Periodic structures due to the photoelectric effect

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It is shown that propagation of two coherent electromagnetic waves of the same frequency but with different wave vector directions produces in a conducting medium a periodic structure whose properties are determined by the carrier dragging. The depth of penetration of this structure is comparable with its period and many considerably exceed the wavelengths and the depth of their damping. This may be ascribed to the presence of a periodic system of currents and magnetic fields due to carrier dragging by the interfering waves. The amplitude of the potential relief of such structures exceeds the familiar Gaponov-Miller potential by several orders of magnitude.

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1. It is known that a strong electromagnetic wave capable of causing nonlinear effects can modulate the dielectric constant, the pressure, the temperature, the carrier density, and other parameters of a medium, and this leads to a number of such phenomena as stimulated scattering of light due to absorption ^[1], self-focusing ^[2], excitation of electroacoustic waves ^[3], etc. A special role is played by effects connected with modulation of the material parameters by the field of a strong standing wave. Mention should be made here of the Kapitza-Dirac effect^[4] and of recent work by Bass^[5]. The latter has considered a plasma in the field of three standing waves and, using the Gaponov-Miller force^[6], has shown that the motion of charged particles occurs in the field of a certain superlattice, the periods of which in different directions coincide with the lengths of the corresponding waves.

The purpose of the present paper is to show that dragging of the charges by electromagnetic waves can lead to the onset of superstructures that differ significantly in their properties from those that can be obtained by using the Gaponov-Miller forces. In particular, we shall show that the superstructures considered by us are connected with a system of stationary currents and magnetic fields that are periodically distributed in space, and this in turn causes the depth of penetration of the superstructure in a conducting medium to exceed greatly the depth of the damping of the electromagnetic waves. We shall show furthermore that heating of the carriers, which leads to their dragging, a factor likewise not accounted for in earlier studies. leads to an appreciable enhancement (by several orders of magnitude) of the amplitude of the potential relief of the structure.

2. Assume that two coherent electromagnetic waves of equal frequency ω but with differently directed wave vectors \mathbf{k}_1 and \mathbf{k}_2 propagate in a conducting isotropic medium. In the approximation quadratic in these fields, a drag current $\mathbf{j}_1(\mathbf{r})$ is produced in the medium. In addition to the terms proportional to the power of each of the waves separately, $\mathbf{j}_1(\mathbf{r})$ includes an interference term that contains a factor of the form

$$\cos (\mathbf{k_1 r} - \omega t + \varphi_1) \cos (\mathbf{k_2 r} - \omega t + \varphi_2)$$

=¹/₂ cos(($\mathbf{k_1 - k_2}$) r+ $\varphi_1 - \varphi_2$) +¹/₂ cos(($\mathbf{k_1 + k_2}$) r-2 $\omega t + \varphi_1 + \varphi_2$).

Averaging this factor with respect to time, we see that a superstructure that is periodic in one dimension with a period $2\pi/|\mathbf{k}_1 - \mathbf{k}_2|$ is produced in space. We shall call it the wave structure.

At arbitrary polarization of interfering waves, both

the curl and the divergence of the vector $\mathbf{j}_1(\mathbf{r})$ differ from zero. It follows therefore that stationary closed currents and spatial modulation of the charge density can exist in the medium. Therefore the resultant structure is characterized by a spatially-periodic stationary distribution of the density of the current and the charge, of the potential, and of the magnetic field. The wave structure produced in the case of a large number of interfering waves is a superposition of one-dimensional wave structures, to the analysis of which we confine ourselves in the present paper.

The drag current can be calculated by solving the critical equation for the electronic distribution function $f(\mathbf{r}, \mathbf{v}, t)$:

$$\frac{\partial f}{\partial t} + \mathbf{v} \nabla_{\mathbf{r}} f + \frac{e}{m} \left\{ \mathscr{B} + \frac{1}{c} [\mathbf{v} \mathscr{H}] \right\} \nabla_{\mathbf{v}} f = S(f), \qquad (2.1)$$

where S(f) is the collision integral, \mathscr{E} and \mathscr{H} are the summary saturations of the high-frequency electric and magnetic fields of the crossed fluxes:

$$\mathscr{E} = \operatorname{Re} \mathbf{E} e^{-i\omega t}, \quad \mathscr{H} = \operatorname{Re} \mathbf{H} e^{-i\omega t}$$

The field amplitudes are assumed to depend weakly on the coordinates.

Using the solution obtained by Perel' and Pinski $i^{(7)}$ for the kinetic equation, we can calculate the density of the constant drag current

$$\mathbf{j}_{1}(\mathbf{r}) = -\frac{ne^{3}}{2m^{2}} \left\langle \left(\frac{\chi}{\nu}\right) - \frac{2}{15} \frac{\nu}{(\nu^{2}+\omega^{2})} \frac{1}{\nu^{4}} \frac{\partial}{\partial \nu} \left(\frac{\nu^{3}}{\nu\nu^{*}}\right) \right\rangle \nabla |\mathbf{E}|^{2} \\ - \frac{ne^{3}}{5m^{2}} \operatorname{Re} \left\{ \left[\left\langle \frac{\nu}{(\nu^{2}+\omega^{2})} \frac{1}{\nu^{4}} \frac{\partial}{\partial \nu} \left(\frac{\nu^{3}}{\nu\nu^{*}}\right) \right\rangle \right. \\ \left\langle \frac{\nu}{(\nu+i\omega)(\nu^{*}+i\omega)} \frac{\partial}{\partial \nu} \left(\frac{1}{\nu}\right) \right\rangle \right] (\mathbf{E}\nabla) \mathbf{E}^{*} \right\} + \frac{ne^{3}}{6m^{2}} \operatorname{Re} \left\{ \left[\left\langle \frac{\chi'}{\nu^{2}} \frac{\partial}{\partial \nu} \left(\frac{\nu^{3}}{\nu}\right) \right\rangle \\ \left. + \frac{6}{5} \left\langle \frac{\nu}{(\nu+i\omega)(\nu^{*}+i\omega)} \frac{\dot{\rho}}{\partial \nu} \left(\frac{1}{\nu}\right) \right\rangle \right] \mathbf{E}^{*}(\nabla \mathbf{E}) \right\} \\ \left. + \frac{2}{5} \left\langle \frac{\nu}{(\nu-i\omega)(\nu^{*}-i\omega)} \frac{\partial}{\partial \nu} \left(\frac{1}{\nu}\right) \right\rangle \right] \mathbf{E}^{*}(\nabla \mathbf{E}) \right\} \\ \left. + \frac{ne^{3}}{2m^{2}c} \operatorname{Re} \left\{ \left[\left\langle \frac{1}{\nu(\nu-i\omega)} \right\rangle + \frac{1}{5} \left\langle \frac{i\omega\nu}{(\nu+i\omega)(\nu^{*}+i\omega)} \frac{\partial}{\partial \nu} \left(\frac{1}{\nu}\right) \right\rangle \right] \left[\mathbf{E} \times \mathbf{H}^{*} \right] \right\}.$$

The angle brackets denote the following averaging with the Maxwellian distribution function $\Phi_0 = n(m/2\pi T)^{3/2} \times \exp(-mv^2/2T)$:

$$\langle \varphi \rangle = \frac{m}{3nT} \int \Phi_0 \varphi v^2 \, d^3 v,$$

where n is the electron concentration and T is the temperature in energy units, and both vary slowly with the coordinates. The quantities ν and ν^* are the reciprocal relaxation times of those parts of the distribu-

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tion functions which contain the first and second Legendre polynomials.

The functions $\chi(v)$ and $\chi'(v)$ are expressed in terms of the functions A(v) and A'(v) of the article by Perel' and Pinskiï^[7] and satisfy the equations

$$\frac{1}{2v^{2}}\left\{\frac{\partial}{\partial v}\left(v_{*}v^{2}\frac{T}{m}\frac{\partial\chi}{\partial v}\right)-v_{*}v^{3}\frac{\partial\chi}{\partial v}\right\} = \operatorname{Re}\left\{\frac{1}{v-i\omega}\left(1-\frac{mv^{2}}{3T}\right)+\frac{v}{3}\frac{\partial}{\partial v}\frac{1}{v-i\omega}\right\},$$

$$\frac{1}{2v^{2}}\left\{\frac{T}{m}\frac{\partial}{\partial v}\left[v_{*}v^{2}\frac{\partial}{\partial v}(v^{2}\chi')\right]-v_{*}v^{3}\frac{\partial\chi'}{\partial v}\right\}+(i\omega-v_{*})v^{2}\chi'=\frac{v^{2}}{v-i\omega}.$$
(2.4)

Here ν_{ϵ} is the reciprocal energy relaxation time. If the collision frequencies do not depend on the carrier energy, then these equations have a simple solution

$$\chi = \frac{\nu}{\nu_{\epsilon}(\nu^2 + \omega^2)} \left(\frac{m\nu^2}{3T} - 1\right),$$

$$\chi' = \frac{1}{(i\omega - \nu_{\epsilon})(\nu - i\omega)} \left(1 - \frac{3\nu_{\epsilon}T}{i\omega m\nu^2}\right).$$
(2.5)

In formula (2.2), the term proportional to the gradient contains two terms. The first is connected with the heating of the carriers in a high-frequency magnetic field, and the second is of the same type as the Miller force. Since the curl of this term is equal to zero, it cannot lead to the appearance of a periodic current structure in space, and its role reduces to the formation of a potential relief analogous to that investigated by Bass^[5], but with an essentially larger amplitude. The remaining terms of formula (2.2) describe the onset of a periodic current structure.

When the collision frequencies do not depend on the energy, we can obtain from formula (2.2) the following expression for the density of the drag current excited by the transverse waves:

$$j_{i} = -\frac{ne^{3}}{m^{2}} \operatorname{Re} \left\{ \frac{1}{3(v^{2}+\omega^{2})} \left(\frac{1}{v_{\epsilon}} - \frac{1}{v^{*}} \right) \nabla |\mathbf{E}|^{2} \right.$$

$$\left. + \frac{1}{v^{*}(v^{2}+\omega^{2})} (\mathbf{E}\nabla) \mathbf{E}^{*} - \frac{1}{2cv(v-i\omega)} [\mathbf{E} \times \mathbf{H}^{*}] \right\}.$$
(2.6)

Substituting in this expression the fields

$$\begin{split} \mathbf{E}(\mathbf{r}) = & \mathbf{E}_1 \exp \{i\mathbf{k}_1\mathbf{r}\} + \mathbf{E}_2 \exp \{i\mathbf{k}_2\mathbf{r}\}, \\ & \mathbf{H}(\mathbf{r}) = & \mathbf{H}_1 \exp \{i\mathbf{k}_1\mathbf{r}\} + & \mathbf{H}_2 \exp \{i\mathbf{k}_2\mathbf{r}\}, \end{split}$$

we separate from (2.6) the expression for the interference term, which we represent in the form

$$j_{i}=2 \operatorname{Re} \{j_{0} \exp \{i(\mathbf{k}_{1}-\mathbf{k}_{2}^{*})\mathbf{r}\}\}, \qquad (2.7)$$

$$j_{0}=-\frac{ne^{3}}{m^{2}c(\mathbf{v}^{2}+\mathbf{\omega}^{2})}\left\{\frac{i}{3}\left(\frac{1}{v_{*}}-\frac{1}{v^{*}}\right)c(\mathbf{k}_{1}-\mathbf{k}_{2}^{*})(\mathbf{E}_{1}\mathbf{E}_{2}^{*})\right. \\ \left.+\left(ic/2v^{*}\right)\left[\left(\mathbf{E}_{2}^{*}\mathbf{k}_{1}\right)\mathbf{E}_{1}-\left(\mathbf{E}_{1}\mathbf{k}_{2}^{*}\right)\mathbf{E}_{2}^{*}\right]^{-1}/\left[\left(1+i\omega/v\right)\left[\mathbf{E}_{1}\times\mathbf{H}_{2}^{*}\right]\right. \\ \left.+\left(1-i\omega/v\right)\left[\mathbf{E}_{2}^{*}\times\mathbf{H}_{1}\right]\right\}.$$

This is precisely the formula which we shall use in the estimates. We note that the last term of (2.7) can be interpreted as a high-frequency Hall effect in electric and magnetic fields of different waves.

3. Consider a plate of finite thickness d, on the surface of which are incident two coherent electromagnetic waves of equal frequency. The drag current $j_1(\mathbf{r})$ produced in this case in the electron plasma leads to a redistribution of the charges both inside the crystal and on its surface, and excites in the medium a secondary current $j_2(\mathbf{r})$. The total current is the sum of these two currents:

$$\mathbf{j}(\mathbf{r}) = \mathbf{j}_1(\mathbf{r}) + \mathbf{j}_2(\mathbf{r}) = 2 \operatorname{Re} \{ \mathbf{j}_0 e^{i\mathbf{k}\mathbf{r}} \} + \mathbf{j}_2(\mathbf{r}).$$
 (3.1)

Here $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2^*$ is the wave vector of the structure. In this expression we have left out terms connected with

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the currents of the carriers dragged by each wave separately. The last effect, which has been quite well investigated in a number of studies, leads to the appearance of a certain constant potential difference between the faces of the sample and cannot influence the properties of the investigated structure in any way.

Assume that the spatial period of the structure is much larger than the carrier mean free path. Then the distribution of the quantities of interest to us is determined from Maxwell's equations together with the material relations

Here φ , μ , and Φ are respectively the electrostatic, chemical, and electrochemical potentials, **H** is the magnetic field and **E** will be called the effective electric field of the structure, σ is the static conductivity, ϵ is the dielectric constant of the medium, and ρ is the charge density of the free carriers, which varies periodically in space.

Let the z axis be normal to the surface of the plate and assume that the vector k lies in the xz plane. We can then assume all the quantities to be independent of y. It is convenient to start the solution of the problem with a definition of the quantity Φ . The continuity equation

$$\operatorname{div} \mathbf{j} = -\sigma \Delta \Phi + i \mathbf{k} \mathbf{j}_0 e^{i\mathbf{k}\mathbf{r}} = 0 \tag{3.3}$$

together with the boundary conditions

$$\mathbf{j}_{z}|_{z=0,d} = \left[-\sigma \frac{\partial \Phi}{\partial z} + \mathbf{j}_{0}^{z} e^{i\mathbf{k}\mathbf{r}}\right]_{z=0,d} = 0$$
 (3.4)

leads to the expression

$$\Phi = \frac{\exp\{ik_{\mathbf{x}}\mathbf{x}\}}{\sigma k^{2}k_{\mathbf{x}}} \left\{ F_{z} \frac{\left[\operatorname{ch} k_{\mathbf{x}}(d-z) - \exp\{ik_{z}d\} \operatorname{ch} k_{\mathbf{x}}z\right]}{\operatorname{sh} k_{z}d} - ik_{z}k\mathbf{j}_{0}e^{ik_{z}z} \right\},$$

$$\mathbf{F} = \left[\mathbf{k} \times \left[\mathbf{k} \times \mathbf{j}_{0}\right]\right].$$
(3.5)

The total current density is determined from expressions (3.2) and is equal to

$$\mathbf{j} = -\frac{\exp\{ik_{\mathbf{x}}\mathbf{x}\}}{k^2} \left\{ \mathbf{F} \exp\{ik_{\mathbf{x}}\mathbf{z}\}\right\}$$
(3.6)

$$+\frac{F_{\mathbf{z}}}{\operatorname{sh} k_{\mathbf{z}} d} \left[i \mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}} \frac{1}{k_{\mathbf{z}}} \frac{\partial}{\partial z} \right] \left[\operatorname{ch} k_{\mathbf{z}}(z-d) - \exp\left\{ i k_{\mathbf{z}} d\right\} \operatorname{ch} k_{\mathbf{z}} z \right] \right\}.$$

where \boldsymbol{e}_{X} and \boldsymbol{e}_{Z} are unit vectors in the corresponding directions.

We consider individually the case of a half-space $(d \rightarrow \infty)$

$$j = -k^{-2} \exp \{ik_x x\} \{F \exp \{ik_z z\} + F_z \exp \{-k_z z\} (ie_x - e_z)\}.$$
(3.7)

The terms of these formulas have different characteristic damping lengths. The first term attenuates over a distance (Im k_Z)⁻¹, whereas the second term attenuates over a length $|k_X|^{-1}$ close to the period of the wave structure. The last quantity is connected with the frequency and direction of propagation of the waves incident on the crystal by the following formula:

$$k_{x} = \omega c^{-1} (\sin \theta_{1} - \sin \theta_{2}), \qquad (3.8)$$

where θ_1 and θ_2 are the incidence angles of the interfering waves.

The second term of (3.7) describes a system of closed currents connected with the condition that the normal component of the total current vanish on the boundary of the sample. The components of these currents j_x and j_z are shifted in phase by $\pi/2$, i.e., they are analogous to a certain degree to a system that is circularly polarized around the y axis, namely, as the

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closed current moves along the z axis it rotates about the y axis, and remains constant in magnitude. In the case of a plate of finite thickness, the closed currents exist on both surfaces of the plates and therefore the correponding term in the expression for the current has a term that attenuates with increasing distance from the front face of the plate, and a term that attenuates with increasing distance from the rear face.

The current structure leads to the appearance of a stationary magnetic field that is distributed in space. To determine this field it is necessary to use Maxwell's equation (3.2) and the expression for the current (3.6) with boundary conditions that ensure continuity of the field on the boundary.

Outside the crystal we have curl $\mathbf{H} = 0$, so that the magnetic field is potential, $\mathbf{H} = -\operatorname{grad} \Psi$, where the magnetic potential satisfies the Laplace equation $\nabla^2 \Psi = 0$. Since all the quantities are independent of y, the magnetic field component outside the crystal is $H_y = -\partial \Psi / \partial y = 0$. At zero boundary conditions we obtain for H_y inside the crystal the obvious solution

$$H_{\mathbf{y}}(\mathbf{r}) = -4\pi i j_z(\mathbf{r}) / c k_{\mathbf{x}}. \tag{3.9}$$

The equations for H_X and H_y under the continuity boundary conditions lead in the interior of the crystal to the solution

$$\mathbf{H}_{\perp} = \mathbf{e}_{x}H_{x} + \mathbf{e}_{z}H_{z} = \frac{2\pi}{ck^{2}}j_{o}^{v}\exp\left[ik_{x}x\right]\left\{\left(k_{z}-ik_{z}\right)\left(\mathbf{e}_{z}+i\mathbf{e}_{z}\right)\exp\left[ik_{z}d\right.\right.\right.\right.} \\ \left.\left.\left(3.10\right)\right.\right. \\ \left.\left.\left.\left.\left.\left(k_{z}-ik_{z}\right)\left(\mathbf{e}_{z}-i\mathbf{e}_{z}\right)\exp\left[-k_{z}z\right]\right\right\}\left(k_{z}-ik_{z}\right)\exp\left[ik_{z}z\right]\right\right\}\right\}\right\}\right\}$$

Outside the crystal, the magnetic field \mathbf{H}_{\perp} is described by the following formulas:

$$\begin{aligned} \mathbf{H}_{\perp}(\mathbf{r}) = \mathbf{H}_{\perp}(0) & \exp\{ik_{x}x + k_{z}z\}, \quad z < 0, \\ \mathbf{H}_{\perp}(\mathbf{r}) = \mathbf{H}_{\perp}(d) & \exp\{ik_{x}x + k_{x}(d-z)\}, \quad z > d. \end{aligned}$$

The formulas obtained for the magnetic field have a number of singularities that should be noted. First, H_{\parallel} differs from zero only when the dragging-current component j_0^y differs from zero. Second, in addition to the plane-polarized power connected with the drag current, the magnetic field has two terms which can be called left- and right-circularly polarized (in the same sense as for the current). These circularly-polarized terms are due to the finite thickness of the plate, and one of them is damped with increasing distance from the front face and the other from the rear face of the plate. Figures 1 and 2 show the distributions of the current lines and of the magnetic field in the xz plane near the boundary of a thick sample. The distribution of the level lines for the magnetic field component H_V can be easily visualized by examining Fig. 1.

The electrostatic potential is also distributed periodically in the wave structure. It is determined by the first equation of (3.2), which assumes, after expansion in the small increment to the chemical potential, the following form

$$\Delta \varphi - q^2 \varphi = -q^2 \Phi, \quad q^2 = \frac{4\pi e^2}{e} \frac{\partial n}{\partial \mu}.$$
 (3.12)

The boundary conditions ensure continuity of the normal component of the electrostatic induction and of the tangential component of the field. Substituting Φ from formula (3.5) in this equation we obtain a solution for the potential inside the crystal:

$$\begin{array}{l} \varphi = \Phi + i \mathbf{k} \mathbf{j}_s e^{i \mathbf{k}_z / \sigma \mathbf{x}^2} \\ -2 \exp \left\{ i \mathbf{k}_z \mathbf{x} \right\} D^{-1} \left\{ A \left[\mathbf{x} \operatorname{ch} \mathbf{x} (d-z) + (\mathbf{k}_z / \varepsilon) \operatorname{sh} \mathbf{x} (d-z) \right] \\ + B \left[\mathbf{x} \operatorname{ch} \mathbf{x} z + (\mathbf{k}_z / \varepsilon) \operatorname{sh} \mathbf{x} z \right] \right\}. \end{array}$$

$$(3.13)$$



FIG. 1. Distribution of the current lines of the structure produced in a thick sample on the surface of which two coherent waves of equal frequency are incident at equal angles.



FIG. 2. Distribution of the force lines of the constant magnetic field produced near the surface of a sample in which a periodic current structure is produced.

Here

$$\begin{aligned} \mathbf{x}^2 &= q^2 + k^2, \\ D &= (\mathbf{x} + k_{\mathbf{x}}/\varepsilon)^2 e^{\mathbf{x}d} - (\mathbf{x} - k_{\mathbf{x}}/\varepsilon)^2 e^{-\mathbf{x}d}, \\ A &= \frac{k_{\mathbf{x}}}{\varepsilon} \Phi\left(0\right) + \frac{1}{\sigma \varepsilon \mathbf{x}^2} \left[\varepsilon F_z - \varepsilon q^2 j_0^z + i k_{\mathbf{x}} \mathbf{k} \mathbf{j}_0\right], \\ B &= \frac{k_{\mathbf{x}}}{\varepsilon} \Phi\left(d\right) + \frac{\exp\left\{i k_z d\right\}}{\sigma \varepsilon \mathbf{x}^2} \left[-\varepsilon F_z + \varepsilon q^2 j_0^z + i k_{\mathbf{x}} \mathbf{k} \mathbf{j}_0\right]. \end{aligned}$$

The density of the uncompensated charge is calculated from the formula

$$\rho = \varepsilon q^{2} (\Phi - \varphi) / 4\pi. \qquad (3.14)$$

The physical meaning of each of the three terms of formula (3.13) is obvious: the second term describes the contribution made to the potential by the charges generated by the two waves, owing to the nonvanishing div j_1 , while the last term describes the screening of the excess space charge which arises near the front and rear faces of the sample. In a real situation the inequality $|\mathbf{q}| \gg |\mathbf{k}|$ is always satisfied. In this case it is convenient to use for the estimates an approximate formula derived for a half-space

$$\varphi = \Phi + \frac{i\mathbf{k}\mathbf{j}_o}{\sigma q^2} e^{i\mathbf{k}\mathbf{r}} - \frac{\exp\left\{-qz + ik_z x\right\}}{\sigma \varepsilon k^2 q} \left[F_z - ik_z \mathbf{k}\mathbf{j}_o - \varepsilon k^2 j_o^2\right]. \quad (3.15)$$

We see that the periodic distribution of the charge density and of the electrostatic potential, which appear in the wave structure in the limit $|q| \gg |\mathbf{k}|$ (i.e., in the case of strong screening), cease to depend on q. This does not pertain to the surface charge, which comes closer to the surface with increasing q and increases in magnitude. The case $|\mathbf{k}| \gg |\mathbf{q}|$ is apparently difficult to realize. We note that the principal terms in the expansion of φ and ρ in the parameter $\mathbf{q}/|\mathbf{k}|$ vanish in this case.

The potential outside the crystal is determined completely by the potential on the surface:

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$$\varphi(\mathbf{r}) = \varphi(0) \exp\{ik_x x + k_x z\}, \quad z < 0,$$

$$\varphi(\mathbf{r}) = \varphi(d) \exp \{ik_x x + k_x(d-z)\}, z > d.$$
 (3.16)

To permit a quantitative comparison of our results with preceding studies, let us compare the amplitude of our potential with the amplitude of the Gaponov-Miller potential^[6], which was used by Bass^[5] to obtain a superlattice in the field of three standing waves. The gradient term in the drag-current density makes the following contribution to the potential φ :

$$\varphi = \frac{e}{3m(\nu^2 + \omega^2)} \left(1 - \frac{2\nu}{\nu_e}\right) |\mathbf{E}|^2. \tag{3.17}$$

The first term differs from the Gaponov-Miller formula by a factor 1/3 but the second contains the larger quantity ν/ν_e . This term is connected with the inhomogeneous heating of the electrons in the field of the electromagnetic wave and makes the main contribution to the amplitude of the potential relief. As a result, the amplitude of the potential of our superlattice is larger by two or three orders of magnitude than that considered by Bass^[5].

The contribution of the gradient term to the effective electric field of the structure can be much larger than the corresponding contribution to the usual optoelectrical field. The reason is that the characteristic spatial scale of the change of the optoelectrical field is equal to the damping length of the wave $(\text{Im } k_{1,2})^{-1}$, and the characteristic spatial scale of the wave structure is equal to its period $2\pi |k_{\rm X}|^{-1}$. In the case of weakly damped waves, the contribution of the gradient term to the field of the structure is $|k_{\rm X}|/(\text{Im } k_{1,2}) \gg 1$ times larger than the corresponding contribution to the optoelectrical field. This circumstance can be used in principle to increase the sensitivity of optoelectrical pick-ups.

The main premises under which our calculation is valid coincides with those used by Perel' and Pinskii, [7], namely:

a) the spatial period λ of the wave structure is large enough to satisfy the inequality

 $\lambda \gg l \sqrt{\nu/\nu_e},$

where l is the electron mean free path;

b) the electron temperature differs little from T;

c) it is assumed that $\hbar \omega \ll T$.

It must be stated that Bass^{(5]} and Gaponov and Miller^{(6]} did not use the carrier mean free path as a parameter of the problem. However, the fact that the formula for the Gaponov-Miller effective potential is obtained from frictionless equations of motion of the electron allows us to conclude that their results are valid for structures whose period is much shorter than the mean free path.

4. We confine ourselves to order-of-magnitude estimates, since the details of the phenomena depend significantly on the polarizations of both waves and on the transmission coefficients. We assume that the power of the waves inside the crystal is $S \sim 1 \text{ W/cm}^3$ (in which case the heating of the sample can be neglected), and consider the isotropic semiconductor Ge or Bi (in which the anisotropy effect can be neglected) at room temperature. We are interested here in the frequency $\omega \sim 10^{12}$ sec⁻¹, corresponding to a structure period 0.1 cm. A. Let the collision frequency in the semiconductor be $\nu \sim \nu^* \sim \omega$ and let the free-carrier density be $n \sim 10^{15}$ cm⁻³. We assume the effective mass to be $m \sim 10^{-28}$ g; if it is smaller, this only enhances the investigated effect. Simple calculations lead to the following estimates of the amplitudes of the quantities of interest to us: $j_0 \sim j_1 \sim j_2 \sim 5 \times 10^{-7} \text{ A/cm}^2$, $H_X \sim H_Z \sim 10^{-10}$ Oe, and $H_y \sim 5 \times 10^{-9}$ Oe. We next estimate the amplitude of the electrostatic potential $\varphi \sim 10^{-8} (\nu/\nu_{\epsilon})$ [volts]. The ratio ν/ν_{ϵ} depends on the mechanism whereby the energy is transferred by the carriers to their lattice, and as a rough estimate we can put $\epsilon \sim 10^3$. Then $\varphi \sim 10^{-5}$ V.

B. For bismuth, $n\sim 2\times 10^{17}~cm^{-3}$ and $\nu\sim 10^{12}~sec^{-1}$ (we consider only the contribution of the electrons, since their mass is much smaller than the mass of the holes). Then a calculation analogous to that carried out in the preceding subsection leads to the following estimates: $j_0\sim j_1\sim j_2\sim 10^{-4}~A/cm^2,~\varphi\sim 10^{-4}~to~10^{-5}~V,$ $H_X\sim H_Z\sim 10^{-7}$ Oe, and $H_Y\sim 10^{-6}$ Oe. All these quantities are proportional to the power of the waves penetrating the crystal.

CONCLUSION

The periodic wave structure produced in a conducting medium as a result of carrier dragging by crossed electromagnetic waves differs both qualitatively and quantitatively from the structures considered earlier^[5]. The qualitative difference lies in the onset of closed currents periodically distributed in space, which generate a stationary magnetic field. This phenomenon makes it possible to modulate in principal the parameters of the medium over a length greatly exceeding the depth of the damping of the electromagnetic waves. In particular, in the case of a plate whose thickness greatly exceeds the depth of damping of the waves, but is comparable with the period of the superstructure, it is possible to observe on its opposite side the presence of a stationary periodic potential relief.

From the quantitative point of view, the potential relief produced as a result of carrier dragging can have an amplitude larger by several orders of magnitude than the potential relief investigated by Bass^[5]. In addition, for weakly damped waves the effective electric field of the structure can greatly exceed in magnitude the usual ferroelectric field.

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