

# Nonlinear high-frequency properties of yttrium-iron garnet at low temperatures

L. A. Prozorova and A. I. Smirnov

Physical Problems Institute, Academy of Sciences, USSR

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By the method developed by Prozorova and Smirnov, the stationary phase  $\psi_{st}$  of parametric spin waves (PSW) is determined directly. The measurements were made at  $T = 1.65$  K and at pumping frequency  $\nu_p = 35.5$  GHz. The dependences of  $\sin \psi_{st}$  on the excess over the PSW threshold were obtained for various values of the magnetic field and orientations of it with respect to the axes of the YIG crystal. It follows from the experiment that in the absence of nonlinear damping of the PSW and of self-oscillations, the phase mechanism of limitation of the PSW amplitude operates. The regions of existence of self-oscillations are identified with the regions of instability of collective PSW oscillations. The threshold for occurrence of self-oscillations has sharp peaks of large amplitude at points of intersection of the PSW and phonon spectra. From the position of the peaks of the threshold for excitation of PSW and of self-oscillations, the value  $\eta = 4.13 \cdot 10^{-9}$  Oe cm<sup>2</sup> is determined for the exchange constant. A fine structure is detected in the magnetic-field dependence of the threshold power  $p_c$  for excitation of PSW: near the field  $H_c$  at which uniform precession (PSW with  $k = 0$ ) is parametrically excited, there are about ten minima and maxima in the  $P_c(H)$  dependence.

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## INTRODUCTION

Among the problems connected with parametric excitation of spin waves in magnetically ordered media (ferro- and antiferromagnets), one of significant interest concerns the mechanism of limitation of the number of parametric spin waves (PSW) and of establishment of a stationary state or its absence (the self-oscillation mode). For basic theoretical models there are two mechanisms: a mechanism connected with nonlinear damping<sup>[1]</sup> and a "phase" mechanism, which arises because of nonlinear interaction of pairs of waves and which leads to limitation of the PSW amplitude, in the absence of nonlinear damping, by phase detuning of the PSW with respect to the pumping<sup>[2,3]</sup>.

The beyond-threshold state of PSW is usually investigated by the method of measurement of the nonlinear high-frequency susceptibility  $\chi = \chi' + i\chi''$ . When the dissipative mechanism dominates, the susceptibility is basically determined by the term  $i\chi''$ , which determines the deterioration of the Q of the microwave resonator containing the specimen. In the "phase" mechanism, the important contribution to the susceptibility will be made by  $\chi'$ , and it shows up in the experiment through a change of the natural frequency of the resonator.

But investigations of PSW on the basis of the resonator parameters do not give high accuracy: in the experiments described<sup>[4,5]</sup>, the detuning of the resonator because of excitation of PSW in the specimen does not exceed the half-width of the resonance curve. Use of larger specimens leads to strong distortion of the resonance curve, and in addition there is an increase in the error resulting from distortion of the microwave field corresponding to the normal mode of the resonator.

Authors of experimental investigations of the nonlinear properties of PSW in the ferromagnet iron-yttrium garnet (YIG), in their interpretation of the data on beyond-threshold high-frequency susceptibility, incline toward use both of the dissipative<sup>[4]</sup> and of the phase<sup>[5]</sup> mechanisms. In a later paper<sup>[6]</sup>, the  $\chi''(H)$  dependence was investigated in detail. The difference between the data obtained in this paper and the relations

predicted by a theory<sup>[1]</sup> based on the dissipative limitation mechanism are explained by the authors through the presence, in the actual situation, of the second mechanism along with the first.

In<sup>[7]</sup> another method was proposed for investigation of the beyond-threshold state of PSW: a method based on study of the transitional processes in a change of the pumping parameters. The power being absorbed by the specimen is

$$W \propto N \sin \psi_k, \quad (1)$$

where  $N$  is the number of spin waves, and where  $\psi_k$  is the time phase of a parametrically excited pair of spin waves, reckoned from the pump. In the stationary state,  $\psi_k = \psi_{st}$ . In a change of phase of the pump that is rapid in comparison with the lifetime  $\tau_m$  of a magnon, by the amount  $\alpha$ , the power being absorbed changes in accordance with (1), and then a transitional process is observed; and after a time  $\sim \tau_m$ , the system again goes into a stationary mode. From the position of the maxima of the periodic function  $W(\alpha)$  it is possible to determine  $\psi_{st}$  with accuracy 10%.

By the method described, the antiferromagnetic crystal  $MnCO_3$  was investigated in<sup>[7]</sup>; in the same place are described the details of the experiment and of the treatment of the data. The results show that in this material the phase mechanism is decisive in the limitation of PSW, and good agreement with theory<sup>[8,2,3]</sup> was found.

In connection with what was said above, it was of interest to apply the method of direct measurement of  $\psi_{st}$  developed in<sup>[7]</sup> to YIG. Part of the present paper is devoted to this. Also studied was, incidentally, the threshold of excitation and of self-oscillation of PSW. The measurements were made at helium temperature.

## EXPERIMENT

The apparatus used was described in detail in<sup>[7]</sup>. The specimen under study was placed in a cylindrical resonator at the antinode of the microwave magnetic field  $h$ . The quality factor of the resonator was  $Q \approx 1500$ .

The source of microwave oscillations was a klystron operating in the long-pulse ( $\sim 20$  msec) mode. The signal that passed through the resonator was applied to a crystal detector and was examined on the screen of an oscillograph. The recording of the excitation and response of the PSW on change of phase of the pump was done on the basis of the absorption of power by the specimen, that is on the basis of the change of signal at the detector.

In the YIG specimen that we had (a sphere of diameter  $1.03 \text{ mm}$ ) the lifetime of PSW at  $T = 1.65 \text{ K}$  was, according to our measurements,  $\tau_m \gtrsim 1 \mu\text{sec}$ . Switching of the phase of the microwave pump was accomplished by a change of the klystron frequency for a time  $\sim 0.1 \tau_m$ ; for this purpose, an additional short pulse was fed to the klystron repeller plate. The shift of phase  $\alpha$  thus obtained could be easily varied from zero to a few radians, depending on the strength of the additional pulse.

The investigations were carried out for two orientations of the specimen with respect to the static magnetic field: 1) magnetic field directed along the axis of easiest magnetization;  $\mathbf{H} \parallel \langle 111 \rangle$ , 2) magnetic field directed along the hard axis,  $\mathbf{H} \parallel \langle 100 \rangle$ .

The PSW were excited by the method of "parallel pumping"; that is, the microwave field  $h$  was parallel to the static. The specimen was located in a liquid-helium bath at  $T = 1.65 \text{ K}$ .

## RESULTS OF THE MEASUREMENTS

### 1. Magnetic field $\mathbf{H} \parallel \langle 111 \rangle$

a) The dependence of the threshold power  $P_c$  on the magnetic field  $H$  has a "butterfly" form (Fig. 1). The minimum of  $P_c$  is attained at field  $H_c = 5.83 \text{ kOe}$ . This value is close to the calculated,  $H_c = 5.88 \text{ kOe}$ . In the calculations we used the values of quantities given in the paper of Nilsen, Comstock, and Walker<sup>[9]</sup>:

$$M = 2470 \text{ Oe}, \quad \gamma/K/M = 418 \text{ Oe}, \quad \gamma = 2.8 \text{ GHz/kOe}.$$

On the  $P_c(H)$  curve there are two peaks, at fields  $H_1$  and  $H_2$ , which are a consequence of intersection of the spin-wave and phonon spectra.

In fields  $H_2 < H < H_c$  nonmonotonicity is observed in the  $P_c(H)$  dependence: about 10 maxima with field intervals of order  $10 \text{ Oe}$  and height  $\sim 1 \text{ dB}$  (Fig. 2). The experimental error in  $P_c$ ,  $\pm 0.5 \text{ dB}$ , prevents us from plotting the exact form of the  $P_c(H)$  dependence in its

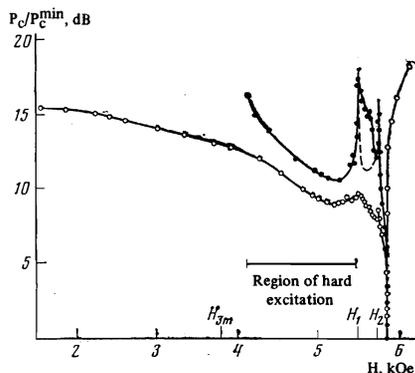


FIG. 1. Dependence on magnetic field  $\mathbf{H} \parallel \langle 111 \rangle$  of threshold power for excitation of PSW ( $\circ$ ) and self-oscillation threshold ( $\bullet$ ). The dotted curve shows the threshold for "weak" self-oscillations.

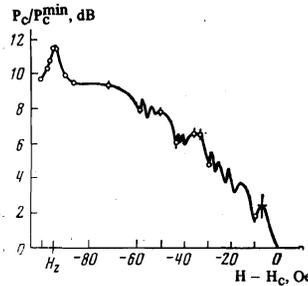


FIG. 2

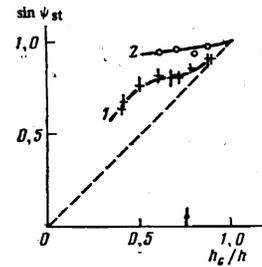


FIG. 3

FIG. 2. Dependence of threshold power for excitation of PSW near  $H_c$ , for  $\mathbf{H} \parallel \langle 111 \rangle$ .

FIG. 3. Results of experiment for determination of  $\psi_{st}$ ;  $\mathbf{H} \parallel \langle 111 \rangle$ . Curve 1,  $H = 5500 \text{ Oe}$ ; curve 2,  $H = 3700 \text{ Oe}$  (large nonlinear damping).

chopped-up part, but observation of the time shift of the start of the distortion of the microwave-power pulse enables us to establish the position of the minima and maxima.

b) There are observed "strong" self-oscillations, 50 to 100% of the absorbed power directly after the threshold for their excitation, and "weak," of order 10% of the absorbed power. The threshold for self-modulation of the first type is shown in Fig. 1 by the solid line, of the second by the dotted line. The automodulation threshold is 1 to 3 dB distant from  $P_c$ . This gap greatly increases at fields  $H_1$  and  $H_2$ . For  $H < 4.1 \text{ kOe}$ , at a resonator input power exceeding the threshold value by 5 dB, self-oscillations were not observed. A larger excess was not attainable in this field range in our experiment.

c) In the interval  $4.1 \text{ kOe} < H < H_1$ , "hard" excitation is observed: on gradual increase of the microwave power from zero, the PSW oscillation begins at a power exceeding by several tens of decibels the power at which the excitation of PSW discontinues when the power is decreased.

d) The presence of strong self-oscillations impedes the measurement of the value of  $\psi_{st}$ . Therefore the study of the  $\psi_{st}(h/h_c)$  dependence can be carried out only in those ranges in which the threshold for strong self-oscillations is appreciably separated from  $h_c$ . Here  $h_c$  is the threshold (for excitation of PSW) amplitude of the magnetic field of the microwave pump. In connection with this, we performed two series of experiments: in fields  $H = 3.7 \text{ kOe}$  (self-oscillations absent) and  $H = 5.55 \text{ kOe}$  (only weak self-oscillations present). The results of the experiments are shown in Fig. 3. At field  $H = 3.7 \text{ kOe}$ ,  $\psi_{st}$  remains practically unchanged with increase of the pumping power (curve 2). At field  $H = 5.55 \text{ kOe}$  (curve 1), a  $\psi_{st}(h)$  dependence is observed. The arrow marks the amplitude of the microwave field on exceeding of which (on the graph, to the left of the arrow) weak self-oscillations occur.

### 2. Field $\mathbf{H} \parallel \langle 100 \rangle$

a) The dependence of the threshold power on the magnetic field has the same form as for the case  $\mathbf{H} \parallel \langle 111 \rangle$ . But the  $P_c(H)$  curve near  $H_c$ , instead of being broken, has only a single maximum (Fig. 4). The value of  $H_c$  is  $6.3 \text{ kOe}$ , which is also close to the calculated value  $H_c = 6.28 \text{ kOe}$ . Two ultrasonic peaks are also observed on the  $P_c(H)$  curve.

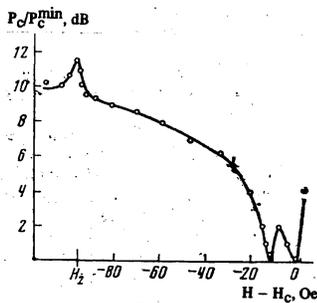


FIG. 4

FIG. 4. Dependence of threshold power for excitation of PSW near  $H_C$  for  $H \parallel \langle 100 \rangle$ .

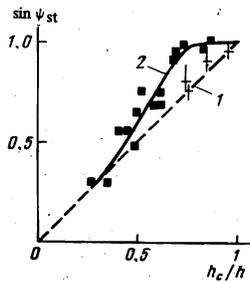


FIG. 5

FIG. 5. Results of experiment on determination of  $\psi_{st}$  for  $H \parallel \langle 100 \rangle$ . 1,  $H = H_C - 185$  Oe (no self-oscillations); 2,  $H = H_C - 30$  Oe (weak oscillations present immediately beyond the PSW threshold).

b) "Strong" self-oscillations are absent. "Weak" self-oscillations are observed in the field range  $H_C - 130$  Oe  $< H < H_C$ ; they occur immediately beyond the threshold of parametric excitation.

c) "Hard" excitation is not observed.

d) Experiments to determine the stationary phase were conducted at two values of the field  $H$  (Fig. 5). At  $H = H_C - 185$  Oe (self-oscillations absent),  $\psi_{st}$  depends on the amplitude of the pumping field immediately beyond the threshold for excitation of PSW (curve 1); at  $H = H_C - 30$  Oe (weak self-oscillations present), the change of  $\psi_{st}$  begins at 2 dB (of power) above the threshold.

## DISCUSSION OF RESULTS

1. From the position of the ultrasonic peaks on the  $P_C(H)$  curve, it is possible to calculate the exchange constant in the spin-wave spectrum,  $\eta = \omega_{ex} a^2$ . The velocities of sound in monocrystalline YIG at  $T = 4.2$  K are given in [9]:

$$v_{l(100)} = 7.209 \cdot 10^3 \text{ cm/sec}, \quad v_{l(110)} = 7.153 \cdot 10^3 \text{ cm/sec}, \\ v_t = 3.843 \cdot 10^3 \text{ cm/sec}.$$

According to our experimental data

$$\eta = 4.13 \cdot 10^{-9} \text{ Oe} \cdot \text{cm}^2.$$

This value is 20% smaller than the value obtained in measurements by the same method at room temperature,  $\eta = 5.19 \times 10^{-9} \text{ Oe} \cdot \text{cm}^2$  [10,11] and close to the value  $\eta = 4.2 \times 10^{-9} \text{ Oe} \cdot \text{cm}^2$  obtained in [6] at  $T = 4.2$  K and  $\nu_p = 34$  GHz. LeCraw and Walker [11] noted a diminution of  $\eta$  on lowering of the temperature from room temperature to 4.2 K, but only by 5%.

2. The existing theory of self-oscillations in YIG [12,3] predicts the presence of self-modulation as it depends on concrete experimental conditions (anisotropy field, demagnetizing factor, pumping frequency, magnetic field, etc.). Zautkin and Starobinets [12], on the basis of an analysis of the stability of the ground state of PSW, determined ranges in which self-oscillations should be observed; these they interpret as an instability of various modes of oscillation of the phase and number of PSW with respect to the stationary values. On applying these results to the conditions of our experiments, we found:

a) For  $H \parallel \langle 111 \rangle$  at all magnetic fields, the uniform mode ( $m = 0$ ) of collective oscillations is unstable; this should lead to strong self-modulation,

b) For  $H \parallel \langle 100 \rangle$ , the mode  $m = 0$  is stable; but for  $m = -2$  (nonuniformity with respect to the azimuthal angle relative to the direction of the magnetic field), instability is predicted in the magnetic-field range  $H_C - 140$  Oe  $< H < H_C$ .

Thus there is good agreement of the theory with experimental data: apparently the principal self-modulation for  $H \parallel \langle 111 \rangle$  is produced by instability of the collective oscillations with  $m = 0$ , and in the case  $H \parallel \langle 100 \rangle$  of those with  $m = -2$ .

Unexpected is the abrupt increase of the threshold of strong self-oscillations at fields  $H_1$  and  $H_2$  with a comparatively small increase of  $P_C$  at these points.

3. The dotted straight lines in Figs. 3 and 5 show the result of stationary nonlinear theory for the threshold of the phase of PSW with respect to the pump in the absence of nonlinear damping. It was of interest to establish experimentally the influence of nonlinear damping and of self-oscillations on the "phase" mechanism of limitation of the PSW amplitude.

Curve 2 in Fig. 3 was drawn according to the results of the experiment at field 3.7 kOe. It is evident that in this case the value of  $\psi_{st}$  remains almost unchanged on increase of the power; that is, the "phase" mechanism plays an insignificant role. This was to be expected, since for  $H < H_{3m}^{(2)}$  ( $H_{3m} = 3.79$  kOe for our experimental conditions) there is strong nonlinear damping, caused by processes of three-magnon fusion of parametric magnons [6,3]. From lines 1 in Fig. 3 and 2 in Fig. 5 it is possible to trace the influence of "weak" self-oscillations on the dependence  $\psi_{st}(h_c/h)$ . Upon occurrence of self-oscillations ( $(h/h_c)^2 = 2$  dB), the slope of the  $\sin \psi_{st}(h_c/h)$  dependence changes (Fig. 3). But if self-oscillations occur immediately beyond the threshold  $h_c$ , then there is a noticeable "retardation" (Fig. 5) of the phase threshold on increase of the pumping power, by comparison with the case in which self-oscillations are absent (line 1 in Fig. 5).

The small departure of curve 1 in Fig. 3 from the dotted straight line is apparently explained by small nonlinear damping. Under the given conditions (close to the intersection of the spin-wave and acoustical spectra), it may be caused by processes of fusion of two parametric magnons into a single phonon.

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<sup>1</sup>The authors are grateful to G. A. Melkov for kindly providing the monocrystalline sphere of YIG.

<sup>2</sup> $H_{3m}$  is the upper bound of the magnetic-field range,  $H < H_{3m}$ , in which the conservation laws permit processes of fusion of two parametric magnons into a third (see, for example, [13]).

<sup>1</sup>P. C. Gottlieb and H. Suhl, J. Appl. Phys. **33**, 1508 (1962).

<sup>2</sup>V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **59**, 1200 (1970) [Sov. Phys.-JETP **32**, 656 (1971)].

<sup>3</sup>V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys.-Usp. **17**, 896 (1975)].

- <sup>4</sup>Y. A. Monosov, V. V. Surin, and V. A. Shakhunov, Zh. Eksp. Teor. Fiz. **65**, 1905 (1973) [Sov. Phys.-JETP **38**, 951 (1974)].
- <sup>5</sup>V. V. Zautkin, V. E. Zakharov, V. S. L'vov, S. L. Musher, and S. S. Starobinets, *ibid.* **62**, 1782 (1972) [**35**, 926 (1972)].
- <sup>6</sup>V. L. Grankin, G. A. Melkov, and S. M. Ryabchenko, *ibid.* **67**, 2227 (1974) [**40**, 1105 (1975)].
- <sup>7</sup>L. A. Prozorova and A. I. Smirnov, *ibid.* **67**, 1952 (1974) [**40**, 970 (1975)].
- <sup>8</sup>V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Fiz. Tverd. Tela **11**, 2047 (1969) [Sov. Phys.-Solid State **11**, 658 (1970)].
- <sup>9</sup>W. J. Nilsen, R. L. Comstock, and L. R. Walker, Phys. Rev. **139A**, 472 (1965).
- <sup>10</sup>H. E. Turner, Phys. Rev. Lett. **5**, 100 (1960).
- <sup>11</sup>R. C. LeCraw, and L. R. Walker, J. Appl. Phys. Suppl. **32**, 167S, (1961).
- <sup>12</sup>V. V. Zautkin and S. S. Starobinets, Fiz. Tverd. Tela **16**, 672 (1974) [Sov. Phys.-Solid State **16**, 442 (1974)].
- <sup>13</sup>E. Schlömann, J. Appl. Phys. **33**, 527 (1962).

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