Nonlinear effects in the emission of interstellar molecules

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The effect of depolarizing collisions and interference of molecular states on the emission parameters of cosmic OH masers (intensity, polarization and spectral line shape) is considered. It is shown, in particular, that interaction between the saturated radiation of the principal lines and the satellite lines gives rise to narrow interference dips of the Raman type on the satellite gain contour, and the widths and amplitudes of the dips increase with increasing gain. Circular polarization of solitary satellites is interpreted as being due to the action of impact broadening.

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It is known that information on physical processes in galactic clouds can be obtained by observing phenomena that occur in the emission of interstellar molecules. Molecular resonances of cosmic sources serve as a tool for the investigation of the composition, velocity, origin, and other important characteristics of interstellar gas condensations.

Recently, great interest has been aroused by the anomalies of maser radio emission of cosmic OH molecules (see the reviews [1–3]). The purpose of the present paper is to analyze the influence of depolarizing collisions and the interference of molecular states on the output characteristics of the cosmic OH masers, such as the intensity, polarization, and shape of spectral lines. This influence was not considered in earlier theoretical papers[4–8]. It will be shown below, however, that the depolarizing collisions and nonlinear interference effects (NIE) can make a decisive contribution to the formation of the polarization of cosmic sources and influence in a crucial manner the intensities of the molecular lines and the shape of the spectrum.

1. We consider a system of four electric dipole transitions occurring in the ground state of the hydroxyl $^2\Pi_{3/2}$ as a result of the Λ doubling and hyperfine splitting (Fig. 1). These transitions are distinguished by the index s = {a, b, c, d} and the levels are labeled by the index i = {f, m, n, λ}. Each of the dipole transitions is acted upon by a strong traveling wave of amplitude $\mathbf{E}_s$ and wave vector $\mathbf{k}_s$ (the waves have the same direction).

As first shown by Rautian[9] (see also [10,11]), the resonant interaction of strong waves with neighboring transitions leads even in first order in the saturation to the appearance of nonlinear interference effects connected with oscillations of the polarization induced on forbidden transitions. For the ground state of an OH maser, transitions of this type, which are forbidden in the dipole approximation, are f → m and n → l (Fig. 1).

The general relaxation of the transitions and of the levels is taken into account by the constants

$$\Gamma_{ij\varkappa}=\Gamma_{ij}+\gamma_{ij\varkappa}\quad (i,\ j=f,\ m,\ n,\ l), \tag{1}$$

where Γ_{ij} are the natural widths and $\gamma_{ij\kappa}$ are the impact-broadening parameters. To describe the collision relaxation we use a model of isotropic collisions, in which the impact widths can be relatively simply expressed in terms of the scattering S matrices[12]

$$\gamma_{ij\varkappa}=8\pi^2 n_0\int\rho\,d\rho\int u^3\varphi(\mathbf{u})\,du\sum_{\substack{q,M,M'\\ M_1,M_1'}}\begin{pmatrix}J_i & J_j & \varkappa\\ -M & M' & q\end{pmatrix}\begin{pmatrix}J_i & J_j & \varkappa\\ -M_1 & M_1' & q\end{pmatrix}$$
$$\times(\delta_{MM_1}\delta_{M'M_1'}-S^{i}_{MM_1}S^{j*}_{M'M_1'})(-1)^{2J_j+M+M_1}. \tag{2}$$

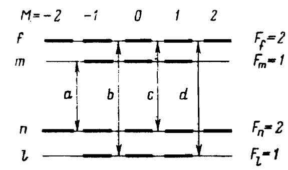

FIG. 1. Electric dipole transitions in the $^2\pi_{3/2}$ state of the hydroxyl molecule.

Here n_0 is the density of the particles, $\varphi(\mathbf{u})$ is the distribution function with respect to the relative velocities $\mathbf{u}$, J_i are the total angular momenta of the levels ($J_i + J_j \geq \kappa \geq |J_i - J_j|$), and M is the magnetic quantum number. The operator $\hat{S}^i_{MM1}$ denotes the scattering matrix in the plane where the impact parameter $\boldsymbol{\rho}$ and the relative velocity $\mathbf{u}$ are located. This operator is connected with the particle interaction potential V by the Lippman-Schwinger equation

$$\hat{T}=\hat{V}(\hat{1}+\hat{\mathscr{G}}_0\hat{T}), \tag{3}$$

where $\hat{T} = \hat{S} - 1$ and $\hat{\mathscr{G}}_0$ is the Green's function for the diverging waves and corresponds to the Hamiltonian of the free particles. The formal solution of (3) is

$$\hat{T}=\hat{V}(\hat{1}+\hat{\mathscr{G}}\hat{V}) \tag{4}$$

($\hat{\mathscr{G}}$ is the Green's function corresponding to diverging waves and to the total Hamiltonian). Thus, the concrete values of the constants $\gamma_{ik\kappa}$ are determined by the character of the interaction and by the concentrations of the partners in the collisions. Estimates for the concentrations of the different particles in galactic clouds[1,2] show that the most probable partners in collisions are HI, HII, H_2, OH, and H_2O.

If only the first-order corrections to the saturation are taken into account, and at exact resonance, the equations describing the interaction of the wave a with the waves b and c, can be written in the form

$$\frac{dG_q^a}{dz}=R_a\sum_{\varkappa=0,1,2}\Biggl\{N_aG_q^a\Biggl[1-\frac{|G_q^a|^2p_\varkappa+|G_{-q}^a|^2p_\varkappa'}{2\Gamma_a}\Biggl(\frac{B^{nn}_{mm\varkappa}}{\Gamma_{nn\varkappa}}+\frac{B^{mm}_{nn\varkappa}}{\Gamma_{mm\varkappa}}\Biggr)\Biggr]$$
$$-\frac{N_b}{\Gamma_a+\Gamma_b}\Biggl[G_q^a|G_q^b|^2p_\varkappa\Biggl(\frac{B^{ln}_{mm\varkappa}}{\Gamma_{ln\varkappa}}+\frac{B^{mm}_{ln\varkappa}}{\Gamma_{mm\varkappa}}\Biggr)+G_q^a|G_{-q}^b|^2p_\varkappa''\Biggl(\frac{B^{mm}_{ln\varkappa}}{\Gamma_{ln2}}+\frac{B^{mm}_{ln\varkappa}}{\Gamma_{mm\varkappa}}\Biggr)$$
$$+G_{-q}^aG_q^bG_{-q}^{b*}p_\varkappa''\Biggl(\frac{B^{ln}_{mm\varkappa}}{\Gamma_{ln\varkappa}}+\frac{B^{ln}_{mm\varkappa}}{\Gamma_{mm2}}\Biggr)\Biggr]-\frac{N_c}{\Gamma_a+\Gamma_c}\Biggl[G_q^a|G_q^c|^2p_\varkappa\Biggl(\frac{B^{fm}_{nn\varkappa}}{\Gamma_{fm\varkappa}}+\frac{B^{nn}_{fm\varkappa}}{\Gamma_{nn\varkappa}}\Biggr)$$
$$+G_q^a|G_{-q}^c|^2p_\varkappa''\Biggl(\frac{B^{nn}_{fm\varkappa}}{\Gamma_{fm2}}+\frac{B^{nn}_{fm\varkappa}}{\Gamma_{nn\varkappa}}\Biggr)+G_{-q}^aG_q^cG_{-q}^{c*}p_\varkappa''\Biggl(\frac{B^{fm}_{nn\varkappa}}{\Gamma_{fm\varkappa}}+\frac{B^{fm}_{nn\varkappa}}{\Gamma_{nn2}}\Biggr)\Biggr]\Biggr\},$$
$$R_s=2\pi^{1/2}|D_s|^2/3\hbar\bar{v}, \tag{5}$$

where

$$p_0=p_0'=p_0''=2,\quad p_1=-p_1=-p_1''=3,\quad p_2=p_2''=1,\quad p_2'=7. \tag{6}$$

Here $G^S_q = E_{Sq}|D_S|^2/2\hbar$ ($q = \pm 1$) denotes the slowlyvarying circular amplitudes of the field that is at resonance with the transition s, D_s is the reduced dipole moment, N_s characterizes the difference between the level excitation rates for the transition s, and Γ_s is the collision-broadened natural width ($\Gamma_a = \Gamma_{mn1}$ etc.). In cosmic rays, Γ_s is usually small in comparison with the Doppler width $k_s\bar{v}$. This was taken into account in the derivation of (5).

It should be noted that Eqs. (5) describe the changes of the intensity and of the polarization picture only for the line a acting on the transition between levels with total angular momenta $J_m = 1$ and $J_n = 2$. However, the symmetry of the coefficients

$$B_{jk\varkappa}^{ii'} = (-1)^{J_j - J_k} \begin{Bmatrix} \varkappa & 1 & 1 \\ J_j & J_i & J_{i'} \end{Bmatrix} \begin{Bmatrix} \varkappa & 1 & 1 \\ J_k & J_i & J_{i'} \end{Bmatrix}, \tag{7}$$

expressed in terms of Wigner 6J symbols makes it possible to obtain from (5) equations for the other lines by means of a simple permutation of the indices. Equations (5) are a generalization of the results of earlier studies[4–7] and at $\gamma_{ij\kappa} = \Gamma_{l n\kappa}^{-1} = \Gamma_{fm\kappa}^{-1} = 0$ they can be reduced to the equations of Heer and Settles[6].

2. According to the Wigner-Exkart theorem we have $|D_a|^2 : |D_b|^2 : |D_c|^2 : |D_d|^2 = 1:5:9:1$. Therefore in many cosmic sources the principal lines b and c, which act on the transitions J = 1 → 1 and J = 2 → 2, are saturated earlier than the satellite lines acting on the transitions J = 1 → 2, and the saturation of the principal lines depends relatively weakly on the resonances a and d. In this situation, the principal lines are circularly polarized, and the position of indifferent equilibrium for the line J = 1 → 1 is eliminated by the depolarizing collisions.

If the principal lines have identical circular polarizations, then we can write for the difference between the amplification coefficients of the σ components of the satellite χ_q^a the following relation:

$$\begin{aligned}\chi_{+1}^a - \chi_{-1}^a = {} & \frac{2N_a(|G_{-1}^a|^2 - |G_{+1}^a|^2)}{25\Gamma_a\Gamma_{mm1}\Gamma_{nn1}} \left[\Gamma_{mm1} + \Gamma_{nn1} + \frac{21}{24}\Gamma_{mm1}\left(1 - \frac{\Gamma_{nn1}}{\Gamma_{nn2}}\right)\right. \\ & \left. + \frac{\Gamma_{nn1}}{24}\left(1 - \frac{\Gamma_{mm1}}{\Gamma_{mm2}}\right)\right] + \frac{N_b}{12}\frac{|G_{+1}^b|^2}{\Gamma_a + \Gamma_b}\left(\frac{2}{\Gamma_{mm1}} - \frac{1}{\Gamma_{ln1}} + \frac{3}{\Gamma_{ln2}}\right) \\ & - \frac{N_c}{20}\frac{|G_{+1}^c|^2}{\Gamma_a + \Gamma_c}\left(\frac{2}{\Gamma_{nn1}} + \frac{3}{\Gamma_{fm1}} - \frac{1}{\Gamma_{fm2}}\right).\end{aligned} \tag{8}$$

We see therefore that at $N_{b,c} \neq 0$ a nonlinear interaction of the line a with the lines b and c takes place, namely, saturation of the principal lines influences the saturated radiation of the transition a. This influence is determined to a considerable degree by the action of the nonlinear interference effect, inasmuch as in the general case the widths of the forbidden transitions, $\Gamma_{nl\kappa}$ and $\Gamma_{jm\kappa}$, and the level widths $\Gamma_{nn\kappa}$ and $\Gamma_{mm\kappa}$ are of the same order of magnitude. At low pressures, when the role of impact broadening is slight, the contribution of the line J = 1 → 1 to the difference of the coefficient χ_q^a contributes to enhancement of the circular component of the satellite with the index q = +1 (i.e., the same that acts on the transition b), whereas the contribution of the line J = 2 → 2 enhances the opposite circular component G_{-1}^a. The latter fact has already been explained in [6], and the physical cause of the former is the fact that the transition M = −1 → −2 in the line a is more probable than the transition M = −1 → 0 (see Fig. 1). Under the conditions of local equilibrium, the ratio of these probabilities is equal to 6. In many sources of class I after Turner[3], the line J = 2 → 2 is the first to be saturated, so that usually the transitions and b emit the same polarization as J = 2 → 2. We note, however, that with increasing pressure, at certain ratios of the concentrations of the different particles, the above-noted tendencies can apparently be reversed, since Γ_{ij1} differs from Γ_{ij2}.

In some sources, the satellites are saturated autonomously ($N_{b,c} = 0$). If the relaxation constants for the magnetic dipole moment and for the electric quadrupole moment are the same ($\Gamma_{jj1} = \Gamma_{jj2}$, collisions strong in M[13]), then the expression in the square brackets of formula (8) is positive and the larger gain is possessed by the weaker circular component. This leads to equalization of the intensities of the opposite circular components of the satellites and to the appearance of a stable linear polarization. In the general case, the constants Γ_{jj1} and Γ_{jj2} are different. The most interesting is the situation wherein $\Gamma_{jj1} > \Gamma_{jj2}$ and the tendency to equalization decreases. If the difference between Γ_{jj1} and Γ_{jj2} is large enough, the effect of equalization can give way to competition and to nonlinear suppression of one of the circular components of the satellite. This case corresponds, for example, to the picture observed in the maser sources of Orion A.[3]

Thus, circular polarization of isolated satellites can result from impact broadening.

3. It is convenient to analyze the change of the width of the spectrum in the model of nondegenerate states. In this model, the relaxation constants do not depend on the index κ and the equations for the radiation transport are noticeably simplified.

A. In the case of moderate saturation, when the Doppler limit remains valid, in spite of the field broadening, and in the case of independent amplification of the line (in cosmic OH masers this is most frequently the line J = 2 → 2), the equation for the radiation transport takes the form

$$\frac{d\theta}{dz} = \frac{\theta}{(1+\theta)^{1/2}}\alpha_0(z)\mathscr{E}(\Omega), \quad \mathscr{E}(\Omega) = \exp\left\{\frac{-\Omega^2}{(k\bar{v})^2}\right\}. \tag{9}$$

Here $\theta = I/I_{sat}$ is the saturation parameter expressed in terms of the spectral intensity I; $I_{sat} = (\hbar^2 c/4\pi^2)|D_c|^2 \times (\Gamma_{nn}^{-1} + \Gamma_{ff}^{-1})$ is the intensity at which allowance for the saturation becomes essential; $\alpha_0 = R_c N_c(z)$ is the unsaturated gain; $\Omega = \omega - \omega_{fn}$ is the detuning of the radiation frequency ω relative to the frequency of the working transition ω_{fn}. We note that the transport equation obtained by Litvak[8] differs from (9) in that the gain $\alpha_0/(1+\theta)^{1/2}$ is replaced by $\alpha_0/(1+\theta)$. The validity of Eq. (9) is due to the approximation $\Gamma_{fn}(1+\theta)^{1/2} \ll k\bar{v}$ employed in the present paper.

The amplification of the spontaneous radiation corresponds to the boundary conditions

$$\theta|_{z=0} = \theta_0 = \theta_0'\mathscr{E}(\Omega), \quad \theta_0' \ll 1. \tag{10}$$

Integration of (9) under these initial conditions yields

$$2(\sqrt{1+\theta} - 1) + \ln\left(\frac{4}{\theta_0}\frac{\sqrt{1+\theta} - 1}{\sqrt{1+\theta} + 1}\right) \approx \beta\mathscr{E}(\Omega), \tag{11}$$

$\beta = \int_0^z \alpha_0(z)dz$ is the optical thickness for unsaturated radiation.

We seek the solution of Eq. (11) in the form of a Gaussian function

$$\theta(\Omega, z) = \theta'(z)\exp\{-\Omega^2/v^2(z)\}, \tag{12}$$

and the standard procedure of the expansion of the exponential in powers Ω^2 is employed.

As follows from (11), prior to saturation ($\theta'' \ll 1$) the amplification of the spontaneous emission leads to the usual narrowing of the Doppler profile of the line and to an exponential growth of the intensity.

$$\theta' = \theta_0' e^{\beta}, \quad \nu = k\bar{v}/\sqrt{1+\beta}. \qquad (13)$$

The situation is significantly altered when saturation becomes noticeable. In the case of sufficiently large gain

$$\nu = \frac{k\bar{v}(1+\theta)^{1/4}}{\beta^{1/2}}, \qquad (14)$$

$$\theta' = \left(y + \ln\frac{y-1}{y+1}\right)^2 - 1, \quad y = \frac{\beta-\beta_{sat}}{2} + 1 - \ln 2, \qquad (15)$$

The parameter $\beta_{sat} = -\ln\theta_0' = \ln(I_{sat}/I_0)$ characterizes the optical distance in which the radiation intensity increases exponentially from an initial value I_0 to I_{sat}. If $\theta' \gg 1$, then the intensity depends on the difference $(\beta - \beta_{sat})$ in accordance with a parabolic law.[1)] The dependence of θ' and of ν on β is illustrated in Fig. 2. We see that as $\beta \to \infty$ the line-narrowing effect is offset by the field broadening and $\nu \to k\bar{v}$. The source brightness temperature T duplicates the variation of the intensity. With the aid of the Rayleigh-Jeans formula we obtain

$$T = 2\hbar\omega\theta'/k_B A_{fn}(\Gamma_{ff}^{-1} + \Gamma_{nn}^{-1}), \qquad (16)$$

where k_B is the Boltzmann constant and A_{fn} is the Einstein probability of spontaneous transitions.

B. As already mentioned, as the satellite lines are enhanced their parameters are frequently transformed under conditions when the saturated radiation of the principal lines exerts a strong influence. Let us consider a simple example, in which the enhancement of the unsaturated line $m \to n$ is influenced only by the saturated radio emission of the transition $f \to n$. In the spontaneous relaxation approximation ($2\Gamma_{ij} = \Gamma_{ii} + \Gamma_{jj}$), this process is described, accurate to the first nonlinear correction, by the equation

$$\frac{d\theta_a}{dz} = \theta_a R_a\left[N_a(z)e^{-\Omega^2/(k\bar{v})^2} - N_c(z)\theta_c(z)\frac{\Gamma_{ff}}{4\Gamma_{mf}} \times \frac{\mathscr{E}(\Omega_a)+\mathscr{E}(\Omega_c)}{1+\varepsilon^2/\Gamma_{mf}^2}\right], \quad \varepsilon = \Omega_a - \Omega_c. \qquad (17)$$

The notation employed here have the same meaning as before. The additional subscripts a and c merely identify the corresponding lines. It is seen from (17) that in this case the gain contour contains, against the Doppler background, a narrow interference dip of the Raman type with a center at the frequency $\omega_a = \omega_{mn} + \omega_c - \omega_{fn}$. Since the nonlinear increment is small in comparison with the linear gain, Eq. (17) can be solved by the perturbation method with respect to θ_c. In the zeroth order of perturbation theory, the solution for $\theta_c^{(1)}$ takes the form (13). The next order yields the change of the width $\Gamma(z)$ and of the depth $r(z)$ of the Lorentz dip in the course of enhancement

$$\frac{1}{\Gamma^2(z)} - \frac{1}{\Gamma^2(0)} \approx -\frac{1}{\Gamma_{mf}^2}\int_0^z \bar{a}(z)\,dz < 0, \qquad (18)$$

$$r(z) = -\theta_a^{(1)}(\varepsilon=0) = \int_0^z \bar{a}(z)\,dz, \quad \bar{a} = \frac{R_a\Gamma_{ff}}{2\Gamma_{mf}}N_c\theta_a^{(0)'}\theta_c'. \qquad (19)$$

from which we easily see that with increasing gain there occurs, besides the narrowing of the Doppler profile of the $m \to n$ line, also an increase of the depth and width of the Lorentz structure, due to the resonant interaction of the strong field with the transition $f \to n$.

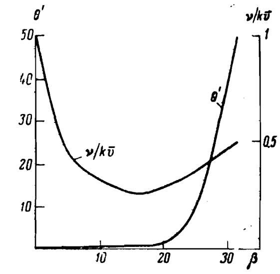

FIG. 2. Dependence of the saturation parameter and of the line width on β ($\beta_{sat} = 20$).

Usually the density of the particles in the amplifying regions is so large, that the spontaneous approximation does not hold ($2\Gamma_{ij} \neq \Gamma_{ii} + \Gamma_{jj}$). In this case there appears a second Lorentzian with width $\Gamma_{mm} + \Gamma_{fn}$, and the amplitudes of the Lorentzians $L(\Gamma_{mn} + \Gamma_{fn})$ and $L(\Gamma_{fm})$ are in the ratio $(\Gamma_{mn} + \Gamma_{fn} - \Gamma_{fm} - \Gamma_{nn})\Gamma_{fm}/\Gamma_{nn}(\Gamma_{mn} + \Gamma_{fn})$. Allowance for the influence of the line $J = 1 \to 1$ also complicates the amplification picture.

We note in conclusion that the theory proposed in the paper is based on the use of the model of a one-dimensional active medium and in this sense is strongly idealized. It can yield only a qualitative description of the phenomena, whereas quantitative estimates vary within a rather wide range determined by the concrete conditions, such as the geometry of the cloud, the spatial distribution of the inversion, the intensity and direction of the magnetic field, etc. A more detailed investigation of the role of these factors is a separate and quite interesting group of problems.

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1)This result was pointed out to the author by S. G. Rautian. The analogous relation in the Litvak model [8] is linear.

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1