Excitation of sound in a superconductor by an electromagnetic wave

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We study the problem of the transformation of an electromagnetic wave into a sound wave in superconductors. We consider the limits of London and Pippard superconductors. We show that the main contribution to the transformation coefficient in Pippard superconductors comes from volume forces, while the surface forces play a lesser role. This also leads to an appreciable weakening of interference effects.

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The problem of the transformation of an electromagnetic wave (EMW) into a sound wave has been studied in a number of papers.[1-4] However, much still remains unexplained, in particular, the justification for the approximations used in the elasticity equations for superconductors with allowance for the forces that the electrons exert on the lattice. In this connection a number of models which have been invoked to explain the experimental data[2] on the transformation coefficient at low temperatures appear to be unconvincing. It is therefore necessary to analyze the whole problem altogether more rigorously.

We consider the case usually realized in an experiment,[1] when an EMW of frequency ω and vector potential $A_x(z)$ is normally incident upon a plane superconducting plate of thickness $d (d > \delta, d < l$; δ is the penetration depth of a weak magnetic field and l the electron mean free path). The z-axis is at right angles to the plane of the slab which occupies the region $0 < z < d$ and is directed into the dielectric half-space. We study here a sufficiently pure superconductor ($l > \xi_0$, $\delta/l \ll 1$). Other limits as far as the impurity concentration is concerned are discussed separately. The Hamiltonian of the system has the form

$$H=\int d^3x H(\mathbf{x}), \qquad H(x)=H_0+H_{\mathrm{BCS}},$$
$$H_0=\left[\varphi^+(\mathbf{x}')\sigma_z\left(\frac{1}{2m}\left(\hat{p}-\sigma_z\frac{e}{c}\mathbf{A}(\mathbf{x})\right)^2-\mu+U(\mathbf{x})\right)\varphi(\mathbf{x})\right]_{\mathbf{x}'\to\mathbf{x}}, \tag{1}$$
$$H_{\mathrm{BCS}}=\Delta\varphi^+(\mathbf{x})\sigma_x\varphi(\mathbf{x}),$$

$U(\mathbf{x})$ is the potential connected with the scattering by the randomly distributed impurities, $\varphi(\mathbf{x})$ are electron operators in the Nambu representation:

$$\varphi(\mathbf{x})=\begin{pmatrix}\varphi_1(\mathbf{x})\\ \varphi_{-1}(\mathbf{x})\end{pmatrix}=\begin{pmatrix}\psi_\uparrow(x)\\ \psi_\downarrow^+(x)\end{pmatrix} \qquad (\varphi_1^+(x)\varphi_{-1}^+(x))=(\psi_\uparrow^+\psi_\downarrow),$$

and σ_x and σ_z are Pauli matrices.

The Hamiltonian of the interaction with the ion oscillations can conveniently be written in a system of coordinates moving with the deformed lattice (see below).

The basis of the following analysis is given by the Kontorovich equations[5] which describe the oscillations of the ions under the influence of an EMW and the forces which the electrons exert upon the lattice. These equations are the appropriate ones to describe both the normal and the superconducting[4] state of the metal. One obtains them very simply by using the local conservation laws[4] and they have the following form:

$$\rho\ddot{u}_x-\frac{\partial}{\partial z}\sigma_{xz}=-\left\langle -i\omega m j_x(\mathbf{x}t)+\frac{\partial}{\partial z}\tau_{xz}(\mathbf{x}t)\right\rangle, \tag{2}$$

$$j(\mathbf{x}t)=\frac{1}{2m}(\hat{p}_{\mathbf{x}}-\hat{p}_{\mathbf{x}'})(\varphi^+(\mathbf{x}'t)\varphi(\mathbf{x}t)-\delta(\mathbf{x}'-\mathbf{x}))|_{\mathbf{x}'\to\mathbf{x}}-\frac{e}{m}n(\mathbf{x}t)A_x(\mathbf{x}t),$$
$$\tau_{mn}(\mathbf{x})=-\frac{1}{4m}\langle(\nabla_{\mathbf{x}}-\nabla_{\mathbf{x}'})_m(\nabla_{\mathbf{x}}-\nabla_{\mathbf{x}'})_n(\varphi^+(\mathbf{x}')\sigma_z\varphi(\mathbf{x})+\delta(\mathbf{x}'-\mathbf{x}))|_{\mathbf{x}'\to\mathbf{x}}\rangle$$
$$-\delta_{mn}|g^{-1}|\Delta^2, \qquad n(\mathbf{x}t)=(\varphi^+(\mathbf{x}t)\sigma_z\varphi(\mathbf{x}t)-\delta(\mathbf{x}'-\mathbf{x}))_{\mathbf{x}'\to\mathbf{x}}, \tag{3}$$

where σ_{xz} is the stress tensor and g the electron-electron interaction constant. Terms quadratic in the field have been omitted in (3).

It is important that the averaging in (2) and (3) is over the non-equilibrium state which is described by the total Hamiltonian (1) in the laboratory frame of reference (moreover, the usual averaging over the random distribution of impurity atom is performed here). It is convenient to go over to the comoving frame of reference for the explicit calculations. It is well known[6,7] that the new Hamiltonian in that frame has the form

$$H'=H+H_{\text{зв}},$$
$$H_{\mathrm{int}}=\int u_x(\mathbf{x}t)\left(i\omega m j_x(\mathbf{x}t)-\frac{\partial}{\partial z}\tau_{xz}(\mathbf{x}t)\right)d^3x. \tag{4}$$

The operators in (2) and (3) are then also transformed. As a result the right-hand side of (2) takes for transverse sound oscillations the form

$$\langle\ldots\rangle_{\mathrm{lab}}=-\omega^2 m N_0 u_x+\langle\ldots\rangle_{\mathrm{comov}}, \tag{5}$$

where N_0 is the number of electrons per unit volume.

In the approximation that is linear in H_{int} and in the electromagnetic field we get correlators proportional, respectively to $u_x(\mathbf{x}t)$ and $A_x(\mathbf{x}t)$. All terms containing u_x must be transferred to the left-hand side of Eq. (2) and if the sound wavelength $\lambda > \xi_0$ (ξ_0 is the coherence length) they enter into the renormalization of the elasticity moduli. If $\lambda < \xi_0$ it is necessary to take into account the dispersion of the elastic moduli. Neglecting furthermore terms $\sim m/M$ we get the equation

$$k_1^2 u_x+\frac{d^2u_x}{dz^2}=\frac{1}{\rho s^2}f(z), \qquad f(z)=f_i+f_g, \tag{6}$$

$$f_i=-i\omega m\frac{c}{4\pi e}\int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ikz}A(k)Q(k\omega), \tag{7}$$

$$f_g=ie\int_{-\infty}^{\infty}\frac{dk}{2\pi}A(k)e^{ikz}k\langle[\tau_{xz},j_x]\rangle_{k\omega}, \tag{8}$$

where we have expressed the stress tensor in terms of the ion displacement

$$\sigma_{xz}=s^2\rho du_x/dz; \qquad k_1=\omega/s;$$

s is the sound velocity in the superconductor, ρ the metal density, and A(k) the Fourier transform of the vector potential. When writing down (7) and (8) we neglected a small contribution from the ion current to

A and also neglected the damping of the sound which in a superconductor is also small. The range of frequencies used for studying the transformation of EMW into sound is usually of the order of 10^8 to 10^9 Hz, i.e., $\omega \ll \Delta(0)$. The frequency dependence of the potential A(k) can thus be neglected. $Q(q\omega)$ is the kernel in the expression for the current $j(q) = -(c/4\pi)Q(q\omega)A(q)$. The correlator in the formula for f_g can be evaluated simply and has the following form:

$$\langle[\tau_{xz}, j_x]\rangle_{q,\omega} = -\langle[\tau_{xz}, j_x]\rangle_{-q,\omega} = -\omega N_n \frac{1}{q}(1-g(ql)), \tag{9}$$

where $g(x) = \frac{3}{2}\{x^{-1} + x^{-3})\arctan x - x^{-2}\}$,

$$N_n = N_0 \int_0^\infty \frac{dx}{\mathrm{ch}^2[x^2+(\Delta(T)/2T)^2]^{1/2}}. \tag{10}$$

The force f_g is connected with an incomplete drag of the electrons by the lattice. In that case after the scattering the electron velocity is on the average the same as the lattice velocity, i.e., the scattering is inelastic and can be connected only with normal excitations and this is described by the appearance of the factor N_n.

We note that in the limit of a normal metal

$$j(q) = \frac{i\omega}{c}\sigma_0 g(ql)A(q), \quad \sigma_0 = \frac{N_0 e^2 \tau}{m},$$

and the expression for $f(z)$ takes the form:[8]

$$f = -e\left[EN_0 - \frac{j(z)}{\sigma_0}(1+i\omega\tau)\right], \quad E = -\frac{1}{c}\frac{\partial A}{\partial t},$$

where τ is the electron mean free flight time.

The boundary conditions for Eq. (6) at the free surface and at the surface separating the superconductor from the dielectric $(z = d)$ have the form

$$\left.\frac{du_x}{dz}\right|_{z=0} = \frac{1}{\rho s^2}\langle\tau_{xz}(0)\rangle, \quad \left.\frac{du_x}{dz}\right|_{z=d} - \frac{1}{\rho s^2}\langle\tau_{xz}(d)\rangle = \frac{\rho_1 s_1^2}{\rho s^2}\frac{du_1}{dz}, \quad u_1(d) = u_x(d), \tag{11}$$

$$\langle\tau_{xz}(z)\rangle = \frac{e}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikz} A(k)\langle[\tau_{xz}, j_x]\rangle_{k,\omega},$$

where ρ_1, s_1, and u_1 are the density of the dielectric, the sound velocity, and the amplitude of the ion oscillations in it.

The amplitude of the oscillations in the dielectric $u_1(z)$ satisfies an equation such as (6) without a right-hand side. The solution of this equation is a traveling sound wave: $\tilde{u}_1(z) = u_1 e^{ik_2 z}$ (we drop the time factor everywhere).

We write down the formula for the transformation coefficient:[1,11]

$$\alpha = \int \frac{w\, df}{P}, \quad w = \frac{1}{2}\rho_1 s_1 \omega^2 |u_1(d)|^2,$$

P is the Poynting vector of the incident EMW,

$$\alpha = \frac{1}{2}\int \frac{|I(\omega k_1)|^2}{Z^* P} df, \tag{12}$$

where

$$I(\omega k_1) = \int_0^d f(z)\cos k_1 z\, dz + \langle\tau_{xz}(0)\rangle - (\cos k_1 d)\langle\tau_{xz}(d)\rangle,$$
$$(Z^*)^{-1} = \rho_1 s_1[(\rho_1 s_1)^2\cos^2 k_1 d + (\rho s)^2 \sin^2 k_1 d]^{-1}. \tag{12a}$$

The integration of $I(\omega k_1)$ in α is over the surface of the film.

In the low-temperature region $\omega \ll T \ll \Delta(0)$, where it follows from (10) and (11) that the surface forces $\langle\tau_{xz}(0)\rangle$ and $\langle\tau_{xz}(d)\rangle$ are small, the nature of the reflection of the electrons from the surface is not very important for London superconductors ($\delta > \xi_0$) and the result for α is the same as the one obtained earlier.[4] When the temperature approaches the critical one the increase in the number of normal excitations and the law for their reflection from the boundary of the slab strongly affects the transformation coefficient. In what follows we shall assume that the scattering from the surface is diffuse. In that case the vector potential A(k) has the form[9]

$$A(q) = i\delta H_0 Y_-(q), \tag{13}$$

$$Y_-(q) = (q^2+Q(q))^{-1/2}\exp i\left[\frac{\pi}{2} - \frac{q}{\pi}\int_0^\infty \frac{\ln(y^2+Q(y))}{y^2-q^2}dy\right],$$

$$Y_-(q) = -Y_-^*(-q); \quad Y_-(q) = \frac{1}{q-i/\delta} \quad \text{if} \quad \delta \gg \xi_0; \tag{14}$$

$$Y_-(q) = \frac{1}{q_0}\left(\frac{x}{1+x^3}\right)^{1/2}\exp i\left[\frac{3\pi}{4} - \frac{x}{\pi}\varphi(x)\right] \quad \text{if} \quad \delta \ll \xi_0, \tag{15}$$

$$\varphi(x) = \int_0^\infty \frac{\ln(1+y^3)}{y^3-x^3}dy, \quad x = \frac{q}{q_0} > 0, \quad q_0^{-1} = \frac{2}{\sqrt{3}}\delta,$$

$$\delta = \delta(T=0)\left(\frac{\Delta(T)}{\Delta(0)}\,\mathrm{th}\,\frac{\Delta(T)}{2T}\right)^{-1/3}.$$

The function $\varphi(x)$ has been tabulated by Shapoval.[9]

From (13) and (11) we get for the surface forces the expressions

$$\langle\tau_{xz}(0)\rangle = -i\omega e H_0 \frac{\delta^2}{2} N_n a_m, \tag{16}$$

$$\langle\tau_{xz}(d)\rangle = i\omega e H_0 \frac{\delta^2}{2} N_n b_m, \tag{17}$$

where m = 1, 2 and correspond to the London and Pippard limits. When $\delta > \xi_0$ we get $a_1 = \frac{3}{8} l/\delta$, $b_1 \approx 0$, $(l/\delta \ll 1)$;

$$a_1 = 1 - \frac{3}{2}\frac{\delta}{l}\ln\frac{\delta}{l}, \quad b_1 = 1 + \frac{3}{2}\frac{d}{l}\ln\frac{d}{l}. \tag{18}$$

The last formulae were obtained using the fact that the mean free path is finite ($\delta/l \ll 1$, $d/l \ll 1$ $\delta/d \ll 1$). a_1 and b_1 can be evaluated for any l. To do this we note that the function $g(kl)$ in the complex k-plane has branch points $(i/l; -i/l)$ and the integrals determining a_1 and b_1 are found exactly. We do not give here the result as it is complicated, but merely note that $\langle\tau_{xz}(d)\rangle$ becomes proportional to $(l/d)\exp(-d/l)$ in the case $l < d$. The same estimate is also valid in the limit $\delta < \xi_0$. We mention also the values $a_1 = b_1 = 1$ used by Abeles.[1]

We now consider pure Pippard superconductors. The expressions for a_2 and b_2 have the form

$$a_2 = \frac{2}{\delta}\int_0^\infty \frac{dk}{\pi}\frac{\mathrm{Re}\,Y_-(k)}{k}, \tag{19}$$

$$b_2 = -\frac{2}{\delta}\int_0^\infty \frac{dk}{\pi k}[\mathrm{Re}\,Y_-(k)\cos kd - \sin kd\,\mathrm{Im}\,Y_-(k)]. \tag{20}$$

If we use Eq. (15) for $Y_-(k)$ in the first integral a very small value of a_2 follows from a numerical calculation. The region $k \sim \xi_0^{-1}$ is thus important in the integral; the kernel $Y_-(k)$ there differs from the extreme of the Pippard limit (15), i.e., a_2 turns out to be a quantity of the order of $\delta/\xi_0 \ll 1$. The same estimate can be obtained in the same way as was done by de Gennes.[10] If $d \gg \delta$, $d > \xi_0$, the main contribution to b_2 is given by Eq. (15) for $Y_-(k)$:

$$b_2 = -\frac{8}{\sqrt{3}\pi}\int_0^\infty \frac{dx}{(1+x^6)^{1/2}}\cos\left(\frac{3\pi}{4}-\frac{x^2}{\pi}\varphi(x^2)+\gamma x^2\right), \quad \gamma = q_0 d. \quad (21)$$

An asymptotic estimate ($\gamma \gg 1$) using the saddle-point method leads to the result $b_2 \sim 2.8\,(\delta/d)^{1/2}$. For sufficiently thick slabs the surface force connected with the second surface also decreases and this leads to a weakening of oscillational effects for the transformation coefficient.

The integral of the volume force $f(z)$ in Eq. (12a) can be expressed in terms of a combination of Fourier components of the vector potential $A(k_1)$ and of the correlators $Q(k_1)$ and $\langle[\tau_{xz}, j_x]\rangle_{k_1\omega}$ (where k_1 is the sound wave wavevector) as follows:

$$\frac{A(k_1)+A(-k_1)}{2} e\left[\frac{-i\omega mc}{4\pi e^2}Q(k_1)+ik_1\langle[\tau_{xz}, j_x]\rangle_{k_1\omega}\right]. \quad (22)$$

As a result, substituting this formula and also (16) and (17) into (12) we get for α

$$\alpha = \frac{\pi\omega^2 N_0^2 e^2 \delta^4}{Z c^3}\left(\frac{c}{8\pi P}\int H_t^2\, dt\right)|L(k_1)|^2, \quad (23)$$

$$L(k_1) = \frac{N_n}{N_0}(a_n + b_n \cos k_1 d) - \frac{2\,\mathrm{Im}\,Y_-(k_1)}{\delta}F. \quad (24)$$

$$F = \frac{N_n}{N_0}(1-g(k_1 l)) + \frac{cm}{4\pi e^2 N_0}Q(k_1). \quad (25)$$

The term with F in (24) is the result of the operation of the volume forces.

We consider a few limiting cases. If the sound wavelength $\lambda > \xi_0$ the kernel $Q(k_1)$ has the London form and when $\delta > \xi_0$ the quantity $L(k_1)$ has the form

$$L(k_1) = \frac{N_n}{N_0}(1+\cos k_1 d) - \frac{2F}{1+(k_1\delta_L)^2}, \quad \delta_L^{-2} = \frac{4\pi e^2 N_s}{mc^2}. \quad (26)$$

As $T \to T_c$ we have, taking the skin effect into account,

$$\delta_L^{-2} \to \delta_L^{-2} + 4i\delta_{\text{skin}}^{-2} n(\Delta), \quad \delta_{\text{skin}} > \delta_L.$$

in the limit $\omega \ll T \ll \Delta(0)$ we can neglect the contribution from the surface forces and only the second term in the last formula remains. The volume force is equal to $-i\omega e N_0 A/c$.[1] The approximation used by Abeles[1] corresponds thus to a pure London superconductor.

At low temperatures most pure superconductors are Pippard superconductors. We have already noted that the contribution from the surface forces in them is appreciably less diminished and the transformation coefficient is determined by the volume forces. If, as before, the condition $\lambda > \xi_0$ holds we get for $L(k_1)$

$$L(k_1) = 2\frac{\delta_L}{\delta}[(k_1\delta_L)^2+1]^{-1}. \quad (27)$$

In the limit $\lambda \ll \xi_0$ we must take into account the dispersion of the elastic moduli and the problem becomes more complicated. For qualitative estimates we can use Eq. (24) with the function $Y_-(k)$ given by Eq. (15).

We note in conclusion that the generation of sound by EMW in superconductors is appreciably weaker than in normal metals and is of interest solely as leading to an additional contribution to the surface resistivity at low temperatures. This last fact plays an important role in superconducting waveguides and resonators.

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