Threshold phenomena in superconductors

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A thin superconducting film in an alternating electromagnetic field of frequency close to Δ is considered. It is shown that the density of states near the threshold $\epsilon = \Delta$ changes strongly. The resultant nonequilibrium electron energy distribution function, the variation of the order parameter, the dissipation current, and the tunneling characteristics are all considered.

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In the interaction between coherent radiation of frequency near the value of a forbidden band and a semiconductor, the spectrum of single-particle excitations in the latter is strongly changed near the band boundaries.[1] There is a correspondence here with the problem of interaction of resonant radiation with a two-level system.

The analogous problem can also be considered in superconductors, where the frequency of the quantity which plays the role of the field should be close to 2Δ. We consider a pure superconducting film, sufficiently thin that all the quantities are homogeneous over its thickness. Let the frequency of the radiation incident on the film be $\omega = 2\Delta$. Then, just as in the case of a semiconductor, the corrections to the Green's function contain resonance singularities in the denominators:

$$G_\varepsilon \mathbf{A}_\omega G_{\varepsilon-\omega} \mathbf{A}_{-\omega} G_\varepsilon = \frac{\varepsilon+\xi}{\varepsilon^2-\Delta^2-\xi^2} \mathbf{A}_\omega \frac{\varepsilon-\omega+\xi}{(\varepsilon-\omega)^2-\Delta^2-\xi^2} \mathbf{A}_{-\omega} \frac{\varepsilon+\xi}{\varepsilon^2-\Delta^2-\xi^2} \tag{1}$$

at $\epsilon - \Delta \ll \Delta$ and at correspondingly small ξ. In the semiconductor, account of this type of interband interaction alone already gives a large effect even for comparatively low intensity of the electromagnetic field. However, the situation is different in the superconductor. Thanks to the specific coherence factors of the type $\epsilon(\epsilon - \omega) + \Delta^2$, which arise at vertices with a vector potential, and which are small at $\epsilon - \Delta \ll \Delta$ and $\omega - 2\Delta \ll \Delta$, one singularity is cancelled out in the expression (1). Account of terms of the type $G_\epsilon \mathbf{A}_{-\omega} G_{\epsilon+\omega} \mathbf{A}_\omega G_\epsilon$ shows that another singularity cancels out in the Green's function that is quadratic in $\mathbf{A}$. It can be shown that for any order of perturbation theory and for the Green's function of the pure superconducting film integrated with respect to ξ no accumulation of resonance singularities takes place with increase in the order of perturbation theory in terms of the vector potential.

We consider the situation in which the frequency of the external field $\omega = \Delta$ or is close to Δ. Under these conditions, the correction to the order parameter is quadratic in the field, has a resonant singularity[2] and, for $\Delta\tau \gg 1$, is given by

$$\Delta_{2\omega} = \Delta_{-2\omega} = -\left(\frac{e}{c} v \mathbf{A}_\omega\right)^2 \left(\frac{2\Delta}{\Delta-\omega}\right)^{1/2} \frac{K(1/2)}{3\pi\Delta^2\tau}, \quad \omega < \Delta. \tag{2}$$

The quantity $\Delta_{2\omega}$ is some external field in addition to $\mathbf{A}_\omega$ and because of the singularity at these frequencies, its role can be important. In this case, graphs are formed which give the contribution to the diagonal in the energy Green's function:

$$G_\varepsilon \mathbf{A}_\omega G_{\varepsilon-\omega} \mathbf{A}_{-\omega} G_\varepsilon, \quad G_\varepsilon \Delta_{2\omega} G_{\varepsilon-2\omega} \Delta_{-2\omega} G_\varepsilon,$$
$$G_\varepsilon \mathbf{A}_{-\omega} G_{\varepsilon+\omega} \mathbf{A}_{-\omega} G_{\varepsilon+2\omega} \Delta_{2\omega} G_\varepsilon,$$

the order of magnitude of which (without external lines) is

$$\frac{(ev/c)^2 \mathbf{A}_\omega \mathbf{A}_{-\omega}}{\Delta^2\tau}, \quad \frac{\Delta_{2\omega}\Delta_{-2\omega}}{\Delta}, \quad \frac{(ev/c)^2 \mathbf{A}_{-\omega}^2 \Delta_{2\omega}}{\Delta^2}.$$

For sufficiently high intensity $(ev\mathbf{A}_\omega/c)^2 \gg \Delta^2 \times [(\Delta - \omega)/\Delta]^{1/2}$, graphs of the latter type predominate. It is important that for these radiation intensities the arising alternating current in the film is much smaller than the pair-breaking current. In addition, $\Delta_{2\omega} \ll \Delta$ always.

Thus, to find the Green's function, it suffices to take into account the minimal degree of intensity of the field for each resonance singularity:

$$G_\varepsilon = G_\varepsilon^{(0)} + \Sigma_1^- G_\varepsilon + \Sigma_1^+ F_\varepsilon^+, \quad F_\varepsilon^+ = F_\varepsilon^{+(0)} + \Sigma_2^- F_\varepsilon^+ + \Sigma_2^+ G_\varepsilon. \tag{3}$$

Typical contributions to Σ_1^- and Σ_2^- are

$$G_\varepsilon^{(0)} \Delta_{2\omega} F_{\varepsilon-2\omega}^{+(0)} \mathbf{A}_{-\omega} G_{\varepsilon-\omega}^{(0)} \mathbf{A}_{-\omega},$$
$$F_\varepsilon^{+(0)} \Delta_{-2\omega} F_{\varepsilon+2\omega}^{+(0)} \mathbf{A}_\omega F_{\varepsilon+\omega}^{(0)} \mathbf{A}_\omega, \tag{4}$$

and analogously to Σ_1^+ and Σ_2^+. If we introduce the notation

$$R_\varepsilon = \varepsilon^2 - \Delta^2 - \xi^2, \quad \lambda = -\frac{\Delta_{2\omega}}{\Delta}\left(\frac{e}{c} v\right)^2 \mathbf{A}_\omega \mathbf{A}_\omega, \quad \varepsilon_0 = 2\Delta - 2\omega, \tag{5}$$

then the series for the Green's function can be represented schematically in the form

$$G_\varepsilon \sim \frac{1}{R}\left[1 + \frac{\lambda}{R} c_1 + \left(\frac{\lambda}{R}\right)^2 c_2 + \ldots\right].$$

The quantities $\epsilon - \Delta$, $\Delta - \omega$, ξ^2/Δ, and λ/Δ are of the same order of smallness. The part of the terms containing additional resonance denominators $R_{\epsilon-2\omega}$ also necessarily contains added small quantity of the type $\Delta(\epsilon - \Delta)$, ξ^2, or $\epsilon(\epsilon - 2\omega) + \Delta^2$ in the numerator, so that the aggregate of the terms of the type (4) is of the same order of smallness.

We have considered the case of a pure film $\Delta\tau \gg 1$; therefore, the scattering must be taken into account only to the extent to which this leads to a correction to the order parameter (2) that is different from zero and is variable in time. In the summation in (3) one can set $1/\tau = 0$. We shall take G and F to mean retarded functions; then Eqs. (3) are valid for any temperature. Account of all the graphs of the form (4) gives

$$\Sigma_{1,2}^+ = -\frac{\xi^2 + 2\Delta(\varepsilon-\Delta) \pm 2\Delta\xi}{\Delta R_\varepsilon R_{\varepsilon-2\omega}}, \quad \Sigma_{1,2}^- = \frac{2\Delta(\varepsilon-\Delta) - \xi^2 \pm 2\Delta\xi}{\Delta R_\varepsilon R_{\varepsilon-2\omega}},$$

and for the Green's function

$$G_\varepsilon = F_\varepsilon^+ = \frac{\Delta}{R_\varepsilon} \frac{1 - 2\lambda x^2/R_{\varepsilon-2\omega}}{[1 - 4\lambda x^2\Delta(\varepsilon-\Delta)/R_\varepsilon R_{\varepsilon-2\omega}][1 + 2\lambda x^2\xi^2/R_\varepsilon R_{\varepsilon-2\omega}]},$$
$$x = \cos\measuredangle(\mathbf{p}\mathbf{A}_\omega). \tag{6}$$

We now find the density of states in such a superconducting film exposed to radiation (ρ is normalized to unity in the normal metal):

$$\rho(\varepsilon) = -\frac{1}{\pi}\int_0^1 dx \int_{-\infty}^{\infty} \operatorname{Im} G_\varepsilon \, d\xi. \tag{7}$$

The introduction of the density of states turns out to be justified on the ground that the Green's functions that are nondiagonal in the energy, integrated over ξ, are small in comparison with those that are diagonal. Actually, if the new vertex, which leads to the collapse of the frequency, does not bring about the appearance of new resonance denominators, then this is obvious, since an additional small quantity $\mathbf{A}_\omega$ or $\Delta_{2\omega}$ appears. The new resonance denominator is always accompanied by an equally small numerator.

The density of states for $\epsilon_0 = 2\Delta - 2\omega > \lambda/\Delta > 0$ is ($\epsilon_1 = \epsilon - \Delta$)

$$\rho(\varepsilon_1) = \int_0^1 dx \frac{\Delta\lambda x^2|\varepsilon_1|}{r_1 - r_2 + \lambda x^2}\left\{\frac{(r_2+\Delta\varepsilon_0)^{1/2}(2\Delta\varepsilon_1+\Delta\varepsilon_0+r_2-2\lambda x^2)}{r_2(r_1+r_2-\lambda x^2)}\frac{\theta(A)}{A^{1/2}} + \frac{[r_1+\Delta(\varepsilon_0-\lambda x^2/\Delta)]^{1/2}(2\Delta\varepsilon_1+\Delta\varepsilon_0+r_1+\lambda x^2)}{r_1(r_1+r_2+\lambda x^2)}\frac{\theta(B)}{B^{1/2}}\right\}, \tag{8}$$

where

$$A = [\varepsilon_1 + {}^1/_2(\varepsilon_0 - \lambda x^2/\Delta)]^2 - {}^1/_4(\varepsilon_0 - \lambda x^2/\Delta)^2,$$

$$B = (\varepsilon_1 + {}^1/_2\varepsilon_0)^2 - {}^1/_4\varepsilon_0^2,$$

θ is the Heaviside function,

$$r_1 = [(\Delta\varepsilon_0 - \lambda x^2)^2 + 4\Delta^2\varepsilon_1(\varepsilon_1+\varepsilon_0)]^{1/2},$$

$$r_2 = [(\Delta\varepsilon_0)^2 + 4\Delta^2\varepsilon_1(\varepsilon_1+\varepsilon_0-\lambda x^2/\Delta)]^{1/2}.$$

In the limiting cases we find the following:

1) $0 < \epsilon_1 \ll \epsilon_0$; here

$$\rho(\varepsilon) = \left(\frac{\Delta}{2\varepsilon_1}\right)^{1/2}\frac{2}{q^{1/2}}\left(\arcsin q^{1/2} - \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{q}{2-2q}\right)^{1/2}\right), \quad q = \frac{\lambda}{\Delta\varepsilon_0}; \tag{9}$$

for $q \ll 1$, we have $\rho(\epsilon) = (\Delta/2\epsilon_1)^{1/2}(1 - q^2/40)$.

2) $\epsilon_2 \ll \epsilon_1$; the effect of the field turns out to be weak, the density of states is identical with its equilibrium value

$$\rho(\varepsilon) = (\Delta/2\varepsilon_1)^{1/2}. \tag{10}$$

3) $\epsilon_1 < 0$, $|\epsilon_1| \gg \epsilon_0$; we have

$$\rho(\varepsilon) = \Delta\lambda^2/10(-2\Delta\varepsilon_1)^{5/2}. \tag{11}$$

4) $\epsilon_1 < -(\epsilon_0 - \lambda/\Delta) \equiv \epsilon_2$, $(\epsilon_2 - \epsilon_1) \ll \epsilon_0$; here

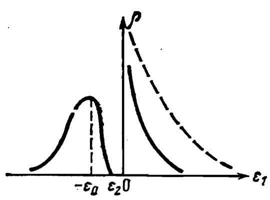
$$\rho(\varepsilon) = (\varepsilon_2 - \varepsilon_1)^{3/2}/3\varepsilon_0[2\varepsilon_0(\Delta\varepsilon_0 - \lambda)]^{1/2}; \tag{12}$$

5) $\epsilon_1 < 0$, $(-\epsilon_0 - \epsilon_1) \ll \epsilon_0$; a discontinuity on the curve of density of states:

$$\delta\rho(\varepsilon) = \frac{\sqrt{\Delta(-\varepsilon_0-\varepsilon_1)}}{2\varepsilon_0}\left(\frac{1}{(1-q)^{1/2}} - \frac{\arcsin\sqrt{q}}{q^{1/2}}\right)\theta(-\varepsilon_0-\varepsilon_1). \tag{13}$$

The density of states is shown schematically in the figure, where the dashed line shows the unperturbed relation. In fact, a certain redistribution of the states occurs near the threshold in the energy scale ϵ_0. Part of the states, which is proportional to the field intensity, goes over from the region behind the threshold $\epsilon > \Delta$ to the forbidden region, where the density of states falls off in power-law fashion. The value at the maximum is of the order of $\lambda/\epsilon_0(\Delta\epsilon_0)^{1/2}$.

However, the role of the alternating field reduces



not only to a redistribution of states as a result of the resonant interaction with the bands. In addition to the retarded Green's function considered above, the anomalous functions which carry information on the change in the energy distribution function of the electrons, are also different from zero.[3] We shall be chiefly interested in the case of temperatures close to zero. Then the creation of new excitations by the field turns out to be ineffective, since its frequency is below the threshold which it would have in the equilibrium state, and the "tail" of the density of states in the forbidden band of energies is proportional to the small quantity λ^2/Δ^4. The equilibrium distribution function under these conditions will change for another reason—because of electron-electron interaction. Each excitation with energy 3Δ breaks up into three with energies Δ, each of which is again built up to an energy 3Δ, and the process repeats itself. Recombination with emission of a phonon prevents the avalanche-like growth of the number of excitations. The balance of these two processes determines the normalization of the nonequilibrium contribution to the distribution function, the shape of which is determined by the balance of the relatively more effective scattering and the pump field. The pump is proportional to the parameter

$$\frac{\eta^5}{\Delta^5} = \left(\frac{e}{c} v\mathbf{A}_\omega\right)^2 \frac{\omega_D^2}{\tau\Delta^5},$$

which is large under our conditions; therefore the nonequilibrium distribution function turns out to be smeared out over an energy scale $\eta \gg \Delta$ and is small in magnitude:[3]

$$n'(\varepsilon) = g_1 \frac{\omega_D^2}{\Delta\varepsilon_F}\frac{\Delta}{\eta} f\left(\frac{\varepsilon}{\eta}\right), \tag{14}$$

where f(x) is some universal function, which falls off at infinity, and g_1 is the ratio of the constants of electron-electron and electron-phonon interactions.

The quantity $\Delta_{2\omega}$ which enters into λ must be determined in self-consistent fashion, with account of the change in the density of states. The expression (2) is valid only under the condition of a small change in the density of states $\lambda \ll \Delta\epsilon_0$; nevertheless, at $\lambda \sim \Delta\epsilon_0$ the singular dependence of $\Delta_{2\omega}$ on the frequency is preserved. In addition, it can be shown that so long as $\rho(\omega) = 0$, the quantity $\Delta_{2\omega}$ remains real, i.e., the obtained systematics of the states is valid for $\epsilon_0 \geq 2\lambda/\Delta$. At high frequencies or high field intensities $\rho(\omega) > 0$ and an imaginary part $\Delta_{2\omega}$ appears, which leads to some smoothing of the spectrum. We shall not consider this question in any further detail here.

The change in the stationary order parameter Δ can be tentatively divided into two parts—a part connected with the change in the density of states, and a part connected with the change in the distribution function. The self-consistent condition here is of the form

$$\frac{1}{g} = \int_0^{\omega_D} \rho(\varepsilon)\frac{1-2n(\varepsilon)}{\varepsilon}d\varepsilon = \int_0^{\omega_D}\rho(\varepsilon)\operatorname{th}\frac{\varepsilon}{2T}\frac{d\varepsilon}{\varepsilon} - 2\int_0^{\infty}\rho(\varepsilon)n'(\varepsilon)\frac{d\varepsilon}{\varepsilon}. \tag{15}$$

For T = 0, the first part can be written in the form

$$\int_0^{\omega_D}\rho(\varepsilon)\frac{d\varepsilon}{\varepsilon} = \int_\Delta^{\omega_D}\rho_0(\varepsilon)\frac{d\varepsilon}{\varepsilon} + \int_{-\infty}^{\infty}\frac{\rho(\varepsilon_1)-\rho_0(\varepsilon_1)}{\Delta}d\varepsilon_1 = \int_\Delta^{\omega_D}\rho_0(\varepsilon)\frac{d\varepsilon}{\varepsilon}$$

(the levels are merely redistributed and their number remains unchanged). Therefore, the fundamental contribution to the change in the order parameter is made by the nonequilibrium distribution function[3]

$$\frac{\Delta_0-\Delta}{\Delta_0} \sim g_1 \frac{\omega_D^2}{\Delta\varepsilon_F} \frac{\Delta}{\eta} \ln\frac{\eta}{\Delta}. \tag{16}$$

The considered effect of the change in the density of states under the action of an alternating field can be observed experimentally by measuring the volt-ampere characteristic of a tunnel junction of an irradiated superconducting film and, for example, of a normal metal. The dependence of the tunnel current on the junction voltage[4] is of the form

$$I=\frac{1}{R}\int_{-\infty}^{\infty}\rho(\varepsilon)\,d\varepsilon\,[n(\varepsilon)-n_N(\varepsilon+V)]=\frac{1}{R}\int_{0}^{V}\rho(\varepsilon)\,d\varepsilon$$
$$+\frac{1}{R}\int_{-\infty}^{\infty}\rho(\varepsilon)\,d\varepsilon\,[n_F(\varepsilon+V)-n_N(\varepsilon+V)], \tag{17}$$

$n_N(\epsilon)$ is the distribution function of the electrons in the normal metal. In writing (17), we have made use of the fact that the total number of particles

$$\int_{-\infty}^{\infty}\rho(\varepsilon)n(\varepsilon)\,d\varepsilon$$

does not change upon deformation of the distribution function, and we can use $n_F(\epsilon)$ for $n(\epsilon)$ in Eq. (17).

The distribution function of the electrons in a normal metal differs generally speaking from a Fermi distribution, since the radiation penetrates into the normal metal. However, in the case of a normal metal, the heating of the electrons in it is of little effect because of diffusion of the carriers into the bulk metal, and the nonlinear distribution function in it is small. Besides, the change in the energy distribution function is a quantity of order Δ or larger; therefore, the second term in (17) certainly makes small contribution to the differential characteristic. The scale of change of the first term $V \sim \epsilon_0 \ll \Delta$. For this reason, $\rho(V) = R\partial I/\partial V$.

Strictly speaking, the resultant state does not have an energy gap between the electrons and holes; however, since the density of states is small in a large part of the forbidden band and the distribution function (14) is also small, the absorption at frequencies $\omega < 2\Delta$ is insignificant. The dissipation current can be expressed in terms of Green's functions integrated over ξ:[5]

$$\mathbf{j}_\omega=\frac{3ne^2}{2m\omega^2\tau}\frac{i}{c}\mathbf{A}_\omega\int_0^1 x^2\,dx\int_{-\infty}^{\infty}\frac{\varepsilon(\varepsilon-\omega)-\Delta^2}{\Delta^2}$$
$$\times[n(\varepsilon-\omega)-n(\varepsilon)][\bar{\nu}_\varepsilon\nu_{\varepsilon-\omega}(x)+\nu_\varepsilon(x)\bar{\nu}_{\varepsilon-\omega}]\,d\varepsilon. \tag{18}$$

The prime denotes averaging over the angles:

$$\nu_\varepsilon(x)=-\frac{\Delta}{\pi\varepsilon}\int_{-\infty}^{\infty}\operatorname{Im}G_\varepsilon(x,\xi)\,d\xi.$$

The contribution to the dissipative current from the change in the density of states ($\mathbf{j}_1$) and the distribution function ($\mathbf{j}_2$) can be represented (for the case $\omega \lesssim 2\Delta$) in the form

$$\mathbf{j}_{1\omega}\sim\frac{ne^2}{m\omega\tau}\frac{i}{c}\mathbf{A}_\omega\left(\frac{\lambda}{\Delta^2}\right)^4,\quad \mathbf{j}_{2\omega}\sim\frac{ne^2}{m\omega\tau}\frac{i}{c}\mathbf{A}_\omega g\frac{\omega_D^2}{\Delta\varepsilon_F}\frac{\Delta}{\eta}. \tag{19}$$

The film considered above was pure in relation to its transport properties and had $\Delta\tau \gg 1$. The effect does not disappear even in the opposite extremely dirty case $\Delta\tau \ll 1$.

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