Elementary excitations in quantum crystals

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The character of the motion in solid helium of two point defects located at a distance considerably smaller than the range of the interaction but greater than the interatomic spacing is investigated. The defects are found to be bound by the interaction potential. Depending on the magnitude of the mutual spacing and the arrangement of the defects with respect to the crystallographic axes, four cases are possible: 1) The defects perform three-dimensional diffusional motion, 2) they are practically stationary, 3) they perform two-dimensional diffusional motion in the basal plane of the crystal, 4) the pair of defects behaves as one two-dimensional quasiparticle, moving as a free particle but only in the basal plane. It is shown that kinks in dislocations in the quantum crystal are transformed into free one-dimensional quasiparticles. Their quantum-mechanical delocalization leads to the delocalization of dislocations with kinks in the slip plane. An estimate of the diffusion coefficients of the kinks at low temperatures is given.

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The quantum nature of crystals of the solid-helium type, with large amplitude of the zero-point vibrations, has a substantial effect on the character of the energy spectrum. In such crystals, associated with any type of point defect or impurity there is a spectral branch of elementary excitations (defectons, impuritons) that move practically freely across the crystal[1,2]. Convincing experimental confirmation of this fact was obtained in the experiments[3-5] on the measurement of the diffusion of He^3 impurities in crystalline He^4 (cf. also the review[6]).

In the present paper it will be shown that distinctive secondary quasi-particles, arising when several defects of one type or another are simultaneously present in the crystal, should exist in quantum crystals. The distinguishing feature of these quasi-particles is the fact that they move practically freely, but only over certain definite planes or along certain axes of the crystal lattice, i.e., they are, in essence, two- or one-dimensional quasi-particles.

1. In the majority of cases the width of the energy band of the defectons, or, which is the same thing, the magnitude of the corresponding exchange integral J, is small compared with the characteristic interaction energy of the atoms in the crystal. This circumstance, as was noted earlier[1], is extremely important when we are concerned with the motion of the quasi-particles under the action of external forces (cf.[7,8]) or under the influence of their interaction with each other (cf.[9-13]). In the latter case the smallness of J gives rise to a larger magnitude of the defecton interaction range R, inasmuch as R is defined by the condition $U(R) \sim J$, where $U(r) \sim U_0(a/r)^3$ is the interaction energy of the point defects, r is the distance between them, U_0 is a certain constant and a is the interatomic spacing. The interaction range $R \sim a(U_0/J)^{1/3}$ for $J \ll U_0$ is large compared with the interatomic spacing. Therefore, even for a very small concentration of defects, we have an appreciable probability that some two of them are separated by a distance r that is small compared with R.

It is interesting to trace the change in the character of the motion of such a pair of defects with decrease of r in the region $R \gg r \gg a$. If $r \gg a(U_0/J)^{1/4}$, then $a\,\partial U/\partial r \ll J$, i.e., on transfer of one of the defects to a neighboring lattice site the energy of the system changes by an amount much smaller than J. Therefore, quantum tunneling processes occur in practically the same way as in a uniform crystal. In addition, in the given case the interaction energy U varies little over distances of the order of the wavelength of the quasi-particles (which, in order of magnitude, coincides with the interatomic spacing). It is clear that in the conditions considered the system can be described semi-classically by means of a Hamiltonian function $\epsilon(\mathbf{p}_1) + \epsilon(\mathbf{p}_2) + U(\mathbf{r}_1 - \mathbf{r}_2)$, where $\epsilon(\mathbf{p})$ is the energy spectrum of an isolated defecton of the type under consideration. Since we assume that $r \ll R$, i.e., that $U \gg J$, the defects cannot, while moving semi-classically, subsequently disperse over an infinite distance, since the total energy should be conserved while the sum of the kinetic energies $\epsilon(\mathbf{p}_1) + \epsilon(\mathbf{p}_2)$ cannot vary by an amount much greater than J. Thus, the defects behave as if bound by the potential U. We emphasize that this conclusion is valid irrespective of the sign of the interaction (attraction or repulsion).

For a more detailed elucidation of the character of the motion of the defects it is convenient to consider first the simplified problem of the motion of a quasi-particle with the dispersion law $\epsilon(\mathbf{p})$ under the action of a constant force $\mathbf{F}$ directed along a certain irrational axis in the crystal. It follows from the equations of motion $\dot{\mathbf{p}} = \mathbf{F}$ that the component of the quasi-momentum along $\mathbf{F}$ increases linearly with time, while the perpendicular components do not change. Since the direction of $\mathbf{F}$ is a random direction in the crystal, all three components of the velocity $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ vary in a random manner and in such a way that the characteristic velocity value $v \sim aJ/\hbar$ while the characteristic time τ of its variation is of the order of the time taken for the quasi-momentum to change by an amount of the order of the reciprocal-lattice period $\hbar/a$, i.e., $\tau \sim \hbar/aF$. The trajectory of the particle in this case is a random curve (with the condition, of course, that its extent along F does not exceed J/F) with a characteristic radius of curvature of the order of J/F. It is now clear how the motion of the pair of defects occurs. The trajectory of each of them will be a random curve with a characteristic radius of curvature of the order of $J/(\partial U/\partial r) \sim a(r/a)^4(J/U_0)$. These two curves are mutually correlated only by the condition that $U(\mathbf{r}_1 - \mathbf{r}_2)$ change by an amount not greater than a quantity of the order of J.

Thus, we can say that a pair of defects placed at a distance $R \gg r \gg a(U_0/J)^{1/4}$ undergoes diffusional motion with a diffusion coefficient characterizing, in particular, the motion of the pair as a whole and equal to

$$D \sim va\left(\frac{r}{a}\right)^4 \frac{J}{U_0} \sim \frac{J^2a^2}{\hbar U_0}\left(\frac{r}{a}\right)^4. \tag{1}$$

For $r \ll a(U_0/J)^{1/4}$ tunneling of one of the defects over an interatomic distance is accompanied by a change in energy by an amount considerably greater than J. In these conditions the probability of tunneling is proportional to J^2, i.e., is negligibly small. Equally small is the probability of simultaneous tunneling of both defects with conservation of energy. Thus, the possibility of motion of the defects practically disappears. There exists, however, an important exception to this result. Namely, we shall assume that the straight line joining the defects is almost parallel to a crystallographic axis of sufficiently high order. In the hcp crystal He^4, e.g., the hexagonal axis is such an a axis.

The quantity $U_0 = U_0(\mathbf{n})$ characterizing the interaction is a function of the orientation of the defect pair, determined by the unit vector $\mathbf{n} = (\mathbf{r}_1 - \mathbf{r}_2)/|\mathbf{r}_1 - \mathbf{r}_2|$. The function $U_0(\mathbf{n})$ has, obviously, an extremum for $\mathbf{n}$ parallel to the hexagonal axis. Therefore, a displacement of one of the defects by a small distance x in a direction perpendicular to $\mathbf{n}$ changes the interaction energy by an amount of the order of

$$\delta U(x) \sim U_0\left(\frac{a}{r}\right)^3\left(\frac{x}{r}\right)^2. \tag{2}$$

If $r \gg a(U_0/J)^{1/5}$, then $\delta U(a) \ll J$. Thus, in the conditions under consideration, motion of the defects along $\mathbf{n}$ is practically impossible, but for motion in the plane perpendicular to $\mathbf{n}$ there arises a situation analogous to that considered above.

The trajectories of the defects are random curves, lying in planes perpendicular to the hexagonal axis. The radius of curvature $\bar{x}$ of the trajectories is determined from the condition $\delta U(\bar{x}) \sim J$, i.e., is equal to $\bar{x} \sim a(J/U_0)^{1/2}(r/a)^{5/2}$. Consequently, two defects positioned at a distance r such that $a(U_0/J)^{1/5} \ll r \ll a(U_0/J)^{1/4}$, in an orientation close to the hexagonal axis, undergo a distinctive two-dimensional diffusional motion in the basal plane of the crystal. The diffusion coefficient is equal in order of magnitude to

$$D \sim v\bar{x} \sim (J^3r^5/\hbar^2U_0a)^{1/2}.$$

We emphasize that an extremum of the function $U_0(\mathbf{n})$ is not sufficient to make such motion possible. It is also necessary that the crystal be periodic in the perpendicular plane.

Finally, let $r \ll a(U_0/J)^{1/5}$. In this case, a displacement of one of the defects to a neighboring lattice site, even in a direction perpendicular to the hexagonal axis, leads, generally speaking, to an energy change greater than J. Here, however, there is also one important exception. Suppose that one of the defects is positioned at a certain point O (see Fig. 1) in a plane perpendicular to the hexagonal axis. We shall assume that the projection of the second defect on this plane coincides with one of the lattice sites nearest to the point O. If we take the point O as the coordinate origin in the plane and denote the lattice constants of the triangular lattice by $\mathbf{a}_1$ and $\mathbf{a}_2$ (see Fig. 1), the possible coordinate projections of the second defect are equal to $\mathbf{a}_1 - \mathbf{a}_2$, $\mathbf{a}_1$, $\mathbf{a}_2$, $-\mathbf{a}_1 + \mathbf{a}_2$, $-\mathbf{a}_1$ and $-\mathbf{a}_2$.

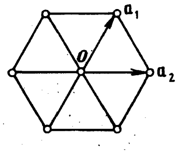

FIG. 1

A transfer of the second defect in its basal plane from one of the positions enumerated to another, neighboring position is not accompanied by any change in energy. The same holds for the displacement of the first defect in the plane of the Figure from the point O to the points $\mathbf{a}_2$ and $\mathbf{a}_1 - \mathbf{a}_2$, if the projection of the second defect coincides, e.g., with the point $\mathbf{a}_1$. As a result of displacements of this type, the system can move as a whole in the basal plane. Since the displacements under consideration do not change the energy at all, in the given case the motion is not diffusional. The pair of defects behave as one distinctive two-dimensional quasi-particle, moving as a free particle but only in the plane perpendicular to the hexagonal axis. In fact, we shall find the possible values of the energy of the system, considering the quantum tunneling processes as a small perturbation.

All configurations of the pair of defects, corresponding to the same energy, can be classified as follows. We shall specify the two-dimensional coordinate $\mathbf{r}$ of the first defect in the plane of Fig. 1. The second defect can then occupy six possible positions in its basal plane. The possible coordinates of its projection on the plane of Fig. 1 are equal to $\mathbf{r} + \mathbf{a}_1 - \mathbf{a}_2$, $\mathbf{r} + \mathbf{a}_1$, $\mathbf{r} + \mathbf{a}_2$, $\mathbf{r} - \mathbf{a}_1 + \mathbf{a}_2$, $\mathbf{r} - \mathbf{a}_1$ and $\mathbf{r} - \mathbf{a}_2$. We shall label them by the index $\alpha = 1, 2, \ldots, 6$. The wavefunctions $\psi_\alpha(\mathbf{r})$ describe the complete set of states of the pair of defects in the absence of tunneling.

Let V be the operator of the perturbation corresponding to quantum tunneling of one of the defects to a neighboring lattice site. Its action on the functions $\psi_\alpha(\mathbf{r})$ is determined by the formulas

$$\begin{aligned}
V\psi_1(\mathbf{r}) &= J\{\psi_6(\mathbf{r}) + \psi_2(\mathbf{r}) + \psi_6(\mathbf{r}+\mathbf{a}_1) + \psi_2(\mathbf{r}-\mathbf{a}_2)\},\\
V\psi_2(\mathbf{r}) &= J\{\psi_1(\mathbf{r}) + \psi_3(\mathbf{r}) + \psi_1(\mathbf{r}+\mathbf{a}_2) + \psi_3(\mathbf{r}-\mathbf{a}_2+\mathbf{a}_1)\},\\
V\psi_3(\mathbf{r}) &= J\{\psi_2(\mathbf{r}) + \psi_4(\mathbf{r}) + \psi_2(\mathbf{r}-\mathbf{a}_1+\mathbf{a}_2) + \psi_4(\mathbf{r}+\mathbf{a}_1)\},\\
V\psi_4(\mathbf{r}) &= J\{\psi_3(\mathbf{r}) + \psi_5(\mathbf{r}) + \psi_3(\mathbf{r}-\mathbf{a}_1) + \psi_5(\mathbf{r}+\mathbf{a}_2)\},\\
V\psi_5(\mathbf{r}) &= J\{\psi_4(\mathbf{r}) + \psi_6(\mathbf{r}) + \psi_4(\mathbf{r}-\mathbf{a}_2) + \psi_6(\mathbf{r}-\mathbf{a}_1+\mathbf{a}_2)\},\\
V\psi_6(\mathbf{r}) &= J\{(\psi_1(\mathbf{r}) + \psi_5(\mathbf{r}) + \psi_1(\mathbf{r}-\mathbf{a}_1) + \psi_5(\mathbf{r}+\mathbf{a}_1-\mathbf{a}_2)\},
\end{aligned} \tag{3}$$

where J is the exchange integral. The structure of the expressions in the right-hand sides of the formulas (3) is extremely simple. For example, the first two terms in the first of formulas (3) correspond to tunneling of the second defect, with the first defect stationary, from the position with projection $\mathbf{r} + \mathbf{a}_1 - \mathbf{a}_2$ to the positions $\mathbf{r} - \mathbf{a}_2$ and $\mathbf{r} + \mathbf{a}_1$ respectively. The second two terms describe the possibility of tunneling of the first defect from $\mathbf{r}$ to $\mathbf{r} + \mathbf{a}_1$ and $\mathbf{r} - \mathbf{a}_2$ respectively.

The solution of the Schrödinger equation

$$(\varepsilon - \varepsilon_0)\psi(\mathbf{r}) = V\psi(\mathbf{r}), \tag{4}$$

where ϵ_0 is the energy of the system in the absence of tunneling, is a linear combination of the functions $\psi_\alpha(\mathbf{r})$:

$$\psi(\mathbf{r}) = \sum_{\alpha=1}^{6} A_\alpha\psi_\alpha(\mathbf{r}), \tag{5}$$

and, by virtue of the periodicity of the crystal in the basal plane, the coordinate dependence of the functions $\psi_\alpha(\mathbf{r})$ is determined by specifying the two-dimensional quasi-momentum p:

$$\psi_\alpha(\mathbf{r}+n\mathbf{a}_1+m\mathbf{a}_2)=\exp[i\mathbf{p}(n\mathbf{a}_1+m\mathbf{a}_2)]\psi_\alpha(\mathbf{r}).$$

Substituting (5) into (4), taking (3) into account we obtain the following system of equations for the coefficients A_α:

$$(\varepsilon-\varepsilon_0)A_1/J=(1+e^{i\varphi_2})A_2+(1+e^{-i\varphi_1})A_6,$$
$$(\varepsilon-\varepsilon_0)A_2/J=(1+e^{-i\varphi_2})A_1+(1+e^{i\varphi_2-i\varphi_1})A_3,$$
$$(\varepsilon-\varepsilon_0)A_3/J=(1+e^{i\varphi_1-i\varphi_2})A_2+(1+e^{-i\varphi_1})A_4,$$
$$(\varepsilon-\varepsilon_0)A_4/J=(1+e^{i\varphi_1})A_3+(1+e^{-i\varphi_2})A_5,$$
$$(\varepsilon-\varepsilon_0)A_5/J=(1+e^{i\varphi_2})A_4+(1+e^{i\varphi_1-i\varphi_2})A_6,$$
$$(\varepsilon-\varepsilon_0)A_6/J=(1+e^{i\varphi_1})A_1+(1+e^{i\varphi_2-i\varphi_1})A_5,$$

where $\varphi_1 = \mathbf{p}\cdot\mathbf{a}_1$ and $\varphi_2 = \mathbf{p}\cdot\mathbf{a}_2$.

The possible values of the energy $\epsilon = \epsilon(\mathrm{p})$ are determined as the roots of the corresponding secular equation. After straightforward calculations we find

$$\varepsilon(\mathbf{p})=\varepsilon_0+2J\lambda(\mathbf{p}). \tag{6}$$

The quantity $\lambda(\mathrm{p})$ appearing here satisfies the following cubic equation in λ^2:

$$\lambda^2\left(\lambda^2-\cos^2\frac{\varphi_1}{2}-\cos^2\frac{\varphi_2}{2}-\cos^2\frac{\varphi_1-\varphi_2}{2}\right)^2=4\cos^2\frac{\varphi_1}{2}\cos^2\frac{\varphi_2}{2}\cos^2\frac{\varphi_1-\varphi_2}{2}. \tag{7}$$

Thus, the energy spectrum of the two-dimensional quasi-particles corresponding to the motion of a pair of defects in the conditions under consideration consists of six bands. For small p, from (6) and (7) we can easily obtain explicit expressions for the six roots:

$$\varepsilon_{1,2}(\mathbf{p})=\varepsilon_0\pm4J\{1-{}^1/_{12}(\varphi_1^2+\varphi_2^2-\varphi_1\varphi_2)\},$$
$$\varepsilon_{3,4,5,6}(\mathbf{p})=\varepsilon_0\pm2J\{1-{}^1/_6[\varphi_1^2+\varphi_2^2-\varphi_1\varphi_2\pm\sqrt{(\varphi_1^2+\varphi_2^2)(\varphi_1-\varphi_2)^2+\varphi_1^2\varphi_2^2}]\}.$$

We emphasize that the exchange integral J occurring here corresponds to quantum tunneling of one of the defects. The width of the energy bands (6) coincides, therefore, in order of magnitude, with the bandwidth of an isolated defecton.

To complete the picture, we must mention also the interesting situation which, as noted by Richards, Smith and Tafts[1)], arises when two point defects are positioned at neighboring sites of the hcp lattice of solid He^4. It is easy to see that in the given case there exist several configurations of defects that can be obtained from the original one by transferring one of the defects to a nearest-neighbor site, these configurations being equivalent to the original one, i.e., the defects are nearest neighbors as before and the energy of the system does not change. By successive application of such transfers it is possible to effect motion of the defects over the entire crystal, i.e., to transfer them to any two lattice sites that lie next to each other. It is clear that, in this case, owing to quantum tunneling, the two defects behave as one quasi-particle with an energy bandwidth of the order of the bandwidth of an ordinary defecton, and this quasi-particle is three-dimensional, i.e., can move in all directions. For a small concentration of defects, however, the probability of formation of such a three-dimensional quasi-particle is small compared with the probability of formation of the two-dimensional quasi-particles that were discussed above.

2. We shall consider a linear dislocation in a quantum crystal, positioned in the slip plane at a certain angle to the crystallographic directions. As is well-known, in this case the dislocation line is not a straight line. It consists of separate straight lengths, positioned along the direction in which the energy of the dislocation is a minimum (this direction coincides with one of the crystallographic axes), and a certain number of kinks, in the vicinity of which the dislocation goes over from one valley to another (see Fig. 2). Each such kink can be regarded as a point defect on the dislocation. Since the crystal is periodic along a crystallographic axis, in a quantum crystal such a point defect is transformed into a quasi-particle whose states are classified by the values of the one-dimensional quasi-momentum. Let there be one kink on the dislocation, with a certain value of the quasi-momentum. In this case, a kink is found with equal probability at all points of the dislocation, and this means that the dislocation is evenly distributed between two neighboring valleys. In the presence of a large number of kinks, the dislocation is distributed over a large number of valleys. Thus, quantum-mechanical delocalization of the kinks leads to delocalization of the dislocation in the slip plane.

If the width of the energy band of the kinks is small, i.e., if the probability, per lattice constant a in the direction of the dislocation, of quantum hopping of the kink under the barrier is small, we can apply the well-known tight-binding approximation to calculate the energy spectrum. As a result we obtain the following dependence of the kink energy ϵ on the quasi-momentum p:

$$\varepsilon(p)=\varepsilon_0+2J\cos(pa/\hbar),$$

where ϵ_0 is the energy of a localized kink in the classical limit and J is the exchange integral.

Let n be the number of kinks per unit length of the dislocation. If the mean distance n^{-1} between them is large compared with the interaction range x_0, the kinks can be regarded as a dilute gas of quasi-particles. The interaction range is determined by the condition $U(x_0) \sim J$, where $U(x) = \alpha/x$ is the interaction energy of two kinks, inversely proportional, as is well-known[14], to the first power of the distance x between them. Here α is a certain constant, equal in order of magnitude to $\alpha \sim \mu a^4$, where μ is the shear modulus.

An important experimentally observable characteristic (e.g., in measurements of the internal friction) of the gas of kinks is their diffusion coefficient. In the high-temperature region this is determined by collisions of the kinks with phonons and with vibrations of the dislocation. As the temperature is lowered the number of phonons and vibrations decreases and the diffusion coefficient increases rapidly, until it attains a limiting value determined by the mutual collisions between the kinks. To estimate this limiting value it is important to note the following. In the one-dimensional case the momenta acquired by the quasi-particles as a result of a binary collision are uniquely determined by

FIG. 2

the conservation laws. In this case, the only solution is the trivial one corresponding to the fact that the quasi-particles exchange their momenta as a result of the collision. The quasi-particle momentum distribution function is not changed at all in this case. In other words, the binary collision integral is equal to zero and all kinetic effects are determined by triple collisions. The corresponding mean free path l_3 differs from the mean free path $l_2 \sim n^{-1}$ characterizing the binary collisions by the large factor $(nx_0)^{-1}$. Thus, the diffusion coefficient is equal to

$$D \sim l_3 \frac{\partial \varepsilon}{\partial p} \sim \frac{J^2 a}{\hbar \alpha n^2},$$

i.e., in place of the usual law $D \propto n^{-1}$, in the given case a reciprocal dependence of the diffusion on the square of the quasi-particle density should be observed.

3. In conclusion, we shall make a few remarks concerning the temperature and concentration dependences of the diffusion coefficient of the point defects. This question is of special interest in connection with the experiments[3-5] on the diffusion of He^3 impurities in solid He^4. In the work of I. Lifshitz and the author[1] it was ascertained that in the region of sufficiently small concentrations of impurities, their diffusion coefficient, determined by the collisions of impuritons with phonons, is inversely proportional to the ninth power of the temperature T. Subsequently, Kagan, Maksimov, and Klinger[11,12] showed that the region of applicability of the T^{-9} law is considerably broader than was indicated in[1], and extends into the region of temperatures in which the mean free path of the impuritons is much shorter than the lattice constant. We shall show that this result can be obtained without any calculations, by ascertaining the region of applicability of the classical kinetic equation.

We are concerned with the motion of a quasi-particle, interacting with phonons and with the spectrum

$$\varepsilon(\mathbf{p}) = \varepsilon_0 + \Delta(\mathbf{p}),$$

where ϵ_0 is a constant and $\Delta(\mathbf{p})$ is a function of the quasi-momentum that is small compared with the temperature T, which, in its turn, is small compared with the Debye temperature Θ.

The kinetic equation for the classical quasi-particle distribution function $f(\mathbf{r}, \mathbf{p}, t)$ has the following form:

$$\frac{\partial f}{\partial t} + \frac{\partial \Delta}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \frac{\partial f}{\partial \mathbf{p}} = I, \tag{8}$$

where $\mathbf{F}$ is the external force acting on the quasi-particle and I is the collision integral for the collisions with the phonons:

$$\begin{aligned} I = -\int d^3p' \int d^3k\, d^3k'\, W(\mathbf{k}, \mathbf{k}') \{ n(\mathbf{k}) [1 + n(\mathbf{k}')] f(\mathbf{p}) \\ - n(\mathbf{k}') [1 + n(\mathbf{k})] f(\mathbf{p}') \} \delta(\mathbf{k} + \mathbf{p} - \mathbf{k}' - \mathbf{p}') \delta[\omega(\mathbf{k}) - \omega(\mathbf{k}')]. \end{aligned} \tag{9}$$

Here $n(\mathbf{k})$ is the phonon distribution function, $\omega(\mathbf{k})$ is the phonon energy spectrum and W is the probability of a collision of an impuriton with a phonon. In the arguemnt of the δ-function, which expresses the energy conservation law, we have neglected the quantity $\Delta(\mathbf{p})$ in comparison with the phonon energy, since the latter coincides, in order of magnitude, with the temperature.

It is easy to see that in this form the kinetic equation is valid under the condition $\hbar/\tau \ll T$ (τ is the time between collisions). The more stringent condition $\hbar/\tau \ll \Delta(\mathbf{p})$, which is equivalent to the condition that the interatomic spacing is small compared with the impuriton mean free path, is not required. In fact, if f and $\mathbf{F}$ vary slowly in space and in time and the force $\mathbf{F}$ is sufficiently small, Eq. (8) is also valid in the general quantum case, but the quantity I describing the collisions is, generally speaking, not even expressed in terms of the distribution function. However, if we neglect $\Delta(\mathbf{p})$ in the expression for the defecton spectrum, the condition $\hbar/\tau \ll \omega \sim T$ guarantees the possibility of using formula (9), inasmuch as the uncertainty $\hbar/\tau$ in the energy is small compared with the phonon energy. Then the energy $\Delta(\mathbf{p})$ does not appear at all in the problem. The difference between the cases $\hbar/\tau \ll \Delta$ and $\hbar/\tau \gg \Delta$ lies only in the fact that, in the first of these, it would be possible to take $\Delta(\mathbf{p})$ into account in the argument of the δ-function in (9), whereas in the second this would be excessively accurate. In both cases, to determine the diffusion coefficient we must use Eqs. (8), (9), from which the T^{-9} law is obtained. In this case, the condition $\hbar/\tau \ll T$ is obviously equivalent to the inequality $T \ll \Theta$.

We shall now discuss the question of the concentration dependence of the diffusion coefficient in the case when it is determined by the mutual interaction of the impurities. In the paper by Meĭerovich and the author[8] it was stated that the diffusion coefficient should be proportional to $x^{-1/3}$ in the region of not too small concentrations $J/U_0 \ll x \ll (J/U_0)^{3/4}$. This result is incorrect. The correct law can be obtained if we note that in the concentration range under consideration the mean distance r between impurities satisfies the condition $a(J/U_0)^{1/3} \gg r \gg a(J/U_0)^{1/4}$ and that formula (1) can therefore be used. Substituting $r \sim a/x^{1/3}$ into it, we find

$$D \sim J^2 a^2 x^{-4/3} / \hbar U_0, \tag{10}$$

which coincides with the result of the paper by Landesman and Winter[10]. It should be noted, however, that the concentration range indicated in[10], in which formula (10) should be applicable, differs substantially from $J/U_0 \ll x \ll (J/U_0)^{3/4}$.

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[1)] Report at the International Conference on Quantum Crystals, Tbilisi, November 1974.

[1] A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 56, 2057 (1969) [Sov. Phys.-JETP 29, 1107 (1969)].
[2] R. A. Guyer and L. I. Zane, Phys. Rev. Lett. 24, 660 (1970).
[3] M. G. Richards, J. Pope, and A. Widom, Phys. Rev. Lett. 29, 708 (1972).
[4] V. N. Grigor'ev, B. N. Esel'son, V. A. Mikheev, and Yu. E. Shul'man, ZhETF Pis. Red. 17, 25 (1973) [JETP Lett. 17, 16 (1973)].
[5] V. N. Grigoriev, B. N. Eselson, V. A. Mikheev, V. A. Slusarev, M. A. Strzhemechny, and Yu. E. Shul'man, J. Low Temp. Phys. 13, 65 (1973).
[6] R. A. Guyer, R. C. Richardson, and L. I. Zane, Rev. Mod. Phys. 43, 532 (1971).
[7] A. É. Meĭerovich, Zh. Eksp. Teor. Fiz. 67, 744 (1974) [Sov. Phys.-JETP 40, 368 (1975)].
[8] A. F. Andreev and A. É. Meĭerovich, Zh. Eksp. Teor. Fiz. 67, 1559 (1974) [Sov. Phys.-JETP 40, 776 (1975)].

[9] D. I. Pushkarev, Zh. Eksp. Teor. Fiz. 59, 1775 (1970) [Sov. Phys.-JETP 32, 954 (1971)].
[10] A. Landesman and J. M. Winter, Low. Temp. Phys. 13, 73 (1974).
[11] Yu. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. 65, 622 (1973) [Sov. Phys.-JETP 38, 307 (1974)].
[12] Yu. Kagan and M. I. Klinger, J. Phys. C 7, 2791 (1974).
[13] Y. Yamashita, Diffusion of Dilute He^3 Impurities in Solid He^4, Preprint, Kyoto University, 1974.
[14] J. P. Hirth and J. Lothe, Theory of Dislocations, Sec. 8.5, McGraw-Hill, N. Y., 1967 (Russ. transl., Atomizdat, M., 1972).

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