Kinetic properties of solutions of He^4 in liquid He^3

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It is shown that at low temperatures $T \ll \epsilon_F$ the scattering of He^3 by impurities with $m_i^* \sim m_{He}^*$ is allowed only at small angles $\sim T/\epsilon_F^*$, and the influence of this effect on the value of the kinetic coefficients is determined.

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1. The question of the properties of liquid He^3–He^4 solutions has long been studied both theoretically and experimentally.[1] The change of various characteristics of He^3 and He^4 on introduction of impurities permits a wide variety of information to be obtained on the nature of the spectrum of excitations and their interaction. The effect of He^4 impurities on the kinetic properties of He^3 in the normal state was first discussed by Zharkov and Silin.[2] It follows from their work that at low temperatures $T \ll \epsilon_F$ the impurity viscosity and thermal conductivity vary according to laws

$$\eta_i \sim (T/\varepsilon_F)^2 c\eta_{He}, \quad \varkappa_i \sim (T/\varepsilon_F) c\varkappa_{He}, \tag{1}$$

where c is the impurity concentration and η_{He} and κ_{He} are the helium viscosity and thermal conductivity, which have an ordinary Fermi-liquid temperature dependence. However, the specific nature of scattering by impurities with effective mass $m_i^* \sim m_{He}^*$ leads to an increase in the value of the kinetic coefficients by a factor $\sim\epsilon_F^*/T$ ($\epsilon_F^* \equiv p_F^2/2m_i^*$, $\epsilon_F = p_F^2/2m_{He}^*$). The cause of this is that the characteristic energy of the impurities is $E \sim T$ (for Bose impurities) and $E \sim \max\{T, c^{2/3}\epsilon_F\}$ (for Fermi impurities). Therefore for an effective mass $m_i^* \sim m_{He}^*$ simultaneous account of conservation of energy and momentum leads to the result that scattering of He^3 is possible only at small angles of the order of T/ϵ_F^* and as a result the scattering cross section σ^* becomes T/ϵ_F^* times smaller than the Born scattering cross section $\sigma \sim [m_i^* m_{He}^*/(m_i^* + m_{He}^*)]^2 |V_0|^2$. In turn this leads to an increase in the coefficients κ_i and η_i by a factor ϵ_F^*/T. The dependence (1) is satisfied only for sufficiently heavy impurities $m_i^* \gg m_{He}^*$.

As is well known, on introduction into He^3 of He^4 impurities for $T < 0.872$ K, stratification occurs into phases with different concentrations of He^4, and on further reduction of the temperature the He^4 impurity begins to freeze out rapidly.[1] However, as estimates show, for temperatures of the order of $T \lesssim 0.1\ \epsilon_F$ the concentration of impurities is still sufficiently high that the effect of the increase in the coefficients κ_i and η_i is quite noticeable. In addition, the solubility of He^4 in He^3 rises substantially with increase of the pressure.

2. We will assume that for small momenta p ($p \ll 1/a$, where a is the atomic dimension) the energy spectrum of the impurity has the form

$$E_i = q^2/2m_i^*. \tag{2}$$

The possible presence of a roton minimum in the region $p \sim 1/a$

$$E_i = \Delta_i + (p - p_{0i})^2/2M_i \tag{3}$$

leads only to exponentially small corrections of the order $\exp(-\Delta_i/T)$, both for scattering with participation of quasiroton excitations (3) and for scattering with transition of a quasiparticle from a quadratic spectrum (2) to quasiroton excitation (3). Therefore in what follows we shall not take into account the contribution from region (3).

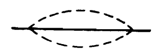

FIG. 1

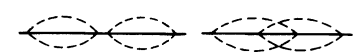

FIG. 2

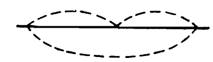

FIG. 3

We shall show first of all that the conclusion that the scattering cross section decreases remains valid if multiple collisions are taken into account. For this purpose we shall limit ourselves to a simple diagram estimate. In the lowest approximation the contribution to the imaginary part of the Green's function of He^3 is given by the diagram of Fig. 1, where the dashed line represents the Green's function of the impurity in the temperature technique[3]: $[i\omega - q^2/2m_i^* + \mu]^{-1}$, and the interaction potential is taken for simplicity in the form $V_0\delta(\mathbf{r} - \mathbf{r}_i)$. The diagrams shown in Fig. 2 have in this case the same order of magnitude and the "cross" technique[3] is inapplicable. However, the imaginary parts of the two diagrams on integration over angle give a small factor $(T/\epsilon_F^*)^2$. Diagrams of the same form as in Fig. 3 arose in discussion of the Kondo effect,[4] where they resulted in a logarithmically large contribution. In this case it is easy to show that Kondo diagrams of Fig. 3 for $m_i^* \sim m_{He}^*$ do not contain large logarithms, and their inclusion leads to replacement of the Born scattering amplitude by the total amplitude. (The logarithmic integral $\int d\xi/\xi$ is replaced in this case by an expression having schematically the form $\int d\xi/(\xi + \epsilon_F^*)$.) As a result of summation of the diagrams for Bose impurities we obtain

$$\frac{1}{\tau_{He-imp}} \sim \frac{T}{\varepsilon_F^*} v_F \sigma n_0, \tag{4}$$

where n_0 is the number of impurities per unit volume and σ is the scattering cross section. Thus, $\sigma^* \sim (T/\epsilon_F^*)\sigma$. For Fermi impurities the factor T/ϵ_F^* must be replaced by $\max\{T/\epsilon_F^*, c^{2/3}\}$.

3. The most noticeable effect of introducing impurities is from the impurity viscosity and thermal conductivity. The impurity corrections to the helium viscosity and thermal conductivity give a relative contribution smaller by a factor $(T/\epsilon_F)^{1/2}$. The temperature dependence of κ_i and η_i can be found from the kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{r}}\frac{\partial E}{\partial \mathbf{q}} - \frac{\partial f}{\partial \mathbf{q}}\frac{\partial E}{\partial \mathbf{r}} = 2\pi \int |V_0|^2 \delta(\mathbf{p}+\mathbf{q}-\mathbf{p}'-\mathbf{q}') \tag{5}$$
$$\times \delta(\xi + E - \xi' - E')[f(1+f')n(1-n') - n'(1-n)f'(1+f)]d\tau\, d\tau'\, dq'/(2\pi)^3.$$

Impurity-impurity and impurity-phonon collisions can be neglected over the entire range of temperatures. In what follows we will limit ourselves to the case of a Boltzmann distribution function f, which is of the greatest practical interest.

We shall look for a solution of Eq. (5) for the impurity distribution function f in the form $f_0 + f_1$, where f_0 is the equilibrium distribution function and $f_0 \gg f_1$. The distribution function of helium excitations n can be assumed equilibrium. In the left-hand side of Eq. (5), utilizing the conservation laws, we separate by standard methods the terms proportional to the gradients of temperature, velocity, and concentration:

$$\frac{1}{2}\frac{f_0}{T}\left(q_i\frac{\partial E}{\partial q_k}-\frac{1}{3}q_l\frac{\partial E}{\partial q_l}\delta_{ik}\right)\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right)+f_0\frac{\partial E}{\partial \mathbf{q}}\frac{\nabla c}{c}+f_0\frac{\partial E}{\partial \mathbf{q}}\left(\frac{E}{T}-\frac{5}{2}\right)\frac{\nabla T}{T}+f_0\frac{\partial E}{\partial \mathbf{q}}\frac{\nabla \rho}{\rho}=I. \tag{6}$$

The factor T/ϵ_F^*, which arises from scattering only at small angles, is obtained automatically from the collision integral (5). For this purpose we shall first integrate over $\mathbf{q}'$. Then, in view of the fact that $\mathbf{q}(\mathbf{p}-\mathbf{p}')/m_i^* \sim T(T/\epsilon_F)^{1/2}$, the δ function of energy takes the form

$$\delta\left(\xi-\xi'-\frac{p_F^2}{m_i^*}\left(1-\frac{\mathbf{p}\mathbf{p}'}{p_F^2}\right)\right),$$

which also leads to appearance of a factor T/ϵ_F^*.

Omitting the standard calculations,[2] we find the corresponding corrections to the distribution function, and after calculation of the heat flux Q, the momentum Π_{ik}, and the mass, we finally obtain

$$\varkappa_i\sim c\frac{p_F^3}{Tm_i^{*2}m_{\mathrm{He}}^{*2}}\frac{1}{|V_0|^2}, \tag{7}$$

$$\eta_i\sim c\frac{p_F^3}{Tm_i^{*}m_{\mathrm{He}}^{*2}}\frac{1}{|V_0|^2}, \tag{8}$$

$$D\sim\frac{m_i^*}{m_{\mathrm{He}}^* c p_F^3}\varkappa_i. \tag{9}$$

We note that the quantity κ_i has a Fermi-liquid temperature dependence $\kappa_i \propto 1/T$ and is small in comparison with the helium thermal conductivity only in proportion to the smallness of the concentration, and therefore the total thermal conductivity $\kappa = \kappa_i + \kappa_{He}$ has a dependence $\kappa \propto \alpha T^{-1}(1+\beta c)$. The impurity part of the total viscosity $\eta = \eta_i + \eta_{He}$ is less than the helium part by a factor $\sim T/\epsilon_F^*$.

4. We now discuss briefly the possible influence of light impurities with $m_i^* \sim m_{He}^*$ on the properties of superfluid He^3. As a working model we will limit ourselves to the Balian-Werthamer solution,[5] which apparently corresponds to the B phase of superfluid He^3. To estimate the effect of light impurities on the thermodynamic properties, it is possible to use the formula in first-order perturbation theory for the change of the free energy on introduction of impurities:

$$\delta F\sim m_{\mathrm{He}}^* p_F\frac{T}{\varepsilon_F^*}\Delta\frac{1}{\tau_{\mathrm{He}-imp}}. \tag{10}$$

The appearance of the factor $(T/\epsilon_F^*)^2$ in comparison with ref. 5 is due to the fact that scattering is allowed only at small angles and leads to the result that impurities have practically no effect on the transition temperature. The kinetic equations for impurities in superfluid He^3 can be obtained by means of a generalized Balian-Werthamer u-v transformation[5] similar to the case of s pairing.[6] In the collision integral in Eq. (5) up to temperatures $T \lesssim 5\times 10^{-4}$ K the dominant role as before is played by collisions of impurities with helium excitations. Then as a result of the rapid falloff of the number of helium excitations with temperature as $e^{-\Delta/T}$, the kinetic coefficients κ_i, η_i, and D will rise as $e^{\Delta/T}$.

A dependence of the form $e^{\Delta/T}$ would be valid also in the case of heavy impurities, and therefore the possibility of introducing such impurities ($m_i^* \gg m_{He}^*$) and also of hydrogen in liquid He^3 at very low temperatures presents definite experimental interest.

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