Elementary particle decays in the field of an intense electromagnetic wave

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The decays $\mu^{\pm}\to e^{\pm}+\nu+\bar{\nu}$, $\pi\to\mu(e)+\nu$, $\pi^{\pm}\to\pi^0+e^{\pm}+\nu$, and also the process $e^{\pm}\to e^{\pm}+\nu+\bar{\nu}$ are considered in the intense electromagnetic field of a plane electromagnetic wave of arbitrary polarization and also in the field of two linearly-polarized waves propagating in the same direction with mutually perpendicular polarizations. Expressions are obtained for the decay probabilities of these processes for both models of the electromagnetic field, and numerical computations are carried out for specific values of the invariant parameters which govern the effect of an external field on elementary particle decays. The characteristic features of the dependence of the total probabilities of particle decays on the frequency and also on the polarization of the external electromagnetic wave are discussed.

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1. INTRODUCTION

The utilization of lasers as sources of high-power electromagnetic radiation enables one to pose the problem concerning the investigation of a new class of phenomena, namely, the quantum effects occurring in the intense field of an electromagnetic wave. Such effects include, for example, the processes of photon emission by an electron, pair production, etc., which have been investigated in many articles.[1-7] An investigation of the effect of an external electromagnetic field on the decays of elementary particles is also of obvious interest.

A number of articles have recently appeared in which one or another aspect of this problem has been considered.[4,5,8-11] In this connection it was found that the field of an electromagnetic wave may either increase the probability of decay, for example, $\pi\to\mu+\nu$, or else decrease the probability, as happens, for example, in the case of the decay $\pi\to e+\nu$.[5,8] Furthermore, the effect of the wave polarization on the total probabilities of decay was investigated[5] using those same decay processes $\pi\to\mu(e)+\nu$ as an example. As a consequence of the nonconservation of parity in the weak interactions, these probabilities turn out to be different for right or left circularly polarized waves. Therefore, an investigation of this effect for other elementary-particle decays is also of interest, for example, decays into three particles:

$$\mu^{\pm}\to e^{\pm}+\nu+\bar{\nu},\quad \pi^{\pm}\to\pi^0+e^{\pm}+\nu.$$

The probabilities for these decay processes in the field of a linearly polarized wave were derived by Ritus.[8] The limiting case of a crossed field was analyzed by him in detail. However, the cases of circular (and in principle any arbitrary elliptical polarization of the wave) were not considered in[8].

We note that some of the weak decay processes involving three particles in the final state have been investigated by a number of authors. For example, Baĭer and Katkov[10] studied the process $e\to e+\nu+\tilde{\nu}$ in a constant magnetic field. This same process was also investigated by Loskutov and Zakhartsov,[12] allowing for polarization of the electrons. Choban and Ivanov[13] carried out a computation of the production of electron-positron pairs by neutrinos, $\nu\to\nu+e^++e^-$, in the field of a laser beam. The decay of the neutron in external fields was investigated by Zharkov[9] and Baranov.[11]

It should be noted that in order to observe the effect of an external field on particle decays, the intensities of the actual fields and the energies of the particles must be sufficiently large. However, as is shown by Ritus,[8] if there is a small difference between the masses of the particles as happens, for example, in the decay $\pi^{\pm}\to\pi^0+e^{\pm}+\nu$, the corresponding intensities of the external fields can be substantially reduced. Decays with a small energy release can occur for ordinary β decays of nuclei, as a consequence of which the investigation of elementary-particle decays may represent a certain model problem for investigation of the effect of an external field on the decay process. We also note that investigation of the decays of particles in external fields may also be of interest in astrophysics.

As is well known,[3,8] the total probability for the decay of a particle in the field of a monochromatic wave depends on the field by means of the two invariants:

$$x=ea/m,\quad \chi=\sqrt{(eF_{\mu\nu}p_\nu)^2}/m^3,$$

where a denotes the amplitude of the potential, $F_{\mu\nu}$ is the field-strength tensor, and p_ν and m denote the momentum and mass of the decaying particle. For $x\ll 1$ we have perturbation theory, whereas the case $x\gg 1$, which is analyzed in detail in[4,8], reduces to an investigation of decays in a crossed field. In the present article we shall assume that the values of the parameters, entering into the problem, are of the order of unity. In this connection there is, just as in the case of a crossed field, an essentially nonlinear dependence of the decay characteristics on the field. This range of values of the parameters requires the application of numerical integration and was not previously investigated in detail (with the exception of the decays $\pi\to\mu+\nu$ and $\pi\to e+\nu$ for a circularly polarized wave[5]).

In addition to the case of a single monochromatic wave, it is also of interest to investigate the behavior of quantum effects in the field of a nonmonochromatic wave consisting of a set of waves of different frequencies, for example, two waves with different frequencies ω_1 and ω_2. We note that the generation of coherent oscillations with different frequencies ω_1 and ω_2 was achieved, for example, in[14,15]. A survey of this problem is contained in the books[16,17]. As is well known,[3] the effect of the wave's field on the behavior of the quantum effects is described in terms of absorption or emission of a definite number of field quanta by the wave from or into the quantum object. In the case of the interaction with the field of two waves, the possibility arises of an exchange of photons with each of the waves separately; in this connection the numbers of these quanta can have different signs, which corresponds to the absorption of a definite number of quanta from one wave and emission into the other wave. The effects of photon emission by an electron in the field of two waves propagating in the same direction were investigated in articles[7,18-20]. Here we shall consider particle decays in such a field. In this connection we shall assume that both waves are linearly polarized, and moreover their polarizations are mutually orthogonal. This greatly simplifies concrete calculations of the effects. We have previously considered such a model for the effects of photon emission by an electron and pair production.[7] The metric and the γ matrices utilized in this article are the same as in the book[21] by Okun'.

2. THE DECAY $\mu \to e + \nu + \tilde{\nu}$ AND THE PROCESS $e \to e + \nu + \tilde{\nu}$

As is well known,[21] the matrix element for the decays $\mu \to e + \nu + \tilde{\nu}$ is of the form

$$M=\frac{G}{\sqrt{2}}\int d^4x[\overline{\psi}_{p'}\gamma_\mu(1+\gamma_5)\psi_{l_2}][\overline{\psi}_{l_1}\gamma_\mu(1+\gamma_5)\psi_p]$$
$$=\frac{G}{\sqrt{2}}\int d^4x[\overline{\psi}_{p'}\gamma_\mu(1+\gamma_5)\psi_p][\overline{\psi}_{l_1}\gamma_\mu(1+\gamma_5)\psi_{l_2}]. \quad (1)$$

The well known Fierz rule was utilized in writing down Eq. (1);[21] p and p′ denote the momenta of the charged μ meson and electron, l_1 and l_2 denote the momenta of the neutrinos. The wave functions of the charged particles are given in[7] in the metric we have adopted. We shall consider the case of a monochromatic wave of arbitrary elliptical polarization with ellipticity parameter ϵ ($-1 \le \epsilon \le 1$), whose potential is given in the form

$$A_\mu(\varphi)=a_{1,\mu}\cos\varphi+\varepsilon a_{2,\mu}\sin\varphi, \quad (2)$$

where

$$a_1a_1=a_2a_2=-a^2,\quad a_1a_2=0,\quad \varphi=kx,\quad k^2=0,$$

and also the two-wave model, considered in[7], with the potential given by

$$A=A_1+A_2,$$
$$A_1=a_1\cos\varphi_1,\quad A_2=a_2\cos(\varphi_2+\varphi_0), \quad (2')$$

where

$$\varphi_i=k_ix,\quad a_1a_2=0,\quad k_ik_j=0,\quad a_1a_1=-a_1^2,\quad a_2a_2=-a_2^2,$$

and φ_0 is the phase shift. Below we shall follow the notation of[7], and for the kinematic variables we adopt the notation of Ritus.[8] For the model (2) we have

$$W_\mu=\frac{G^2n}{96\pi^4q_0}\sum_{s>s_0}\int K(s)\frac{dq'}{q_0'}, \quad (3)$$

where

$$K(s)=|A_0|^2\delta(l)-e^2a^2\varepsilon^2|A_0|^2h_0(k)$$
$$+e^2a^2[|A_1|^2+\varepsilon^2|A_1'|^2-\mathrm{Re}\,A_0A_2^*(1-\varepsilon^2)]h_0(k)$$
$$+ea[2\,\mathrm{Im}\,A_0A_1^*j_1(k)+2\,\mathrm{Im}\,A_0A_1'^*\varepsilon j_2(k)]h_1(k)$$
$$+2\,\mathrm{Im}A_1A_1'^*e^2a^2\varepsilon[h_2(k)j_{12}'(k)+2h_3(k)j_{12}(k)]. \quad (4)$$

In the case of the two-wave model (2′) we have

$$W_\mu=\frac{G^2n}{96\pi^4q_0}\sum_{s_1s_2}\int N(s_1s_2)\frac{dq'}{q_0'}, \quad (5)$$

where

$$N(s_1s_2)=A_0^2B_0^2\delta(l)+e^2[a_1^2(A_1^2-A_0A_2)B_0^2+a_2^2(B_1^2-B_0B_2)A_0^2]h_0(k_i). \quad (6)$$

The following notation has been introduced in formulas (3)—(6):

$$\delta(l)=-2l^4+l^2(m^2+m'^2)+(m^2-m'^2)^2,$$
$$h_0(k_i)=4l^2+\frac{(m^2+m'^2+2l^2)(k_il)^2}{(q'k_i)(qk_i)}$$
$$h_1(k_i)=(m^2-m'^2)\frac{(k_il)}{(q'k_i)(qk_i)}+2l^2\left[\frac{1}{(q'k_i)}+\frac{1}{(qk_i)}\right],$$
$$h_2(k_i)=(m^2-m'^2)\frac{(k_il)^2}{(q'k_i)^2(qk_i)}$$
$$h_3(k_i)=l^2\left[\frac{1}{(q'k_i)}-\frac{(q'k_i)}{(qk_i)^2}\right],$$
$$j_i(k_j)=\langle q'qk_jn_i\rangle,\quad j_{12}'(k_j)=\langle q'n_1k_jn_2\rangle,\quad j_{12}(k_j)=\langle qn_1k_jn_2\rangle. \quad (7)$$

In expressions (7) l denotes the total momentum of the neutral particles (which are neutrinos in the present case): $l = sk + q - q'$ for the model (2), and $l = s_1k_1 + s_2k_2 + q - q'$ for the model (2′); q and q′ denote the quasimomenta of the charged particles—the muon and the electron; n is the average density of the decaying particles;[3,4]

$$\langle a_1a_2a_3a_4\rangle=\varepsilon_{\mu\nu\delta\eta}a_{1\mu}a_{2\nu}a_{3\delta}a_{4\eta},$$

where $\epsilon_{\mu\nu\delta\eta}$ is the completely antisymmetric tensor of fourth rank.[21] In expressions (7) the subscript i may take either of the two values i = 1 or 2, but consistent with the analogous values in (6); the n_i denote spatial unit vectors in the direction of a_i. The functions A_i, B_i, and A_1' are given in[7] for each of the cases (2) and (2′) under consideration.

Using the invariant variables $u = (kl)/(kq')$ and $\lambda = l^2/m^2$, introduced in[3,8], Eq. (3) can be represented in the form

$$W_\mu=\frac{G^2m^6n}{96\pi^4q_0}\sum_{s>s_0}\int_0^{2\pi}d\varphi_0\int_{u_1}^{u_2}\frac{du}{(u+1)^2}\int_{\lambda_1}^{\lambda_2}d\lambda\,\overline{K}(s), \quad (8)$$

where

$$\overline{K}(s)=|A_0|^2[-\lambda^2+{}^1/_2\lambda(1+\mu)+{}^1/_2(1-\mu)^2]$$
$$-x^2\varepsilon^2|A_0|^2\left[2\lambda+(1+\mu+2\lambda)\frac{u^2}{2(u+1)}\right]+x^2[|A_1|^2+\varepsilon^2|A_1'|^2$$
$$-\mathrm{Re}\,A_0A_2^*(1-\varepsilon^2)]\left[2\lambda+(1+\mu+2\lambda)\frac{u^2}{2(u+1)}\right]$$
$$+x(\mathrm{Im}\,A_0A_1^*\sin\varphi_0-\varepsilon\,\mathrm{Im}\,A_0A_1'^*\cos\varphi_0)[(1-\mu)u+2\lambda(u+2)]P$$
$$+x^2\varepsilon\,\mathrm{Im}\,A_1A_1'^*\frac{u}{u+1}[(1-\mu)u+2\lambda(u+2)]. \quad (9)$$

Here we have introduced the notation

$$P=[(\lambda_2-\lambda)/(u+1)]^{1/2},\quad \mu=m'^2/m^2. \quad (10)$$

The limits of the integrations with respect to du and dλ are given by the following expressions, which were derived in Ritus's article:[8]

$$u_{2,1}=\frac{E_*^2-m_n^2-m_*'^2\pm[(E_*^2-m_n^2-m_*'^2)^2-4m_n^2m_*'^2]^{1/2}}{2m_*'^2},$$
$$\lambda_1=\frac{m_n^2}{m^2},\quad \lambda_2=\frac{E_*^2}{m^2}\frac{u}{u+1}-\frac{m_*'^2}{m^2}u. \quad (11)$$

In expression (11) $m_n = m_1 + m_2$ denotes the mass of the neutral particles; in the present case it is equal to zero. Furthermore,

$$\frac{E_*^2}{m^2}=1+\frac{1}{2}x^2(1+\varepsilon^2)+\frac{2s\chi}{x},$$
$$m_*'^2=\mu+{}^1/_2x^2(1+\varepsilon^2). \quad (12)$$

The summation over s in Eq. (3) runs over the number of photons $s > s_0$, where s_0 denotes the minimum possible value of s, given by

$$s_0=\frac{\mu-1}{2\chi}x.$$

For the two-wave model (2′) the expression for the decay probability has a form similar to (3), with the only difference being that $\overline{K}(s)$ is replaced by

$$\bar{N}(s_1s_2)=A_0^2B_0^2\left[-\lambda^2+\frac{\lambda}{2}(1+\mu)+\frac{1}{2}(1-\mu)^2\right]$$
$$+\left[2\lambda+(1+\mu+2\lambda)\frac{u^2}{2(u+1)}\right][x_1^2B_0^2(A_1^2-A_0A_2)+x_2^2A_0^2(B_1^2-B_0B_2)] \quad (13)$$

and, moreover, a double summation over s_1 and s_2 appears instead of the single summation over s, where the summation over s_i is carried out in the following way. For a given value $s_1=\bar{s}_1$, we determine y from the condition

$$\frac{2\chi_1\bar{s}_1}{x_1}+\frac{2\chi_2 y}{x_2}=\mu-1. \quad (14)$$

Let us assume $s_2^{\min}$ is equal to the nearest integer greater than y. Then the summation over s_2 runs over the values $s_2 \geq s_2^{\min}$, but the summation over $\bar{s}_1$ runs over all integers. Furthermore, instead of expressions (12) we have the following relations for the two-wave model:

$$\frac{E_*^2}{m^2}=1+\frac{1}{2}x_1^2+\frac{1}{2}x_2^2+\frac{2s_1\chi_1}{x_1}+\frac{2s_2\chi_2}{x_2},$$
$$\frac{m_*'^2}{m^2}=\mu+\frac{1}{2}x_1^2+\frac{1}{2}x_2^2. \quad (15)$$

The functions A_i and B_i depend on the invariants, for which expressions are given by formula (30) of[7]. In the case of the decay $\mu\rightarrow e+\nu+\tilde{\nu}$, one should take the expression

$$Z_i=\frac{x_i^2}{\chi_i}\sqrt{(u+1)(\lambda_2-\lambda)}.$$

for the quantities Z_i appearing in formula (30) of[7]. $i = 1$ and 2 for the two-wave model (2′); the subscript i should be omitted for the single-wave model (2).

The expressions for $\overline{K}(s)$ in (9) can be greatly simplified for a single monochromatic wave of circular polarization with $\epsilon = 1$ or $\epsilon = -1$. In this case the functions A_i can be calculated analytically, reducing to Bessel functions.[5] Carrying out the simple calculations, we obtain

$$\bar{K}(s)=\left[-\lambda^2+\frac{\lambda}{2}(1+\mu)+\frac{1}{2}(1-\mu)^2\right]J_s^2$$
$$+\frac{x^2}{2}\left[2\lambda+(1+\mu+2\lambda)\frac{u^2}{2(u+1)}\right](J_{s+1}^2+J_{s-1}^2-2J_s^2)$$
$$-\frac{x\varepsilon}{2}[(1-\mu)u+2\lambda(u+2)]\left[P-\frac{xs}{Z}\frac{u}{u+1}\right]J_s(J_{s+1}-J_{s-1}). \quad (16)$$

The quantity Z is the argument of the Bessel function I_s. In the case of circular polarization of the wave, $\overline{K}(s)$ does not depend on the angle φ_0, and in this case the number of integrations in expression (8) for the decay probability reduces to two.

By using expressions (8)–(16) we have carried out numerical calculations of the probabilities of the decays $\mu\rightarrow e+\nu+\tilde{\nu}$ for a single wave and for the two-wave model. The results of the calculations are shown in Fig. 1. The values of the parameters x and χ, which were used in the calculations, are also indicated there. Owing to parity nonconservation, the probabilities turn out to depend on the sign of the wave's polarization. As is clear from the results of the calculations, for a variation of ϵ turns out to be nonmonotonic. We also note that for the values of the parameters under consideration, switching on the external field increases the probability of the decays $\mu\rightarrow e+\nu+\tilde{\nu}$ for both models (2) and (2′). The results of the calculations for the process $e\rightarrow e+\nu+\tilde{\nu}$ are shown in Fig. 2 for the case of a single monochromatic wave.

3. π-MESON DECAYS

Pion decays of the type $\pi\rightarrow\mu(e)+\nu$, and also $\pi^{\pm}\rightarrow\pi^0+e^{\pm}+\nu$ for the case of a crossed field, i.e., for $x \gg 1$, have been analyzed in detail by Nikishov and Ritus[4] and by Ritus.[8] In the latter article an expression is derived for the probability of the decays $\pi^{\pm}\rightarrow\pi^0+e^{\pm}+\nu$ for a linearly polarized wave. Here we shall consider these decays for cases when the values of the invariant parameters, characterizing the effect of the wave's field, are of the order of unity.

Decays of the type $\pi\rightarrow\mu(e)+\nu$ were considered in the article by Narozhnyĭ, Nikishov, and Ritus[5] for the case of a circularly polarized wave, when $\chi = 1$ and the value of $x \sim 1$. Using the decays $\pi\rightarrow\mu+\nu$ and $\pi\rightarrow e+\nu$ as an example, it was shown in[5] that the external field of the monochromatic wave may, depending on the specific ratio of the masses, either accelerate or decelerate particle decays. Consequently, it is of interest to investigate other external-field configurations from this point of view, for example, the two-wave model. In the present article we consider these decays both for the case of a single monochromatic wave of arbitrary polarization—in order to study the dependence of the decay probability on the variation of the wave's ellipticity parameter—and for the two-wave model (2′) considered by us.

In terms of their kinematics, decays into two particles of the type $\pi\rightarrow\mu(e)+\nu$ are the simplest type, and this greatly simplifies concrete numerical calculations. By using standard methods one can obtain the following expressions for the squares of the matrix elements for the decays $\pi\rightarrow\mu(e)+\nu$.

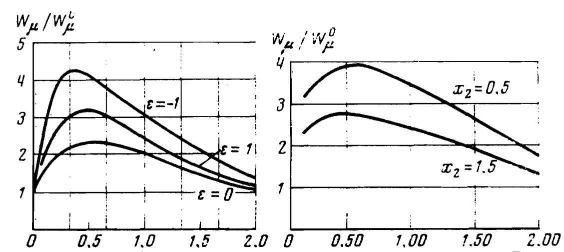

FIG. 1. Probabilities for the decay $\mu+e+\nu+\tilde{\nu}$; Fig. 1a refers to the model (2) with $\chi = 0.5$; Fig. 1b refers to the model (2′) with $\chi_1=\chi_2 = 0.5$; the value of W_μ^0 is equal to $G^2m^6n/192\pi^3q_0$.

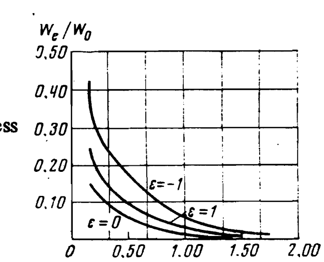

FIG. 2. Probabilities of the process $e\rightarrow e+\nu+\tilde{\nu}$ for the model (2) with $\chi = 0.5$.

In the case of the wave (2) of arbitrary polarization we have

$$|M_{if}|^2=|A_0|^2(m^2-m'^2)-e^2a^2\varepsilon^2|A_0|^2\frac{(kl)}{(kq')}$$
$$+e^2a^2[|A_1|^2+\varepsilon^2|A_1'|^2-\operatorname{Re}A_0A_2^*(1-\varepsilon^2)]\frac{(kl)}{(kq')}$$
$$-\frac{ea}{(kq')}[2\operatorname{Im}A_0A_1^*j_1(k)+2\operatorname{Im}A_0A_1'^*\varepsilon j_2(k)]$$
$$-2\operatorname{Im}A_1A_1'^*e^2a^2\varepsilon\frac{(kl)}{(kq')^2}j_{12}'(k). \tag{17}$$

The notation in Eq. (17) is obvious and does not require special explanations. For the two-wave model (2′) we find

$$|M_{if}|^2=A_0^2B_0^2(m^2-m'^2)+e^2a_1^2\frac{(k_1l)}{(k_1q')}B_0^2(A_1^2-A_0A_2)$$
$$+e^2a_2^2\frac{(k_2l)}{(k_2q')}A_0^2(B_1^2-B_0B_2). \tag{18}$$

Using Eqs. (17) and (18) we obtain the following expressions for the total probabilities of the decays $\pi\to\mu(e)+\nu$. For the model (2) we have

$$W_{2\pi}=\frac{G^2f^2m^2m'^2}{16\pi^2q_0}\sum_{s>s_0}\int_0^{2\pi}d\varphi_0\int_0^{u_{max}}\frac{du}{(u+1)^2}K_{2\pi}(s), \tag{19}$$

where

$$K_{2\pi}(s)=|A_0|^2(1-\mu)-ux^2\varepsilon^2|A_0|^2+ux^2[|A_1|^2+\varepsilon^2|A_1'|^2$$
$$-\operatorname{Re}A_0A_2^*(1-\varepsilon^2)]-2x[(u+1)\operatorname{Im}A_0A_1^*P\sin\varphi_0$$
$$-(u+1)\operatorname{Im}A_0A_1'^*\varepsilon P\cos\varphi_0+ux\varepsilon\operatorname{Im}A_1A_1'^*]. \tag{19'}$$

In formula (19)

$$u_{max}=\left(1-\mu+\frac{2s\chi}{x}\right)\frac{m^2}{m_*'^2},\quad s_0=(\mu-1)\frac{x}{2\chi}.$$

The quantity f is the same as in the book[21].

For the two-wave model (2′) we have

$$W_{2\pi}=\frac{G^2f^2m^2m'^2}{16\pi^2q_0}\sum_{s_1s_2}\int_0^{2\pi}d\varphi_0\int_0^{u_{max}}\frac{du}{(u+1)^2}N_{2\pi}(s_1s_2), \tag{20}$$

where

$$N(s_1s_2)=A_0^2B_0^2(1-\mu)+ux_1^2B_0^2(A_1^2-A_0A_2)$$
$$+ux_2^2(B_1^2-B_0B_2)A_0^2. \tag{20'}$$

Here $u_{max}=E_s^2/m_*'^2-1$, E_s^2, and $m_*'^2$ are given by formulas (12). The summation over s_1 and s_2 is carried out in the same manner as in the investigation of the decay $\mu\to e+\nu+\tilde{\nu}$. Finally, we note that the quantities P and Z_i, entering into formula (19) and into the expressions for the functions A_i and B_i, can also be obtained from the analogous quantities for μ decay, if we set $\lambda=0$ there.

Numerical calculations of the total probabilities of the decays $\pi\to\mu(e)+\nu$ were carried out for both models (2) and (2′) by using expressions (17)—(20). The results are shown in Fig. 3 (where $W_{2\pi}^0=(G^2f^2m^2n/8\pi q_0)\times m'^2(1-\mu)^2$). For the case of circular polarization of the wave in the model (2), the total probabilities of the decays $\pi\to\mu+\nu$ and $\pi\to e+\nu$ were calculated in the article by Narozhnyĭ, Nikishov, and Ritus[5] for this same range of variation of the parameter x and the same value of χ. The curves calculated by us agree with the results of[5].

Furthermore, we carried out calculations of the same probabilities for the case of a linearly polarized wave. The results of the calculation, shown in Fig. 3a, indicate that, just as in the case of circular polarization of the wave, switching on the external field accelerates the decay $\pi\to\mu+\nu$ and slows down the decay $\pi\to e+\nu$,

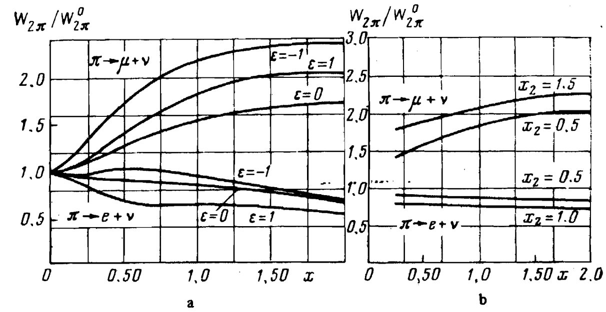

FIG. 3. Probabilities of the decays $\pi\to\mu(e)+\nu$; Fig. 3a is for the model (2) with $\chi=1$; Fig. 3b is for the model (2′) with $\chi_1=\chi_2=1$.

although the nature of the dependence on ϵ of the total probabilities of these decay processes turns out to be different for variation of ϵ within the limits $-1\leq\epsilon\leq 1$.

The results of the calculations for the case of the two-wave model (2′) are shown in Fig. 3b. The specific values of the parameters x and χ, which were used in the computations, are indicated there. Just as in the case of a single monochromatic wave, for the two-wave model the switching on of the external field increases the probability of the decays $\pi\to\mu+\nu$ and decreases for $\pi\to e+\nu$.

Finally, let us consider the decays $\pi^\pm\to\pi^0+e^\pm+\nu$. These decays are characterized by a small energy release; therefore, to a large extent they are exposed to the influence of the external field and, as shown by Ritus,[8] the quantity $\chi\delta^{-2}$ becomes the effective parameter instead of χ, where

$$\delta=(1-\mu-\lambda_1)/2\lambda_1,$$

and for the present decay process $\delta\sim 0.034$.

The matrix element for the decay $\pi^\pm\to\pi^0+e^\pm+\nu$ is given by

$$M_{if}=2^{-1/2}G\int J_\mu^\pi J_\mu^{e+}d^4x, \tag{21}$$

where J_μ^π and J_μ^e are the hadron and lepton currents, respectively, in the field of the wave:

$$J_\mu^\pi=2^{1/2}[\pi_+^*(i\nabla+eA)_\mu\pi_0+(i\nabla+eA)_\mu^*\pi_+^*\pi_0$$
$$+\pi_0^*(i\nabla-eA)_\mu\pi_-+(i\nabla-eA)_\mu^*\pi_0^*\pi_-], \tag{22}$$
$$J_\mu^e=\bar{\nu}\gamma_\mu(1+\gamma_5)e.$$

The square of the matrix element, summed over the appropriate spin variables, is given by

$$|M_{if}|^2=2R\,d_1+\frac{e}{(q'k_i)}(L_1+L_3)+\frac{e^2}{(q'k_i)}\left[L_2d_2+\frac{d_3}{(q'k_i)}L_4j_{12}'\right]. \tag{23}$$

The quantities appearing in formula (23) have the following form. For the case of a single wave

$$R=|A_0|^2,$$
$$L_1=2\operatorname{Re}A_0A_1^*at_1+2\operatorname{Re}A_0A_1'^*a\varepsilon t_2,$$
$$L_2=(|A_1|^2+\varepsilon^2|A_1'|^2)a^2,$$
$$L_3=2\operatorname{Im}A_0(A_1af_1+A_1'a\varepsilon f_2)^*,$$
$$L_4=2\operatorname{Im}A_1A_1'^*\varepsilon. \tag{24}$$

In the case of the potential (2′), $L_3=L_4=0$ and, furthermore,

$$R=C_{00}^2,$$
$$L_1=2C_{00}C_{01}a_1t_1+2C_{00}C_{10}a_2t_2,$$
$$L_2=C_{01}^2a_1^2+C_{10}^2a_2^2. \tag{25}$$

Here, just as in[7], we introduced the functions $C_{ij}=A_iB_j$. Finally, the following notation is used in writing down Eqs. (23)—(25):

$$d_1=8(p'l_1)(ll_2)+4m'^2(ll_2)+(m'^2-4m_1^2)(p'l_2),$$
$$d_2=8(ll_2)(l_1k_i)+(m'^2-4m_1^2)(l_2k_i),$$

$$t_j=[(p'n_j)(l_1k_i)-(p'k_i)(n_jl_1)][8(ll_2)+4m_1^2-m'^2]+[(n_jp)(p'k_i)-(pk_i)(n_jp')](4m_1^2-m'^2),$$
$$f_j=8(ll_2)\langle p'l_1k_in_j\rangle-(m'^2+4m_1^2)\langle p'l_2k_in_j\rangle-4m'^2\langle l_1l_2k_in_j\rangle. \qquad (26)$$

Further, we obtain d_3 from d_2 by the replacement $m'^2 \to -m'^2$. Here l_1 and l_2 denote the momenta of the neutral particles, the π^0 meson and the neutrino, and $l = l_1 + l_2$. Just as in the preceding cases, i may take either of the two values $i = 1$ or $i = 2$ for the potential (2′). This subscript can be omitted for the model (2), that is, one can assume $k_i = k$.

Carrying out the appropriate integrations, in the case of the model (2) we obtain the following result for the probability of the decays $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$:

$$W_{3\pi}=\frac{G^2m^6n}{256\pi^4q_0}\sum_{s>s_0}\int_0^{2\pi}d\varphi_0\int_{u_1}^{u_2}\frac{du}{(u+1)^2}\int_{\lambda_1}^{\lambda_2}d\lambda\,\overline{K}_{3\pi}(s). \qquad (27)$$

Here

$$\overline{K}_{3\pi}(s)=\frac{(\lambda-\lambda_1)^2}{\lambda^2}\{[-4\lambda^2+\lambda(4+3\mu)+\mu(1-\mu)]|A_0|^2-ux^2\varepsilon^2|A_0|^2+ux^2(4\lambda+\mu)[|A_1|^2+\varepsilon^2|A_1'|^2-\mathrm{Re}\,A_0A_2^*(1-\varepsilon^2)]-2x(4\lambda-\mu)[(u+1)P\,\mathrm{Im}\,A_0A_1^*\sin\varphi_0-(u+1)\varepsilon P\,\mathrm{Im}\,A_0A_1'^*\cos\varphi_0+ux\varepsilon\,\mathrm{Im}\,A_1A_1'^*]\}, \qquad (28)$$

In the case of circular polarization of the wave, $|\epsilon| = 1$, the functions A_i are calculated analytically and instead of expression (28) we have

$$\overline{K}_{3\pi}(s)=\frac{(\lambda-\lambda_1)^2}{\lambda^2}\Big\{[-4\lambda^2+\lambda(4+3\mu)+\mu(1-\mu)]J_s^2+\frac{ux^2}{2}(4\lambda+\mu)(J_{s+1}^2+J_{s-1}^2-2J_s^2)+x\varepsilon(4\lambda-\mu)\Big[(u+1)P-\frac{xs}{Z}u\Big]J_s(J_{s+1}-J_{s-1})\Big\}. \qquad (29)$$

Just as in the case of the decay $\mu \to e + \nu + \tilde{\nu}$, the quantity Z is the argument of the Bessel functions. The notation in Eqs. (27)–(29) is also obvious and does not require special explanations.

Using the formulas cited above, we have carried out computations of the decay probabilities for the process $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$ for values $\chi = 10^{-3}y$, $0.5 \le y \le 1$, and $x = 10^{-2}\overline{x}$, $\overline{x} \le 1$. The qualitative effect of the frequency dependence of the decay probability is the same as for μ decay. However, accurate estimates of the probabilities are hindered by the fact that in the present case the range of integration over du in Eq. (27) is rather large, and in order to carry out similar computations, more powerful calculational methods are required. In the case of the two-wave model it is not difficult, by using Eq. (25), to obtain an expression for the decay probability $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$ analogous to expression (27).

[1] H. R. Reiss, J. Math. Phys. 3, 59 (1962).
[2] L. S. Brown and T. W. B. Kibble, Phys. Rev. 133, A705 (1964).
[3] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 776 (1964) [Sov. Phys.-JETP 19, 529 (1964)].
[4] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 1768 (1964) [Sov. Phys.-JETP 19, 1191 (1964)].
[5] N. B. Narozhnyĭ, A. I. Nikishov, and V. I. Ritus, Zh. Eksp. Teor. Fiz. 47, 930 (1964) [Sov. Phys.-JETP 20, 622 (1965)].
[6] I. I. Gol'dman, Zh. Eksp. Teor. Fiz. 46, 1412 (1964) [Sov. Phys.-JETP 19, 954 (1964)].
[7] V. A. Lyul'ka, Zh. Eksp. Teor. Fiz. 67, 1638 (1974) [Sov. Phys.-JETP 40, 815 (1975)].
[8] V. I. Ritus, Zh. Eksp. Teor. Fiz. 56, 986 (1969) [Sov. Phys.-JETP 29, 532 (1969)].
[9] G. F. Zharkov, Yad. Fiz. 1, 173 (1965) [Sov. J. Nucl. Phys. 1, 120 (1965)].
[10] V. N. Baĭer and V. M. Katkov, Dokl. Akad. Nauk SSSR 171, 313 (1966) [Sov. Phys.-Doklady 11, 947 (1967)].
[11] I. G. Baranov, Izv. Vyssh. Uchebn. Zaved. Fiz. 17, No. 4, 115 (1974).
[12] Yu. M. Loskutov and V. M. Zakhartsov, Izv. Vyssh. Uchebn. Zaved. Fiz. 13, No. 11, 45 (1970) [Sov. Phys. J. 13, 1441 (1973)].
[13] E. A. Choban and A. N. Ivanov, Zh. Eksp. Teor. Fiz. 56, 194 (1969) [Sov. Phys.-JETP 29, 109 (1969)].
[14] J. A. Giordmaine and R. C. Miller, Phys. Rev. Lett. 14, 973 (1965).
[15] S. A. Akhmanov, A. I. Kovrigin, A. S. Piskarskas, V. V. Fadeev, and R. V. Khoklov, ZhETF Pis. Red. 2, 300 (1965) [JETP Lett. 2, 191 (1965)].
[16] S. A. Akhmanov and R. V. Khokhlov, Problemy nelineĭnoĭ optiki (Problems of Nonlinear Optics), izd. VINITI, Moscow 1964.
[17] A. Yariv, Quantum Electronics and Nonlinear Optics, (Russ. transl.), Soviet Radio, 1973.
[18] H. Prakash and Vachaspati, Nuovo Cimento B 53, 24 (1968).
[19] V. Ch. Zhukovskiĭ and J. Herrman, Vestnik Moskovskogo Universiteta. Fizika, 25, No. 6, 671 (1970) [Moscow Univ. Phys. Bulletin 25, No. 6, 55 (1970)].
[20] I. V. Lebedev, Opt. Spektrosk. 29, 948 (1970) [Opt. Spectrosc. 29, 503 (1970)].
[21] L. B. Okun', Slaboe vzaimodeĭstvie elementarnykh chastits (Weak Interactions of Elementary Particles), Fizmatgiz, 1963 (English Transl., Israel Program for Scientific Translations, Jerusalem 1965, published in the U.S.A. by Daniel Davey and Co., Inc.).

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