Source-field distribution and energy absorption in inhomogeneous magnetoactive plasma

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(Submitted October 9, 1974)
Zh. Eksp. Teor. Fiz. 69, 142–154 (July 1975)

The distribution of oscillatory fields excited by a source in magnetoactive plasma and the absorption of high-frequency energy are investigated. The effect of plasma inhomogeneity, thermal motion of particles, finite size of source, and other factors, on the structure of the source field is consdered.

PACS numbers: 52.50.D

INTRODUCTION

The field distribution due to a source in plasma is of interest for the understanding of the mechanism responsible for the emission of radiation by antennas, the operation of probes in plasma diagnostics, and the microwave heating of plasma in large systems. In the last case, the full characterization of plasma heating efficiency requires the localization of microwave absorption regions and the elucidation of the effect of source size, plasma inhomogeneity, and thermal motion of the plasma particles. So far, the field distribution has been considered mainly in connection with the excitation, propagation, and diffraction of electromagnetic waves in anisotropic media, and the results of these studies have been reviewed by, for example, Felsen[1] and Andronov and Chugunov[2]. The field distribution was found to exhibit certain remarkable properties. In particular, a resonance cone appears in certain definite frequency bands, and most of the radiated energy is concentrated near this cone. Shadow regions appear in the angular distribution, and an interference structure is found to be present.

Let us consider the appearance of the resonance cone in cold magnetoactive plasma. According to Fisher and Gould[3], the potential associated with oscillations excited by a point source $e\delta(\mathbf{r})\exp(i\omega t)$, is of the form

$$\varphi(r,\ t)=(e/r)\,[\varepsilon_\perp(\varepsilon_\parallel\sin^2\theta+\varepsilon_\perp\cos^2\theta)]^{-1/2}e^{i\omega t},$$

where θ is the angle between the external magnetic field and the radius vector $\mathbf{r}$ of the point of observation. Hence it is clear that, in the absence of dissipation, the potential has a singularity on the surface of the cone, which is defined by the condition $\tan^2\theta = -\epsilon_\perp/\epsilon_\parallel$, if the components $\epsilon_\perp$, $\epsilon_\parallel$ of the permittivity tensor along and at right angles to the external magnetic field have different signs. In addition, we note that the group velocity vectors of plane monochromatic waves for which the hybrid resonance conditions are satisfied lie on the surface of this cone.

To determine the field due to a distributed source, we must first determine the Green function for the field due to a point source. In the case of homogeneous cold magnetoactive plasma, the tensor Green function and its asymptotic behavior in the wave zone were established by Bunkin[4]. In later work,[3,5-11] the efforts of researchers were concentrated on the interference structure of source fields, due to the thermal motion of plasma particles, with the aim of using it in plasma diagnostics. Thus, Fisher and Gould[3] have demonstrated theoretically and experimentally the existence of interference structure in the field radiated by a source of small linear dimensions for electron ($\omega \lesssim \omega_{pe}$) oscillations of magnetized ($\omega_{He} \gg \omega_{pe}$) plasmas. Kuehl[7] has discussed the conditions under which interference structure can be produced both inside and outside the resonance cone. The corresponding experiment has been described by Gonfalone and Beghin[10]. At low frequencies, i.e., when $\omega_{Hi} < \omega \sim \omega_{pi} < (\omega_{He}\omega_{Hi})^{1/2}$, the field structure produced by a source in plasma with smooth density inhomogeneity was investigated experimentally by Briggs and Parker[6].

In the present paper, the field distribution in magnetoactive plasma is investigated largely from the point of view of effective microwave heating of inhomogeneous plasmas both at high ($\omega \sim \omega_{pe}$) and low ($\omega \sim \omega_{pi}$) frequencies. Specifically, we investigate the localization of regions of absorption of high-frequency energy in cold and hot plasmas, and the effect of plasma inhomogeneities and source dimensions on absorption efficiency. Linear transformation of waves in inhomogeneous hot plasmas is taken into account. The difference in the localization of regions of collisional and collisionless absorption is noted. It is shown that the dissipation of high-frequency energy on plasma-particle collisions is localized in the region of the fine jets leaving the source, in which the radiation field has a sharp maximum. At the same time, collisionless dissipation of high-frequency energy is smeared over the plasma volume as a result of Cerenkov and cyclotron damping. It is shown that resonances due to the excitation of standing waves in the inhomogeneous plamsa shell adjacent to the source provide an important contribution to the source field.

It is important to note that, in contrast to the mechanism responsible for high-frequency absorption in inhomogeneous plasma, which is connected with the imposition of regular boundary conditions and is described, for example, by Golant and Piliya[12] and Erokhin and Moiseev[13], in this paper we use a source to define a particular boundary condition, and this radically alters the absorption picture discussed in these reviews. The properties of the field distribution investigated below are due to the properties of the broad wave packet as a whole.

The present paper is divided into two parts which correspond, respectively, to sharply differentiated and smoothly varying plasma inhomogeneity. The effect of two-dimensional inhomogeneity is discussed at the end of the second part.

1. SOURCE-FIELD DISTRIBUTION AND HIGH-FREQUENCY ENERGY ABSORPTION IN PLASMA WITH HIGHLY DIFFERENTIATED INHOMOGENEITY

1. Consider the field structure due to a source with charge distribution $\rho(r)\exp(i\omega t)$ in cold magnetoactive plasma. Since we are interested in the properties of the field, we can use the quasistatic approximation $E = -\nabla\psi$ where ψ is the potential of the oscillations. In that case, plane geometry with z axis parallel to the external magnetic field leads to a Poisson equation of the form

$$\frac{\partial}{\partial x}\varepsilon_\perp\frac{\partial}{\partial x}\psi+\frac{\partial}{\partial z}\varepsilon_\parallel\frac{\partial}{\partial z}\psi=-4\pi\rho(x,z), \tag{1.1}$$

where $\epsilon_\perp(\omega)$ and $\epsilon_\parallel(\omega)$ are the components of the permittivity tensor.

Equation (1.1) is elliptic and hyperbolic for $\epsilon_\perp\epsilon_\parallel > 0$ and $\epsilon_\perp\epsilon_\parallel < 0$, respectively. Consequently, the degeneracy lines for (1.1) are defined by the conditions $\epsilon_\perp = 0$ and $\epsilon_\parallel = 0$. For the sake of clarity, we begin with the point source $\rho = \sigma\delta(x)\delta(z)$, where σ is the running charge density and the source is located in the hyperbolic region occupying the half-space $x < x_0$. In the elliptic region $x > x_0 > 0$. To be specific, we suppose that $\epsilon_\parallel =$ const < 0, and also $\epsilon_\perp = -\epsilon_1$ for $x > x_0$ and $\epsilon_\perp = \epsilon_2$ for $x < x_0$. This is a model of the transition layer which appears, for example, as a result of the rapid change in plasma density due to the discontinuity in the transverse permittivity component.

Solving (1.1) subject to the radiation boundary conditions for, say, the electric field component E_x, we obtain the following expressions: in the elliptic region $(x > x_0)$

$$E_x=\frac{4\sigma}{\varepsilon_2}\frac{(x-x_0-ix_0\operatorname{tg}\alpha)e^{i\alpha}\operatorname{tg}\theta_c\cos\alpha}{[z^2\operatorname{tg}^2\theta_c+(x-x_0-ix_0\operatorname{tg}\alpha)^2\operatorname{ctg}^2\alpha]\operatorname{tg}\alpha} \tag{1.2}$$

and in the hyperbolic region $(x_0 > x > 0)$

$$E_x=\frac{2i\sigma}{\varepsilon_2}\left[\frac{x}{x^2-z^2\operatorname{tg}^2\theta_c}+\frac{2(x_0-x)e^{2i\alpha}}{(2x_0-x)^2-z^2\operatorname{tg}^2\theta_c}\right]\operatorname{tg}\theta_c. \tag{1.3}$$

In these expressions $\alpha = \arctan(\epsilon_1/\epsilon_2)^{1/2}$ and $\theta_c = \arctan(\epsilon_2/|\epsilon_\parallel|)^{1/2}$. It is clear from (1.2) that the source field decreases monotonically in all inward directions in the elliptic region $x > x_0$. At the same time, in the absence of losses ($\operatorname{Im}\hat{\epsilon} = 0$) in the hyperbolic region, the source field is singular on the characteristics $x = \pm z\tan\theta_c$ passing through the source, and also on the characteristics $2x_0 - x = \pm z\tan\theta_c$, i.e., the mirror reflections of the former in the degeneracy line $x = x_0$. These special characteristics are the analogs of the resonance cones in cylindric geometry. We note also that, in the hyperbolic region, equation (1.1) can be looked upon as a one-dimensional wave equation with x as the variable, and the dispersion is linear. Hence it follows that, as x increases, an initial disturbance consisting of a packet of waves propagating in a given direction will do so without change of profile.

Let us now consider another case where the source lies in the elliptic region. Assuming that $\epsilon_\perp =$ const > 0, and also $\epsilon_\parallel = -\epsilon_\parallel^{(1)}$ for $x > x_0 > 0$ and $\epsilon_\parallel = \epsilon_\parallel^{(2)}$ for $x < x_0$, we find from (1.1) that for the hyperbolic region $x > x_0$

$$E_x=\frac{2\sigma}{\varepsilon_\perp}\operatorname{tg}\theta_c\left[\frac{1}{x-x_0+ib+z\operatorname{tg}\theta_c}+\frac{1}{x-x_0+ib-z\operatorname{tg}\theta_c}\right]e^{i\alpha}\sin\alpha, \tag{1.4}$$

where

$$\theta_c=\operatorname{arctg}(\varepsilon_\perp/\varepsilon_\parallel^{(1)})^{1/2},\quad b=x_0\operatorname{ctg}\alpha,\quad \alpha=\operatorname{arctg}(\varepsilon_\parallel^{(1)}/\varepsilon_\parallel^{(2)})^{1/2}.$$

It follows from (1.4) that the source field is finite everywhere in the hyperbolic region, in this case. The layer of plasma of thickness x_0, which corresponds to the elliptic region for (1.1), defines the effective source dimensions: $\Delta x = b$, $\Delta z = x_0(\epsilon_\parallel^{(2)}/\epsilon_\perp)^{1/2}$.

We now use the same formulation of the problem but take into account the finite size l of the source in the z direction. This can be done, for example, by taking $\rho(x, z) = \sigma\delta(x)\exp(-z^2/l^2)/l\pi^{1/2}$. In the hyperbolic region $x > x_0$, instead of (1.4) we have

$$E_x=2\sigma\pi^{1/2}[W(\xi_1)+W(\xi_2)]e^{i\alpha}\sin\alpha/i\varepsilon_\perp l. \tag{1.5}$$

In this expression $\xi_{1,2} = (x - x_0 + ib \pm z\tan\theta_c)\cotan\theta_c/l$ and $W(\xi)$ is the error function defined, for example, in [3]. We note that both (1.5) and (1.4) consist of two terms which correspond to two wave packets propagating in opposite directions along the z axis. In each of these packets the source field is constant on the characteristics $x \pm z\tan\theta_c =$ const. It follows from (1.5) that, in the hyperbolic region, the field is very dependent on the source size l if l is greater than the effective thickness $x_0(\epsilon_\parallel^{(2)}/\epsilon_\perp)^{1/2}$ of the elliptic region. In this case (well away from the point $x = x_0$), the maximum field amplitude is reached on the characteristics $x - x_0 = \pm z\tan\theta_c$, and is given by

$$E_x\approx 2\pi^{1/2}\sigma e^{i\alpha}\sin\alpha/il\varepsilon_\perp.$$

2. Suppose that the field is nonpotential, and consider the effect of this on the field structure near the resonance cone. In plane geometry, and assuming that the thickness of the inhomogeneous plasma layer near the source is negligible, we obtain the following expression for the amplitude of the magnetic field due to the source in cold plasma:

$$H_y(x,z)=\frac{\pi}{c}\int_{-\infty}^{+\infty}dk_z j_s(k_z)\exp(ik_z z-ik_\perp x), \tag{1.6}$$

which is valid for high-frequency oscillations ($\omega \sim \omega_{pe}$) in magnetized ($\omega \gg \omega_{pe}$) plasma excited by a dipole with current density $\operatorname{Re} j_s(z)\delta(x)\exp(i\omega t)$, where $k_\perp = \epsilon_\parallel^{1/2}(\omega^2/c^2 - k_z^2)^{1/2}$. The contour of integration runs around the branch points $k_z = \omega/c$, $k_z = -\omega/c$ above and below, respectively. For frequencies $\omega > \omega_{pe}$, when $\epsilon_\parallel = 1 - (\omega_{pe}/\omega)^2 > 0$, propagating waves correspond to $|k_z| < \omega/c$. In this case, the integral (1.6) for a point source $j_s(z) = J\delta(z)$ can be expressed in terms of the Hankel function

$$H_y(x,z)=\frac{\pi\omega J}{ic^2}H_1^{(2)}\left(\frac{\omega}{c}(z^2+x^2\varepsilon_\parallel)^{1/2}\right)\left(1+\frac{z^2}{x^2\varepsilon_\parallel}\right)^{-1/2}. \tag{1.7}$$

For frequencies $\omega > \omega_{pe}$, the Hankel function $H_y(x, z)$ satisfies an elliptic equation and, therefore, the resonance cone is absent from the source-field distribution. This is clear directly from (1.7). For frequencies $\omega < \omega_{pe}$, when $\epsilon_\parallel < 0$, waves with $|k_z| > \omega/c$ can propagate. The source field is then described by (1.7), as before, in which whenever necessary one can take into account a small negative imaginary part of $\epsilon_\parallel$.

It follows from (1.7) that, when $\epsilon < 0$, the field structure is characterized by the presence of the resonance cone $z^2 = x^2|\epsilon_\parallel|$ on the surface of which the field has a singularity of the form $H_y \sim (z^2 - x^2|\epsilon_\parallel|)^{-1}$ (when dissipation is ignored). The source emits waves into the interior of the resonance cone. In the exterior of the cone, on the other hand, we have the shadow region where the radiation field decreases exponentially with distance from the surface of the resonance cone. The size of the transition layer between these two regions is of the order of $|\epsilon_{\|}|^{1/2}c^2/r\omega^2$. When dissipation is taken into account, the source field on the surface of the resonance cone is limited to $H_y \approx -(2J\omega/r\nu c)\cos\theta_c$, where ν is the collision frequency and $\cot^2\theta_c = |\epsilon_{\|}|$.

We also reproduce the expression for the x component of the electric field due to a point source:

$$E_x = -\frac{\pi\omega J}{c^2}\varepsilon_{\|}^{1/2}H_2^{(2)}\left(\frac{\omega}{c}(z^2+x^2\varepsilon_{\|})^{1/2}\right)xz(z^2+x^2\varepsilon_{\|})^{-1/2}.$$

Let us now take into account the effect of the finite size of the source on the amplitude of the radiated field near the resonance cone. We substitute $j_s(k_z) = (J/\pi)\exp(-k_z^2 l^2/4)$ in (1.6) where, by analogy with the foregoing, the quantity l is the size of the source. We assume that this size is small in comparison with the vacuum wavelength $\lambda_0 = 2\pi c/\omega$. When this is not so, the sharp field maximum near the resonance cone is found to smear out completely. Near the maximum, $|1 - x^2z^{-2}\cot^2\theta_c| \le (l/\lambda_0)^2$, where the main contribution to the field amplitude is provided by $k_z \gg \omega/c$, the integral given by (1.6) can be transformed to read

$$H_y = \frac{2J}{cl}\int_0^\infty ds\exp\left(-s^2-2i\gamma_1 s-\frac{2i\gamma_2}{s}\right). \qquad (1.8)$$

In this expression, $\gamma_1 = (z - x\cot\theta_c)l^{-1}$ defines the distance from the surface of the cone and $\gamma_2 = \pi^2 zl/\lambda_0$ is a parameter representing the departure from the potential character of the oscillations. The expression given by (1.8) is analogous to the integrals encountered in the theory of plasma echo (see, for example, [14]), which describe the spreading of an initial perturbation as a result of the thermal spread of the plasma-particle velocity.

The nonpotential character of the oscillations [i.e., nonlinear dispersion of $k_\perp(k_z)$] thus ensures that the field on the surface of the resonance cone is

$$H_y \approx (\pi/3)^{1/2}(2J/cl)\exp(-3\gamma_2^{2/3}e^{i\pi/3}), \quad |\gamma_2| \gg 1,$$

i.e., it falls exponentially with distance from the source. The attenuation length is of the order of $r \sim \lambda_0^2/\pi^2 l\cos\theta_c$. Here we must note that allowance for the thermal motion of the plasma particles will also lead to a restriction on the field near the resonance cone. This can be characterized by an effective source size $l_{eff} \sim (z\lambda_D^2/\sin^2 2\theta_c)^{1/3}$ where λ_D is the Debye length of electrons. Hence we find that the effect of the thermal motion on the structure of the source field near the resonance cone is unimportant when $l^2 \gg \lambda_0\lambda_D/\sin 2\theta_c$.

3. To investigate the plasma heating efficiency, we write down the formula for the volume absorption, which we can then use to investigate the distribution of absorption throughout the volume of the plasma. This is very important for large systems. In the case of quasimonochromatic wave fields, the power absorbed per unit volume is given by (see, for example, [15])

$$Q = (\omega/8\pi)E_\alpha^* E_\beta \operatorname{Im}\varepsilon_{\alpha\beta}. \qquad (1.9)$$

We also reproduce the formula for the imaginary part of the permittivity tensor. In the case of cold plasma, we have

$$\operatorname{Im}\varepsilon_{zz} = \sum_\alpha \frac{\nu_{\|,\alpha}\omega_{p\alpha}^2}{\omega(\omega^2+\nu_{\|,\alpha}^2)},$$

$$\operatorname{Im}\varepsilon_{xx} = \sum_\alpha \frac{\nu_{\perp,\alpha}\omega_{p\alpha}^2(\omega^2+\omega_{H\alpha}^2+\nu_{\perp,\alpha}^2)}{\omega[4\omega^2\nu_{\perp,\alpha}^2+(\omega^2+\nu_{\perp,\alpha}^2-\omega_{H\alpha}^2)^2]}. \qquad (1.10)$$

In these expressions, $\nu_{\|}$ and $\nu_\perp$ are the collision frequencies along and across the external magnetic field, and the sums are evaluated over the particle species.

In hot plasma, the imaginary part of the permittivity is connected with the collisional wave damping through Cerenkov and cyclotron interactions with plasma particles. For example, at low frequencies, i.e., for $\omega_{Hi} < \omega \sim \omega_{pi} < (\omega_{He}\omega_{Hi})^{1/2}$, and for isothermal plasmas with transverse wavelength greater than the Larmor radius of ions, the imaginary part of the permittivity is

$$\operatorname{Im}\varepsilon_{zz} = 2\pi^{1/2}(\omega_{pe}/\omega)^2(\omega/k_z v_{Te})^3\exp(-\omega^2/k_z^2 v_{Te}^2),$$

$$\operatorname{Im}\varepsilon_{xx} = \frac{\pi^{1/2}\omega_{pi}^2}{2\omega k_z v_{Ti}}\left[\exp\left(-\frac{(\omega-\omega_{Hi})^2}{k_z^2 v_{Ti}^2}\right)+\left(\frac{m_i}{m_e}\right)^{1/2}\exp\left(-\frac{(\omega_{He}-\omega)^2}{k_z^2 v_{Te}^2}\right)\right]. \qquad (1.11)$$

Consider the absorption of oscillations excited by a source in homogeneous cold magnetized plasma. In this case, it is clear from (1.9) and (1.10) that collisional absorption is determined by the z component of the electric field: $Q = \nu_{\|,e}\omega_{pe}|E_z|^2/8\pi\omega^2$. Hence, using (1.7) and the fact that $E_z = (c/i\omega_{\|})\partial H_y/\partial x$, we find that the main contribution to absorption is provided by the plasma region near the resonance cone where the expression for the absorbed power has the form $[x < c/\omega_{pe}(\omega/\nu_{\|,e})^{1/2}]$

$$Q_e \approx \frac{J^2\sin^3\theta_c}{4\pi\omega\cos\theta_c}\frac{\theta_*}{r^4[\theta_*^2+(\Delta\theta)^2]^2}. \qquad (1.12)$$

In this formula, $\theta_* = \frac{1}{2}(\nu_{\|,e}/\omega)\tan\theta_c$ is the angular size of the absorption region $\Delta\theta = \theta - \theta_c$. Using (1.12), we obtain the maximum and the running absorbed power:

$$Q_{max} = \frac{2J^2\omega^2\cos^2\theta_c}{\pi r^4\nu_{\|,e}^3}, \quad \int_0^\pi d\theta\, rQ = \frac{\omega J^2\sin 2\theta_c}{4r^3\nu_{\|,e}^2}.$$

Therefore, in contrast to the absorption mechanism considered in [12,13], the type of absorption considered here, which is due to the presence of the resonance cone, is very dependent on the amount of dissipation and, moreover, the absorption increases rapidly with decreasing collision frequency. When $l > \lambda_0(\nu_{\|,e}/\omega)^{1/2}/\cos \times \theta_c$, the radiation-field distribution near the resonance cone is determined by the linear size l of the source. The angular size of the absorption region is then of the order of $(l/2r)\sin\theta_c$, and the maximum and running absorbed power is given by the following order-of-magnitude expression:

$$Q_{max} \approx \frac{\nu_{\|,e}J^2}{2\pi\omega^2 l^4\cos\theta_c}, \quad \int_0^\pi d\theta\, rQ \sim \frac{\nu_{\|,e}J^2\sin\theta_c}{\pi\omega^2 l^3\cos^2\theta_c}.$$

Hence it follows that, as the size of the source increases, the heating efficiency must fall.

2. SOURCE-FIELD STRUCTURE AND ENERGY ABSORPTION FOR SMOOTHLY VARYING PLASMA INHOMOGENEITY

1. Consider the source-field structure in plasma with smoothly-varying density inhomogeneity across the magnetic field at low frequencies $\omega_{Hi}^2 \ll \omega^2 \sim \omega_{pi} \ll \omega_{He}\omega_{Hi}$. To begin with, take the case of cold plasma. We shall use (1.1) for the oscillation potential $\Phi(\mathbf{r})\exp(-i\omega t)$ in which we shall substitute $\rho = 0$ and will

take $\Phi(\mathbf{r})$ by analogy with the work of Briggs and Parker, [6] so that the boundary condition $\Phi(x_0, z) = f(z)$ will be dictated by the presence of the source. In the chosen frequency band, the components of the permittivity tensor have the form [see (1.10)]

$$\varepsilon_\perp = 1 - \frac{\omega_{pi}}{\omega^2}\left(1 - i\frac{\nu_{\perp,i}}{\omega} - i\frac{\nu_{\perp,e}\omega}{\omega_{He}\omega_{Hi}}\right), \quad \varepsilon_\parallel \approx -\frac{\omega_{pe}{}^2}{\omega^2}. \tag{2.1}$$

Near the hybrid resonance line $\mathrm{Re}\ \epsilon_\perp = 0$, and we suppose that the plasma density can be represented by the linear formula $n(x) = n_0(1 + x/L)$ where $n_0 = m_i\omega^2/4\pi e_i^2$. If the coordinates along and across the external magnetic field are written in the dimensionless form $\zeta = (z/L)(m_e/m_i)^{1/2}$ and $\xi = x/L$, respectively, we can rewrite (1.1) in the form

$$\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi}\Phi + (1+\xi)\frac{\partial^2}{\partial\zeta^2}\Phi = 0. \tag{2.2}$$

We note, to begin with, that the equation given by (2.2) is hyperbolic in the layer $0 > \xi > -1$. Next, in the hyperbolic region, the trajectories of group-velocity rays for a short wave packet are described by the characteristics of (2.2):

$$\pm(\zeta - \zeta_0) = \arcsin(1-\eta)^{1/2} - \eta^{1/2}(1-\eta)^{1/2} \equiv g(\eta), \tag{2.3}$$

where $\eta = |\xi|$ and ζ_0 is the parameter of the family of characteristics. It is clear from (2.3) that the characteristics touch the line $\epsilon_\perp = 0$ and are perpendicular to the line $\epsilon_\parallel = 0$. The analogs of the resonance cones considered in Sec. 1 are the characteristics leaving the point source located in the hyperbolic region. In the absence of losses, the oscillation potential has a singularity on these characteristics (this will be demonstrated below). The characteristics leaving the point source are also singular after reflection from the $\epsilon_\perp = 0$ line (in the case of transverse plasma inhomogeneity). At the same time, in contrast to the case of sharp inhomogeneity, the potential singularity is absent after the singular characteristic touches the line $\epsilon_\perp = 0$ on its continuation and, consequently, the energy of the wave packet is completely absorbed near the point at which the two touch.

Let us now consider Eq. (2.2). For the symmetric boundary condition $f(z) = f(-z)$, the solution of (2.2) which decreases for $\xi > 0$ has the form

$$\Phi(\xi,\zeta) = \int_0^\infty dq \cos q\zeta f(q)\frac{\Phi(q,\xi)}{\Phi(q,\xi_0)},$$
$$\Phi(q,\xi) = \Psi((1+q)/2, 1; 2q\xi)e^{-q\xi}, \tag{2.4}$$

where Ψ is the degenerate hypergeometric function, $q = k_z L(m_i/m_e)^{1/2}$, the boundary condition is specified on the line $\xi = \xi < 0$, and $f(q) = f(k_z)(m_e/m_i)^{1/2}L$. We note that the solution given by (2.4) has a branch point at $\xi = 0$. In view of (2.1), we shall suppose henceforth that $-\pi \le \arg\xi \le 0$.

Consider the excitation of oscillations by the point source $f(z) = f_0\delta(z)$ located at the point $(\xi_0, 0)$ where $0 > \xi_0 > -1$. Under these conditions, $f(k_z) = f_0/\pi$. Since we are interested in the plasma region near the singular characteristics, where the main contribution to the radiation field is provided by harmonics with large values of q, we shall use the asymptotic form of $\Phi(q, \xi)$ for $q \gg 1$ in the hyperbolic region $0 > \xi > -1$:

$$\Phi(q,\xi) \approx \pi^{1/2}q^{-1/2}[\eta(1-\eta)]^{-1/4}\Gamma^{-1}\left(\frac{1+q}{2}\right)\exp\left[i\frac{\pi}{2}\left(q+\frac{3}{4}\right) + iqg(\eta)\right], \tag{2.5}$$

where $g(\eta)$ is given by (2.3). We note that $g(\eta)$ is a monotonically decreasing function with maximum value $g(0) = \pi/2$. Substituting (2.5) into (2.4), we obtain the following expression for the potential near the singular characteristics $|\zeta| = g(\eta) - g(\eta_0)$:

$$\Phi(\xi,\zeta) \approx \frac{f_0}{2\pi iL}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\eta_0}{\eta}\right)^{1/4}\left(\frac{1-\eta_0}{1-\eta}\right)^{1/4}\frac{1}{\zeta + g(\eta_0) - g(\eta)}. \tag{2.6}$$

In view of the symmetry of the problem in ζ, we assume in (2.6) that $\zeta \ge 0$. It follows from (2.6) that, in the absence of losses, the potential diverges on the singular characteristics $|\zeta| = \zeta_s$, where $\zeta_s \equiv g(\eta) - g(\eta_0)$. A more convenient form of this can be obtained by rewriting the resonance factor in the form

$$\frac{1}{\zeta - \zeta_s} = P\frac{1}{(\zeta - \zeta_s)} - i\pi\delta(\zeta - \zeta_s),$$

where P represents the principal value. When dissipation is taken into account, the potential is restricted to a level determined by the condition $|\zeta - \zeta_s| \sim \mathrm{Im}\ \epsilon_\perp/(\mathrm{Re}\ \epsilon_\perp)^{1/2}$ for $\eta < \eta_0 \ll 1$. The thin layer of plasma near the singular characteristic, the thickness of which is of the order $\Delta x \sim L\ \mathrm{Im}\ \epsilon_\perp/(\mathrm{Re}\ \epsilon_\perp)^{1/2}$, is a kind of flute over which most of the radiation energy emitted by the source is propagated. Using (1.11), we obtain the following order-of-magnitude expression for the absorbed power:

$$Q_{max} \sim \frac{\omega}{8\pi}\left(\frac{\Phi_{max}}{L}\right)^2 \mathrm{Im}\ \varepsilon_\perp.$$

In this expression Φ_{max} represents the order of magnitude of the potential in the region of the flute:

$$\Phi_{max} \sim (f_0/2\pi L\ \mathrm{Im}\ \varepsilon_\perp)(m_e/m_i)^{1/2}(\eta\eta_0)^{1/4}.$$

Thus, the absorbed power in the region of the flute increases with decreasing dissipation in inverse proportion to the cube of the collision frequency.

For a source of finite dimension l, we substitute the function

$$f(q) = (f_0/2\pi L)(m_e/m_i)^{1/2}\exp(-q^2/q_0{}^2),$$

into the integral in (2.4) where $q_0 = (2L/l)(m_i/m_e)^{1/2}$. When the condition $q(\mathrm{Im}\ \epsilon_\perp/\epsilon_\perp^{1/2}) < 1$ is satisfied, the source field in the region of the flute is determined by the size of the source. In this case, the amplitude of the potential and the absorbed power in the region of the flute are given by the following order-of-magnitude expressions:

$$\Phi_{max} \sim \frac{f_0}{2\pi l}\left(\frac{\eta_0}{\eta}\right)^{1/4}, \quad Q_{max} \sim \frac{\omega\Phi_{max}^2}{2\pi l^2\eta}\frac{m_i}{m_e}\mathrm{Im}\ \varepsilon_\perp.$$

Hence it is clear that, similarly to the results obtained in Section 1, an increase in the size of the source reduces the heating efficiency for inhomogeneous plasma.

2. It was established above that collisional absorption of oscillations excited by a source in cold plasma is localized in a thin layer of plasma (the flute). Moreover, because of collisional energy dissipation, the region of efficient heating occupies a small fraction of the plasma volume (it takes the form of jets diverging from the source) in which the radiation field strength is a maximum. In hot plasma, on the other hand, the situation is quite different. Here the distribution of collisionless absorption is determined both by the inhomogeneity of the excited fields and the inhomogeneity of $\mathrm{Im}\ \hat{\epsilon}$ which, in turn, depends on the source-field distribution. It then turns out that the maximum-field and maximum-absorption regions are spatially separated. As a result, collisionless heating occurs in a large volume of plasma, mainly outside the flute.

Let us now consider the source-field structure in hot inhomogeneous plasmas. For frequencies $\omega_{Hi} < \omega \sim \omega_{pi} < (\omega_{He}\omega_{Hi})^{1/2}$, and when the wavelength of the oscillations across the magnetic field is greater than the Larmor radius r_{Hi} of the ions, whilst the phase velocity along the external magnetic field is greater than the thermal velocity of the electrons $(\omega/k_z) > 2v_{Te}$, the problem can be solved by solving the fourth-order equation

$$\frac{1}{2}r_{Hi}^2\frac{\partial^4\Phi}{\partial x^4}+\frac{\partial}{\partial x}\varepsilon_\perp(x)\frac{\partial\Phi}{\partial x}+\varepsilon_\parallel(x)\frac{\partial^2\Phi}{\partial z^2}=0, \tag{2.7}$$

where Φ is the oscillation potential, and $\epsilon_\perp$ and $\epsilon_\parallel$ are given by (2.1). It is assumed that the electron and ion temperatures are comparable. The contribution of collisions to Im $\hat{\epsilon}$ is neglected. Equation (2.7) describes two oscillation branches, namely, the electromagnetic and the plasma modes. The corresponding dispersion relation can be written in the form

$$k_x(k_z)=2^{-1/2}k_*[(1+k_z/k_0)^{1/2}\pm(1-k_z/k_0)^{1/2}]\equiv k_{1,2}. \tag{2.8}$$

In this expression the wave vector k_1 corresponds to the plasma mode (Bernstein mode) whilst k_2 corresponds to the electromagnetic mode. Finally, the characteristic values of the wave-vector components are given by

$$k_*=\varepsilon_\perp^{1/2}/r_{Hi},\quad k_0=(\varepsilon_\perp/r_{Hi})|2\varepsilon_\parallel|^{-1/2}.$$

It is clear from (2.8) that the oscillations can propagate for $|k_z| < k_0$. The value $|k_z| = k_0$ corresponds to the merging of the oscillation branches. It can readily be shown that the dispersion of the electromagnetic mode is anomalous, i.e., the components of the phase and group velocities across the magnetic field have different signs. Next, the group-velocity vector of the electromagnetic mode may depart from the direction of the external magnetic field by an angle not exceeding $\theta_c = \arctan|\epsilon_\perp/\epsilon_\parallel|^{1/2}$, and the Bernstein group-velocity vector can have any direction. In inhomogeneous plasma, the point of intersection of the oscillation branches lies in the region corresponding to $\epsilon_\perp \ll 1$ provided $k_z r_{Hi} \ll (m_e/m_i)^{1/2}$.

Consider the field structure and the absorption of source radiation in hot inhomogeneous plasma. By analogy with the foregoing, we shall substitute $\epsilon_\perp = -x/L$ in the region in which we are interested. Without losing sight of the fundamentals, we may suppose that $\epsilon_\parallel = \text{const} = -m_i/m_e$. As before, we shall use the dimensionless coordinates

$$\xi=x/L,\quad \zeta=(z/L)(m_e/m_i)^{1/2}.$$

Equation (2.7) can then be written in the form

$$-\beta^2\frac{\partial^4\Phi}{\partial\xi^4}+\frac{\partial}{\partial\xi}\xi\frac{\partial\Phi}{\partial\xi}+\frac{\partial^2\Phi}{\partial\zeta^2}=0. \tag{2.9}$$

In this expression, $\beta = 2^{-1/2}(r_{Hi}/L)$ is a small parameter. The solution of (2.9) must satisfy the boundary condition $\Phi(x_0, z) = \Phi_b\delta(z)$. We shall also suppose that the electromagnetic mode predominates near the source. The solution of (2.9) which satisfies these conditions is given by the Fourier integral

$$\Phi(\xi,\zeta)=\frac{\Phi_b}{\pi L}\left(\frac{m_e}{m_i}\right)^{1/2}\int_0^\infty dq\cos q\zeta\frac{\Phi(q,\xi)}{\Phi(q,\xi_0)}, \tag{2.10}$$

where

$$\Phi(q,\xi)=\int_0^\infty\frac{ds}{s}\exp\left[i\left(\frac{1}{3}\beta^2s^3+\xi s-\frac{q^2}{s^3}\right)\right],$$

and the integration contour runs along the ray arg s $= \pi/6$. As before, the main contribution in the region of maximum potential (on the flute) is provided by harmonics with large q, for which we shall use asymptotic formulas. For $\xi < 0$ we have

$$\Phi(q,\xi)\approx\sum\left(\frac{2\pi\beta}{uv}\right)^{1/2}(u\pm v)^{-1/2}\exp\left(\pm i\frac{\pi}{4}-\frac{iu^3}{3\beta}\mp\frac{iv^3}{3\beta}\right). \tag{2.11}$$

In this expression u, v $= (\epsilon_\perp \pm \epsilon_*)^{1/2}$, $\epsilon_* = 2q\beta$.

The expression given by (2.11) has the following interpretation. In the case of sufficiently weak wave attenuation, Im $\epsilon_\perp \ll (r_{Hi}/L)^{2/3}$, which we are discussing, the electromagnetic mode excited by the source propagates from the plasma boundary toward the hybrid resonance layer $\epsilon_\perp = \epsilon_*$ where it is completely transformed into the Bernstein mode. The latter then transports energy back to the plasma boundary. In contrast to the case of cold plasma, allowance for linear wave transformation in hot plasma leads to the reflection of energy from the hybrid resonance layer, followed by scattering and absorption of this energy within the volume between the plasma boundary and the hybrid resonance layer. For sufficiently strong absorption, the fraction of reflected energy will, of course, be exponentially small, in accordance with the reduction in the electromagnetic-mode field with distance from the source.

It also follows from (2.11) that, well away from the transformation layer, i.e., in the region $\epsilon_\perp \gg \epsilon_*$, the amplitude of the electromagnetic wave is greater than the amplitude of the Bernstein mode in the ratio $(2\epsilon_\perp/\epsilon_*)^{1/2}$. Consequently, the field structure well away from the transformation layer is determined by the electromagnetic mode. The rapid rise in the fields occurs near the characteristic $\zeta = \zeta_s$ where $\zeta_s = 2(\epsilon_0^{1/2} - \epsilon_\perp^{1/2})$, $\epsilon_0 = \epsilon_\perp(x_0)$.

Substituting (2.11) into (2.10), we obtain the expression for the oscillation potential in the form of a superposition of the long-wave and short-wave parts of the electromagnetic-mode spectrum. Suppose that $\zeta = \zeta_s$ determines the singular characteristic in the case of cold plasma. In hot plasma, in the region where $\zeta < \zeta_s$, the main contribution is due to the long-wave part of the spectrum, which coincides with the expression given by (2.6) for the source potential in cold plasma. However, for $\zeta > \zeta_s$, the potential is determined by the short-wave part of the spectrum, i.e.,

$$\Phi(\xi,\zeta)\approx\frac{\Phi_b}{2L}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{\varepsilon_0^{1/4}\varepsilon_\perp^{1/8}}{(\pi\beta)^{1/2}(\zeta-\zeta_s)^{1/4}}\exp\left[i\frac{\pi}{4}-\frac{2i}{3\beta}\varepsilon_\perp^{3/4}(\zeta-\zeta_s)^{3/2}\right] \tag{2.12}$$

and corresponds to the field of the wave packet in which the direction of the group velocity is parallel to the radius vector of the point of observation. In the region of the flute, $|\zeta - \zeta_s| \lesssim \beta^{2/3}/\epsilon_\perp^{1/2}$, the two contributions are of the same order. The maximum value is reached for $\zeta = \zeta_s$ and is given by

$$\Phi_{max}\approx(\Phi_b/2\pi L)(m_e/m_i)^{1/2}(\varepsilon_0\varepsilon_\perp)^{1/4}\beta^{-2/3}.$$

We shall show that absorption of the short-wave part of the spectrum occurs mainly outside the flute, in the region $\zeta > \zeta_s$. Substituting (2.12) in (1.11), we obtain the Cerenkov absorption of the short-wave part of the oscillation spectrum on plasma electrons:

$$Q_e\approx\frac{\omega_{Hi}\omega^2}{8\pi^{3/2}v_{Te}^2}\frac{\varepsilon_\perp\Phi_b^2}{r_{Hi}L}\varepsilon_0^{1/2}y^2e^{-y^2}, \tag{2.13}$$

where $y = \omega/k_z^{(S)}v_{Te}$ and $k_z^{(S)}$ is the local wave number of the wave packet (2.12). The parameter y and the coordinates of the point of observation are related through the formula

$$1/y = 2^{1/3}(\omega_{Hi}/\omega)\varepsilon_\perp^{3/4}(\zeta-\zeta_s)^{1/3}.$$

Hence it is clear that, as the distance from the flute $\zeta = \zeta_s$ increases, the effective wave number $k_z^{(S)}$ increases and, consequently, there is a reduction in the phase velocity of the wave packet (2.12) along the external magnetic field. This in turn leads to an exponential increase in the absorbed power (2.13) with increasing distance from the flute, and to the heating of the large plasma volume in the region $\zeta > \zeta_s$ outside the flute. In hot plasma, therefore, thermal motion limits the source-field amplitude on the singular characteristics (diverging from the source) whilst, on the other hand, collisionless dissipation leads to plasma heating distributed over a large volume of the plasma.

Similarly, it can be shown that the absorption of the long-wave part of the spectrum, $k_z r_{Hi} < (m_e/m_i)^{1/2}\epsilon_\perp^{1/2}(r_{Hi}/L)^{1/3}$, occurs mainly in the region of the flute.

It is important to note that we have investigated the source-field structure in a class of functions representing the continuous spectrum of longitudinal wave numbers k_z. In some cases, however, this spectrum may be discrete because of, say, the excitation of standing-wave resonances in the inhomogeneous plasma shell near the source. For example, for a probe in magnetized plasma ($\omega_{He} \gg \omega_{pe} \sim \omega$) and when the plasma shell has a symmetric density profile relative to the probe (x = 0) so that $\omega_{pe}^2(x) = \omega^2|x|/L$ (L is the length of the density inhomogeneity), the magnetic field of the radiation is given by

$$H_y(z,x) = \frac{2\pi}{c}\int_{-\infty}^{\infty} dk_z j_y(k_z)\exp(ik_z z)\frac{v'(R)}{v'(R_0)}, \qquad (2.14)$$

instead of (1.6), where v(R) is the Airy function, the prime represents differentiation with respect to the argument, $R_0 = R(0)$, and

$$R(x) = (\omega L/c)^{2/3}(1-c^2k_z^2/\omega^2)^{1/3}(x/L-1).$$

Let γ_n be the zeros of the derivative of the Airy function, arranged in an increasing sequence. It is then readily seen from (2.14) that the radiation field in the transparency region $x < L$, $|k_z| < \omega/c$ is determined by the discrete part of the spectrum

$$k_z^{(n)} = \pm\frac{\omega}{c}\left(1-\frac{c^2\gamma_n^3}{\omega^2L^2}\right)^{1/2},$$

corresponding to the contribution of the residues at the zeros of the function $v'(R_0)$. The number m of levels in the plasma resonator depends on the quasiclassical parameter $\rho = \omega L/c$ and is found from the condition $\gamma_m < \rho^{2/3} < \gamma_{m+1}$. We note, moreover, that the thickness L of the plasma shell in the present case determines the maximum field amplitude in the hyperbolic region $x > L$ on the characteristics

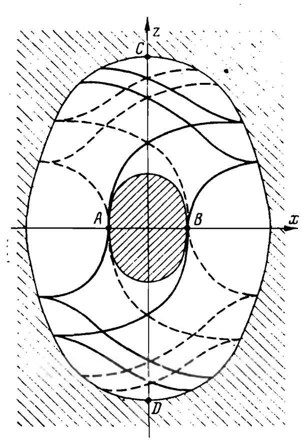
$$z/L = \pm{}^2/_3(1-x/L)^{3/2}.$$

CONCLUSION

We conclude by noting a number of points. Firstly, we have discussed sources of variable size. However, roughly speaking, we fixed the source power. For example, the total, i.e., integrated over the entire volume, source charge was fixed, so that as the source size was reduced, the charge density increased. Under real conditions this may not be the case, and as the source size l is reduced, its strength may fall. This may be taken into account by supposing that the source parameters J and Φ_b are functions of l. Since the problem is linear, this leads to a variation in the source fields which is uniform in all space, but the field structure investigated above will not change. In particular, there will be no change in the localization of the absorption region. From this point of view, it would be desirable to have a source for which the density (for example, the electric field in the gap of the exciting waveguide) is independent of the source size. Under these conditions, an increase in the source size will not be accompanied by a reduction in the absorbed power and, consequently, there will be no deterioration in plasma heating efficiency.

Secondly, it is essential to note the importance of the two-dimensional plasma inhomogeneity. The field singularities in this case were first investigated by Piliya and Fedorov [16] for regular boundary conditions. As noted above, the trajectories of group-velocity rays of a wave packet coincide with the characteristics of the wave equation. Hence it follows that it is very important to know the behavior of characteristics if we are to understand the spatial evolution of the oscillations excited by the source. Thus, the characteristics of (1.1) have the form $(dz/dx)^2 = -\epsilon_\parallel(x,z)/\epsilon_\perp(x,z)$. For a plasma bunch with finite linear dimensions, the typical distribution of characteristics is as shown in the Figure. The hyperbolic region lies between the inner ($\epsilon_\perp = 0$) and outer ($\epsilon_\parallel = 0$) curves. It is important to note that any characteristic will pass through a point such as C or D after an infinite number of reflections from the $\epsilon_\perp = 0$, $\epsilon_\parallel = 0$ lines. Therefore, perturbations are eventually reduced to these points, and this leads to the formation of field singularities at them. According to [16], a field singularity will also appear at the saddle points A and B. For a point source located in the hyperbolic region, the points at which characteristics leaving the source meet the $\epsilon_\perp = 0$, $\epsilon_\parallel = 0$ lines are singular points. The distribution of absorption throughout the volume of the plasma will then be highly inhomogeneous and will take the form of jets (in the region of plasma near the singular characteristics and their continuations).

Finally, we must note the importance of nonlinear effects. In the nonlinear case, the behavior of the characteristics will also depend on the field amplitude and this will lead, in particular, to the possible intersection of characteristics outside the singular points considered in the linear case. For a source of size l, exciting oscillations with energy density W, a very rough qualitative



estimate of the convergence length l_s of characteristics due to nonlinearity yields $l_s \sim l(nT_e/W)$. Of course, this is not a very quantitative result.

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Translated by S. Chomet
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