms with high powers of $\varphi_i^{\pm}$ influence the interactions with a small number of pomerons (Fig. 5) since the shift (8) separates from the higher powers of $\varphi_i^{\pm}$ any lower power. The vacuum equations of type (9) simultaneously contain coefficients for forms of arbitrary order, and the relations obtained by means of simple invariant forms could be modified. However, it turns out that for the interactions which can change the stability of the pomeron and worsen the infrared situation, the interdictions and the relations among the coupling constants are not changed. Changes occur only in those P and P′ vertices which cannot lead to infrared-divergent diagrams.

We carry out the proof for terms up to $(\varphi_i^{\pm 2})^4$. Forms of the eighth degree are of interest since in them there appear for the first time interactions containing an odd number of pomerons (from terms with ϵ_{ik}). Thus, we add to (6) the invariant forms of sixth order

$$\frac{2^6}{6!}\{g_1(\varphi_l^{-2})^3+g_2(\varphi_l^{-2})^2\varphi_k^{+2}+g_3\varphi_l^{-2}(\varphi_i^-\varphi_i^-)^2+g_4\varphi_l^{+2}(\varphi_i^-\varphi_i^+)^2 +g_5\varphi_i^{-2}(\varphi_l^{+2})^2+g_6(\varphi_i^{+2})^3\} \tag{13}$$

and of eighth order

$$\frac{2^8}{8!}\{h_1(\varphi_i^{-2})^4+h_2(\varphi_i^{-2})^3\varphi_k^{+2}+h_3(\varphi_i^{-2})^2(\varphi_k^{+2})^2+h_4(\varphi_i^{-2})^2(\varphi_k^-\varphi_k^+)^2 +h_5\varphi_i^{-2}(\varphi_k^-\varphi_k^+)^2\varphi_l^{+2}+h_6(\varphi_i^-\varphi_i^+)^2(\varphi_k^{+2})^2+h_7(\varphi_i^{+2})^3\varphi_k^{-2} +h_8(\varphi_i^{+2})^4+h_9(\varphi_i^-\varphi_i^+)^4\}, \tag{14}$$

as well as terms proportional to ϵ_{ik} which lead to vertices with an odd number of pomerons

$$V_4(\varphi_i^-\varphi_i^+)\varepsilon_{kl}\varphi_k^-\varphi_l^+,$$
$$V_4=\tilde\Delta+\tilde\Delta_1\varphi_i^{-2}+\tilde\Delta_2\varphi_i^{+2} +\frac{1}{4!}\{\tilde\lambda_1(\varphi_i^{-2})^2+\tilde\lambda_2(\varphi_i^{+2})^2 +\tilde\lambda_3\varphi_i^{-2}\varphi_k^{+2}+\tilde\lambda_4(\varphi_i^-\varphi_i^+)^2\}. \tag{15}$$

The vacuum equations for (15) are separated from (6), (13) and (14) and are absolutely analogous to (9) with the exception of the fact that they contain an extra constant $\tilde\Delta$ and an additional equation for it

$$\tilde\Delta+{}^1/_4\tilde\Delta_1\varphi^2+\tilde\lambda_1\varphi^4/4!. \tag{16}$$

The terms (15) lead to interactions, the simplest among which are represented in Fig. 8. We only note that the vacuum conditions are forbidding the existence of pomeron interactions of Fig. 8e in the Lagrangian, and are allowing the presence of interactions of the type of Fig. 8c. They also remove from (15) all the pomeron decay vertices.

As regards the interactions with an even number of pomerons, the forms (6), (13), (14) considered together for them yield a set of 14 vacuum equations for 21 coefficients and for the shift φ in (8).[3)] We list these equations directly in the form of their solutions:

$$\begin{gathered}
-h_4'=4h_1'+{}^1/_4h_9', \quad h_8'=h_1'+{}^1/_2[g_1'-g_6'+{}^1/_4(g_3'-g_4')],\\
h_2'=2h_1'-{}^1/_4h_5'-{}^1/_8h_9', \quad h_9'=16h_1'+8g_1'+2g_3',\\
h_3'=h_1'+h_8'-{}^1/_2h_5'-{}^1/_4h_9', \quad h_5'=-16h_1'-10g_1'+2g_2'-{}^3/_2g_3'+{}^1/_2g_4',\\
-h_6'=4h_8'+{}^1/_4h_9', \quad -h_1'={}^1/_2(\lambda_1+{}^1/_4\lambda_4)+{}^5/_8g_1'+{}^1/_{32}g_3',\\
h_7'=2h_8'-{}^1/_4h_5'-{}^1/_8h_9', \quad g_1'-g_2'+g_5'-g_6'=0,\\
-\lambda_1+\lambda_2+{}^1/_4(g_1'-g_6')-{}^1/_2(g_2'-g_5')+{}^3/_{16}(g_3'-g_4')=0,\\
\lambda_1+\lambda_2-\lambda_3+{}^3/_8(g_1'+g_6')-{}^1/_8(g_2'+g_5')=0,\\
\Delta_1+{}^1/_3\lambda_1\varphi^2+{}^1/_2g_1'\varphi^2+{}^2/_3h_1'\varphi^2=0,\\
\Delta_2+{}^1/_6\lambda_3\varphi^2+{}^1/_{12}g_2'\varphi^2+{}^1/_2h_2'\varphi^2=0,\\
g_i'=\frac{g_i\varphi^2}{3\cdot5}; \quad h_i'=\frac{h_i\varphi^4}{3\cdot5\cdot7\cdot8}.
\end{gathered} \tag{17}$$

With the help of Eqs. (17) one can express the coefficient of any interaction vertex in terms of independent constants, for which it is convenient to choose the λ_i, any three g_i, and φ. The coefficients of all the decay vertices of the pomeron will vanish, i.e., the pomeron remains absolutely stable. The relations derived for the forms of lower degree are preserved if they are designed to remove the possible infrared singularities. Thus, if one calculates the coefficients of the vertices of the diagrams in Figs. 5a and 5b, one obtains the expressions:

$$\begin{gathered}
r={}^1/_3\varphi[\lambda_1+{}^3/_4\lambda_4+{}^3/_4(g_1'+{}^1/_4g_3')];\\
\lambda=-{}^1/_3[\lambda_1+{}^3/_4\lambda_4+{}^3/_4(g_1'+{}^1/_4g_3')],
\end{gathered} \tag{18}$$

whereas the magnitude of the shift Δ for P′ is

$$\Delta=-{}^1/_3\varphi^2[\lambda_1+{}^3/_4\lambda_4+{}^3/_4(g_1'+{}^1/_4g_3')]. \tag{19}$$

Thus, as in Eq. (11), (18) and (19) imply

$$r^2=\lambda\Delta. \tag{20}$$

At the same time in (11) the coefficient of the vertex of

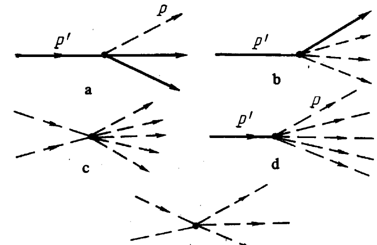

FIG. 8

Fig. 5c (and of Fig. 5b) equals the coefficient $(\Delta\lambda)^{1/2}(\lambda)$ for the diagram in Fig. 5a (Fig. 5e). The loss of this relation does not worsen the infrared situation and it was not maintained in higher-order forms. If one denotes the constant at the vertex of Fig. 5c by $(\Delta\lambda_1)^{1/2}$, then the preceding consideration shows that the vertex of Fig. 5c turns out to be equal to $(\lambda\lambda_1)^{1/2}$. Such a relation among the vertices of Figs. 5a, 5c, 5d leads to a weakening of the infrared singularities in the $2P \to 2P'$ transition channels, analogous to (12). The coupling constant of four P′ reggeons (Fig. 5e) remains completely independent. Relations between the new interactions appear in analogous cases. Thus, the vertices of Fig. 8c and 8d are related to one another and Δ (19) by Eq. (20). For such a relation there appears the possibility of cancellation of diagrams from different interactions, as happened in (12).

Thus, for local interactions which we have considered in Secs. 2 and 3 the nonrelativistic theory yields an absolutely stable Goldstone boson—the pomeron—whose diagrams exhibit a cancellation of the infrared singularities. Nonlocal interactions lead to a weak instability.

4. THE INSTABILITY OF THE POMERON IN ORDER k^2

The interaction of pomerons is nonlocal, since the blocks which describe its interactions may depend on the reggeon momenta $\mathbf{k}_i$. Until now we have considered these blocks in the limit of small momenta and have found those parts of the interactions which do not vanish for $\mathbf{k}_i = 0$. In order to study those vertices which vanish for $\mathbf{k}_i = 0$ (Figs. 2 and 6) we must consider the next order in the expansion with respect to momenta, or, what amounts to the same, take into account in the Lagrangian terms which depend on $\nabla\psi$ and $\nabla\psi^+$.

Let us consider the most general invariant form containing the square of the gradient (without ϵ_{ik} terms). It has the form

$$\begin{gathered}\alpha_1'(\nabla\varphi_i^-)^2+\alpha_2'(\nabla\varphi_i^+)^2+(\nabla\varphi_i^-)^2[\sigma_1(\varphi_k^-)^2\\ +\sigma_2(\varphi_k^+)^2]+(\nabla\varphi_i^+)^2[\sigma_3(\varphi_k^-)^2+\sigma_4(\varphi_k^+)^2]\\ +\sigma_5(\nabla\varphi_i^-\nabla\varphi_i^+)(\varphi_k^-\varphi_k^+)+\sigma_6(\nabla\varphi_i^-\varphi_i^-)^2\\ +\sigma_7(\nabla\varphi_i^-\varphi_i^-)(\nabla\varphi_k^+\varphi_k^+)+\sigma_8(\nabla\varphi_i^+\varphi_i^+)^2+\sigma_9(\nabla\varphi_i^-\varphi_i^+)^2\\ +\sigma_{10}(\nabla\varphi_i^-\varphi_i^+)(\nabla\varphi_k^+\varphi_k^-)+\sigma_{11}(\nabla\varphi_i^+\varphi_i^-)^2.\end{gathered} \tag{21}$$

This form also includes the terms with α' in (5) and therefore it must determine the relation between the slopes of P and P′.

We carry out in (21) the shift (8) and equate to zero the vacuum transition terms. This leads to the equations:

$$\begin{gathered}\alpha_1'+\alpha_2'+{}^1/_4(\sigma_1+\sigma_3)\varphi^2=0,\quad \sigma_6+\sigma_{11}=0,\\ \sigma_1+\sigma_3+{}^1/_2\sigma_5=0,\quad \sigma_7+\sigma_{10}=0,\\ \sigma_1+\sigma_2+\sigma_3+\sigma_4+\sigma_5=0,\quad \sigma_8+\sigma_9=0.\end{gathered} \tag{22}$$

A calculation of the interactions remaining in (21), after the relations (22) are taken into account, shows that the pomeron becomes unstable, since it leads to vertices represented in Fig. 9. The sign Δ in these diagrams indicates to the momenta of which lines the vertex is proportional. Under these conditions the role of the group in these conditions reduces to forbidding all the transitions which could lead to a shift of the pomeron position for $k^2 = 0$, i.e., which would require a renormalization of the energy gap. These are the vertices of the type of Fig. 9d and 9e. Indeed, their disappearance from (21) is a direct consequence of Eqs. (22). The group properties permit a change of the slope of

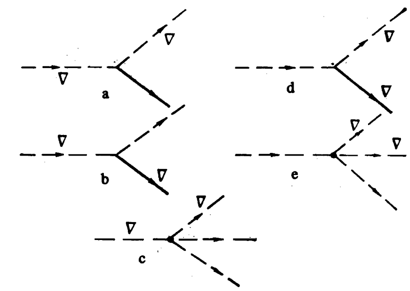

FIG. 9

the pomeron (a renormalization of α'), a change of the trajectory in the $\mathbf{k}^4$ terms, but strictly interdict a renormalization of the pomeron position, it being a Goldstone boson.

It is interesting to note that (21) and (22) naturally yield a difference in the slopes of P and P′. Indeed, a computation of the coefficients of $\nabla\psi_2^+\nabla\psi_2(\mathrm{P})$ and of $\nabla\psi_1^+\nabla\psi_1(\mathrm{P}')$ leads to the following result:

$$\alpha_P=\alpha_1'+4\sigma_1\varphi^2,\quad \alpha_{P'}=\alpha_1'+4(\sigma_1+\sigma_6)\varphi^2. \tag{23}$$

The same procedure allows one to obtain the dependence of the pomeron decay vertices on the momenta (Fig. 9, a, b, c). The decay vertex of a pomeron into three pomerons (Fig. 9, c) must be proportional to the square of the momentum of the decaying pomeron:

$$V_{P\to3P}=g\mathbf{k}^2, \tag{24}$$

and the decay vertex into P and P′ contains two momentum combinations:

$$V_{P\to PP'}=g_1\mathbf{k}^2+g_2\mathbf{k}\mathbf{k}', \tag{25}$$

where $\mathbf{k}'$ is the momentum of the outgoing reggeon.

We now consider the invariant form for the interactions containing an odd number of pomerons in the vertex (of the type (15)):

$$\begin{gathered}\mu_1(\nabla\varphi_i^-\nabla\varphi_i^+)\varepsilon_{lm}\varphi_l^-\varphi_m^++[\mu_2(\nabla\varphi_i^-\varphi_i^+)+\mu_3(\varphi_i^-\nabla\varphi_i^+)]\varepsilon_{lm}\\ \times\nabla\varphi_l^-\varphi_m^++[\mu_4(\nabla\varphi_i^-\varphi_i^+)+\mu_5(\varphi_i^-\nabla\varphi_i^+)]\varepsilon_{lm}\varphi_l^-\nabla\varphi_m^+\\ +\mu_6(\varphi_i^-\varphi_i^+)\varepsilon_{lm}\nabla\varphi_l^-\nabla\varphi_m^++[\mu_7(\nabla\varphi_i^-\varphi_i^-)+\mu_8(\nabla\varphi_i^+\varphi_i^+)]\\ \times\varepsilon_{lm}\nabla\varphi_l^-\varphi_m^-+[\mu_9(\nabla\varphi_i^-\varphi_i^-)+\mu_{10}(\nabla\varphi_i^+\varphi_i^+)]\varepsilon_{lm}\nabla\varphi_l^+\varphi_m^+.\end{gathered} \tag{26}$$

The equations for the vacuum terms have the form

$$\begin{gathered}\mu_1+\mu_3-\mu_5+\mu_7+\mu_9=0;\quad \mu_1+\mu_4+\mu_5-\mu_7-\mu_8=0,\quad \mu_5-\mu_7=0,\\ \mu_4-\mu_3+2\mu_5-2\mu_7-\mu_8-\mu_9=0,\quad \mu_2+\mu_3-\mu_4-\mu_5+\mu_7+\mu_8+\mu_9+\mu_{10}=0.\end{gathered} \tag{27}$$

In analogy with (22), the conditions (27) exclude from (26) all the interactions which could lead to a renormalization of the position of the pomeron. At the same time in the Lagrangian (26) there remains the three-pomeron vertex (Fig. 2a) and other pomeron decay vertices (e.g., Fig. 6b) which are proportional to the momentum of the decaying pomeron. The three-pomeron vertex is of the form

$$V_{P\to2P}\sim(\mu_5+\mu_7)\mathbf{k}^2. \tag{28}$$

The conditions (27) leave in (26) also the direct $\mathrm{P} \rightleftarrows \mathrm{P}'$ transition vertex, which equals

$$V_{P\rightleftarrows P'}\sim(\mu_5+\mu_7)(\nabla\psi_1^+\nabla\psi_2+\nabla\psi_2^+\nabla\psi_1). \tag{29}$$

It would seem that excluding from the Lagrangian the transition terms (29)—a diagonalization of P and P′ to order $\mathbf{k}^2$—would lead to the diappearance of the three-pomeron vertex (28) in this order ($\mu_5 + \mu_7 = 0$). However, in addition to the direct transitions (29) the transition $\mathrm{P} \rightleftarrows \mathrm{P}'$ can be realized also via diagrams which contain interactions with different parity of the pomerons in the vertex (13), (14), (21) and (15), 26).

A complete diagonalization of P and P′ must include in addition to (29) a consideration of all these diagrams. As a result of this the effective Lagrangian for P and P′ contains only vertices with even numbers of pomerons: (9), (13), (14) and (21), but the coefficients r, λ, Δ etc. turn out to be singular functions of k^2. This is related to the fact that the $P \rightleftarrows P'$ transition diagrams are singular for $t = -\mathbf{k}^2 = 0$. Thus the three-pomeron vertex (28) and the vertex of Fig. 5a give rise already in order $\mathbf{k}^2$ to the term

$$V_{P\to P'}\sim(\mu_5+\mu_7)(\Delta\lambda)^{1/2}\ln(\omega+\mathbf{k}^2/2). \qquad (30)$$

Therefore the diagonalization of the Lagrangian, i.e., the exclusion of vertices with an odd number of pomerons, does not mean the disappearance of the singularities related to the odd vertices, including the three-pomeron vertex. Thus, the effective Lagrangian of the pomeron-P′ system which is analytic in $\mathbf{k}^2$ is not diagonalized in P and P′ and includes vertices of arbitrary parity in the pomeron number.

The possibility of full diagonalization of P and P′ in the pomeron problem without the appearance of singular t-dependences in the effective Lagrangian seems to be interesting. This could be realized if after spontaneous symmetry breakdown all vertices with an odd number of pomerons, (15) and (26), were absent in all orders of $\mathbf{k}^2$. Such a conservation law (even number of Goldstone particles in each vertex) occurs in the relativistic models[7,8]. From the point of view of the theory of phase transitions this case would correspond to a realization of the Landau theory.

CONCLUSION

Thus, a theory with broken symmetry of two Regge excitations with correctly defined vacuum necessarily leads to a quasistable pomeron which manifests itself as a Goldstone boson. The method developed here allows to find its interactions and to establish relations between them.

The first to discuss symmetry breaking in the reggeon problem was Abarbanel[11]. However, his discussion does not involve the continuous group and, if carried to the end, would not lead to the appearance of a genuine nonrelativistic Goldstone boson.

In conclusion I would like to thank A. A. Ansel'm, V. N. Gribov, A. B. Kaĭdalov, and O. V. Kancheli for useful discussions.

APPENDIX

In the simplest model without a continuous symmetry group, and therefore without the occurrence of the pomeron (as a Goldstone boson) as an HR[11] the fluctuations of Fig. 4a and b can be described in terms of parton distributions in the space of impact parameters and parton rapidities by means of the following function of ϵ and ρ[10]:

$$G_0(\xi,\rho)=\frac{\exp[-\rho^2/4\alpha'\xi-\Delta\xi]}{4\pi\alpha'\xi}. \qquad (A.1)$$

The quantity $\xi = \ln(s/s_0)$ defines something like a fluctuation length: the difference between the parton rapidities at the beginning and the end of the fluctuation. In terms of the operators ψ and ψ^+, $G_0(\xi,\rho)$ is the free Green's function of the PR. The shift Δ for the PR may be smaller than zero (an anti-Froissart pole).

For simplicity we assume that all fluctuations have the same length $(\xi - \xi_0)$, i.e., the partons interact with the target or go off into the vacuum for small rapidities $\xi_0 \sim 1$. In addition we consider that one may not take into account the production of fluctuations with large $(\xi - \xi_0)$. Then in the tree approximation for the interactions of Fig. 2 the equation for $f(\xi - \xi_0) = \langle 0|\psi^+|0\rangle$ takes the form (Fig. 10):

$$f(\xi-\xi_0)=\exp\{-\Delta(\xi-\xi_0)\}+r\int_{\xi_0}^{\xi}\exp\{-\Delta(\xi-\xi')\}f^2(\xi'-\xi_0)d\xi' + \frac{\lambda}{3!}\int_{\xi_0}^{\xi}\exp\{-\Delta(\xi-\xi_0)\}f^3(\xi'-\xi_0)d\xi'. \qquad (A.2)$$

A similar equation for the Green's function of the HR is given by the equation (Fig. 11)

$$g(\xi-\xi_0)=\exp\{-\Delta(\xi-\xi_0)\}+r\int_{\xi_0}^{\xi}\exp\{-\Delta(\xi-\xi')\}f(\xi'-\xi_0)g(\xi'-\xi_0)d\xi' + \frac{\lambda}{2}\int_{\xi_0}^{\xi}\exp\{-\Delta(\xi-\xi')\}f^2(\xi'-\xi_0)g(\xi'-\xi_0)d\xi',$$
$$G(\xi-\xi_0,\mathbf{k}^2)=g(\xi-\xi_0)\exp\{-\alpha'\mathbf{k}^2(\xi-\xi_0)\}. \qquad (A.3)$$

The solution of this system is quite simple for λ = 0 (the initial conditions are: f(0) = 1, g(0) = 1):

$$f=\left[\frac{r}{2\Delta}+\left(1-\frac{r}{2\Delta}\right)e^{\Delta x}\right]^{-1},$$
$$g(x)=\exp\left\{\int_0^x[rf(x')-\Delta]dx'\right\},\quad x=\xi-\xi_0. \qquad (A.4)$$

It can be seen from (A.4) that for Δ < 0 (and values of r which do not lead to a pole in the denominator of f)

$$f\to 2\Delta/r \text{ for } x\to\infty,$$

and g(x) turns into a simple pole satisfying the Froissart theorem:

$$g(x)\to e^{\Delta x}.$$

For Δ > 0 f(x) → 0 for x → ∞ there is no redefinition of the vacuum and of the reggeon g(x):

$$g(x)\to e^{-\Delta x}.$$

Results of these two variants are obtained also for λ ≠ 0. The presence of a four-reggeon interaction makes them completely analogous to those obtained in the usual discussion of the minima of the field potential energy:

$$V(\varphi)=\Delta\varphi^2-{}^1/_3r\varphi^3-{}^1/_4\lambda\varphi^4. \qquad (A.5)$$

The solution (A.2) and (A.3) can now be obtained only implicitly:

$$f(x)\left(\frac{f(x)-f_+}{1-f_+}\right)^{\alpha_+}\Big/\left(\frac{f(x)-f_-}{1-f_-}\right)^{\alpha_-}=e^{-\Delta x},$$
$$f_\pm=-\frac{3r}{2\lambda}\pm\sqrt{\frac{9r^2}{4\lambda^2}+\frac{6\Delta}{\lambda}},\quad \alpha_\pm=\frac{f_\mp}{f_+-f_-},$$
$$g(x)=\exp\left\{\int_0^x\left[rf(x')-\Delta+\frac{\lambda}{2}f^2(x')\right]dx'\right\}. \qquad (A.6)$$

It follows from Eq. (A.6) that for λ < 0 (stable vacuum for (A.5)) the solution of the equations (A.2) and (A.3) exists for arbitrary signs of Δ and r and for arbitrary relations between them. For Δ < 0 spontaneous symmetry breakdown occurs:

$$f(x)\to f_+\neq 0,\quad g(x)\to e^{-\gamma x}\text{ as } x\to\infty, \qquad (A.7)$$
$$\gamma=-2\Delta+{}^1/_2rf_+. \qquad (A.8)$$

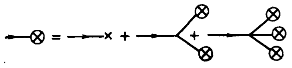

FIG. 10

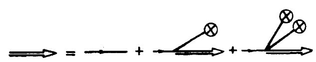

FIG. 11

For $\Delta > 0$ the solution corresponds to an absence of redefinition of the vacuum:

$$f(x)\to 0, \quad g(x)\to e^{-\Delta x} \text{ as } x\to\infty. \tag{A.9}$$

Only in the region $\Delta > 0$, $0 < f_- < 1$, $r > 0$ the solution (A.7) may exist again. It is easy to verify that for $\lambda < 0$ the redefined pole $g(x)$ always satisfies the Froissart theorem ($\gamma > 0$).

The asymptotic behavior of $g(x)$ for $x \to \infty$, after separation of the contributions of the principal poles (A.7) and (A.9), must correspond to the asymptotic behavior of the other Regge poles of the problem, i.e., must differ from (A.7) and (A.9) by a power of $s(e^x)$. This circumstance is related to the fact that the solution of the model is obtained in the tree approximation, neglecting branch points which can appear only on account of loop diagrams. One can verify that his condition is satisfied for the solutions (A.4) and (A.6).

For $\lambda > 0$ (absence of vacuum for (A.5)) the solution (A.2) and (A.3) ($f(0) = 1$, $g(0) = 1$ does not exist for all values of Δ and r. The solution of the type of a spontaneous symmetry breaking exists only in the region

$$\Delta<0, \quad r<0, \quad f_+>1 \tag{A.10}$$

and equals

$$f\to f_-, \quad g(x)\to e^{-\gamma_1 x} \text{ as } x\to\infty, \qquad \gamma_1=-2\Delta+{}^1/_2 rf_-. \tag{A.11}$$

For $\lambda > 0$, γ_1 may change sign depending on the relation between the constants, i.e., the pole of the HR may be anti-Froissart in this case.

Thus, the conclusions which can be reached in the model under consideration reduce, essentially, to two points (for stable vacuum $\lambda < 0$):

1) The Regge poles which can appear in a theory without continuous symmetry group have a shift $\Delta R > 0$ from $j = 1$. As it should be, the pomeron is not formed.

2) The role of the negative squared mass which is necessary in field theory for spontaneous symmetry breakdown is played in the reggeon problem by the anti-froissart bare pole $\Delta < 0$[11].

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1)A real shift leading to spontaneous symmetry breaking appears for the real operator φ_1^-. Without introducing (4) such a shift is hard to understand in terms of the operators ψ^+ and ψ.

2)It is interesting to note that in theories with a Goldstone particle massive bosons are always unstable (the first diagram in Fig. 5) and therefore cannot exist as free particles. Consequently a theory with Goldstone bosons is in fact a consistent theory of the self-action of massless particles. Their interaction is expressed by means of the resonance state.

3)One can show that the number of variables increases faster than the number of equations so that the problem has a solution (but does not necessarily make sense) for forms of any degree.

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