Resistance of intermediate-state indium with an uneven concentration of the normal phase

I. P. Krylov

Institute of Physics Problems, USSR Academy of Sciences
(Submitted December 3, 1974)
Zh. Eksp. Teor. Fiz. **68** 1557–1568 (April 1975)

An increase of the resistance of high-purity indium single crystals in the intermediate state is observed if an uneven concentration of the normal phase is produced. The dependence of the effect on the introduced concentration inhomogeneity, on the temperature, and on other factors is investigated. The experimental results are in agreement with the theoretical calculations based on the equations of macroscopic electrodynamics of the intermediate state. The observed phenomenon is due to the Hall effect and to the "hydrodynamic" properties of the flow of normal superconducting domains, and is close to the resonant wave-resistance growth predicted theoretically by Andreev.

PACS numbers: 72.10.E

In 1967 Andreev considered a new mechanism of dc resistance of pure superconductors in the intermediate state.[1] This mechanism is closely connected with excitation, in the intermediate state, of helical electromagnetic waves (helicons). The new resistance was therefore called "wave resistance."

In the intermediate state, the volume of the metal breaks up into alternating regions of normal (n) and superconducting (s) phases. When direct current J is made to flow, a situation is readily obtained wherein the structure of the n and s domains is set in motion with velocity $v \sim J$.[2] in the coordinate system connected with the moving domains, any statistical inhomogeneity of the magnetic field becomes a time-varying perturbation. Such perturbations excite helicon waves in the intermediate-state structure. The current-source energy is consumed in the excitation of the helicons. It is this which leads to the additional resistance. A similar effect is well known in hydrodynamics: the flow of a liquid with a free surface around an obstacle leads to excitation of waves on the surface of the liquid and to the appearance of additional "wave" resistance.

In the intermediate state, the electromagnetic-type waves (helicons) are excited most effectively by inhomogeneities of the magnetic fields, which can be produced near roughnesses on the sample surface, near cavities in the volume, near inhomogeneities of the thickness, and can finally be produced artificially. In the case of inhomogeneities that are periodically distributed in space along the direction of the direct durrent, a time-periodic perturbation acts in the system of the moving n and s domains. When the frequency of this perturbation coincides with natural frequency of the helicons, a resonant increase of the field amplitudes should be observed in the sample, and accordingly also a maximum of the wave resistance. Thus, at certain values of J there should be observed maxima of the sample resistance. The positions of these maxima are determined by the spectrum of the natural oscillations of the helicons in the sample, and the width is determined by the helicon damping.

Attention should be called to two circumstances. First, all the fields are static in the laboratory frame. Second, for the wave resistance to appear, the motion of the structure of the n and s domains as a unit is not a necessary conditions. The existence of wave resistance follows from the solution of the equations of macroscopic electrodynamics.[1] Macroscopic electrodynamics describes electromagnetic waves that vary over distances greatly exceeding the period d of the intermediate-state structure, i.e., the configuration of the n and s domains has no significance. For example, within the framework of macroscopic electrodynamics of a pure uncompensated metal it is possible to have, besides the case of moving n and s layers perpendicular to the current, also a configuration of immobile layers extending along the current direction.[3] In this situation, wave resistance can also set in. It is necessary only in each case to ensure free displacement of the boundaries between the phases, i.e., absence of pinning of the n–s boundaries by defects in the crystal. If there is no such dry friction, then the hydrodynamic properties of the flow of domains of the normal and superconducting phases will become manifest.

The natural helicon oscillations in an unbounded plane-parallel plate are characterized by a wave vector $\mathbf{k}$ lying in the plane of the plate, and by an integer number n (n = 1, 2, 3,. . .) of half-waves that are spanned by the plate thickness. Andreev[1] calculated the spectrum of the natural oscillations in a plate of thickness a. The natural oscillation frequency $\omega_n(\mathbf{k})$ is given by

$$\omega_n(\mathbf{k}) = \frac{cH_c}{\pi Nea^2}\varphi_n^2(k) + \frac{1}{Ne}\mathbf{jk}. \qquad (1)$$

Here N is the absolute value of the difference between the hole and electron densities, e is the electron charge, H_c is the critical field, c is the speed of light, $\mathbf{j}$ is the density of the electric current, and $\varphi_n(k)$ is a certain monotonically increasing function of k, lying in the range $n\pi/2 \leq \varphi_n(k) < (2n+1)\pi/4$. For a statistical perturbation that is periodic along the plate with a wave vector $\mathbf{k}$, the maxima of the wave resistance should be observed at current values corresponding to the roots of the equation $\omega_n(\mathbf{k}) = 0$.

In addition to the natural-oscillation spectral modes described above, there exists in the intermediate state one more mode determined by the solution $\varphi \equiv 0$. This can naturally be called the zero mode. The equation $\omega = 0$ for the zero mode has a trivial solution $J = 0$ for all k. This case was not considered by Andreev[1] specially. We shall therefore write down in the next section the calculation results for the distribution of the electromagnetic field and of the plate resistance as $J \to 0$. It turns out that in this case, when the perturbation is turned on, an additional resistance

is likewise produced and reaches a maximum at J = 0. This zero maximum is on a par with the resonant maxima of the wave resistance at $J \neq 0$. The fundamental reason for their appearance, besides the hydrodynamic properties of the intermediate state, is the large Hall angle in pure metals. At the same time, it should be noted that on going from the maxima at $J \neq 0$ to the zero maximum, the resistance increment loses its wave character. This is analogous, to some degree, to the transition from experiments with helicons to observation of the Hall effect in direct current.

In this paper we present the results of observation of the zero maximum of the resistances of pure indium samples. A preliminary communication concerning these experiments has already been published.[4] These experiments were performed on cylindrical samples, for which calculations were not made because of the considerable mathematical difficulties. The comparison of the experimental data with the theory of the zero maximum of resistance, developed for a plate, will therefore be only qualitative in character.

THE ZERO MAXIMUM OF RESISTANCE (THEORY)

We consider a plane-parallel plate of thickness a placed in a constant external field $\mathscr{H} \leq H_c$ perpendicular to the surface of the plate. We introduce a coordinate frame with the x axis along the direct current J flowing in the plate and with a z axis along $\mathscr{H}$ so that on surfaces of the plate are given by $z = \pm a/2$. We consider for simplicity the case of a very pure isotropic metal, in which the off-diagonal component of the conductivity tensor of the normal phase are given by $\sigma_{xy} = -\sigma_{yx} \approx Nec/H_c$, and the diagonal components by $\sigma_{xx} = \sigma_{yy} \approx \sigma_{xy}/\Omega\tau \ll \sigma_{xy}$. Here Ω is the cyclotron frequency of the carriers in the critical field, and τ is the carrier relaxation time. We shall henceforth assume throughout that the temperature T is much lower than the critical temperature and that $J \ll J_c$, where J_c is the critical current producing the field H_c on the surfaces $z = \pm a/2$.

Assume that a constant inhomogeneous magnetic field $\mathscr{H}_1$ is produced on the surface of the plate, such that at $z = \pm a/2$ we have $\mathscr{H}_{1z} = \mathscr{H}_1 e^{ikx}$, $\mathscr{H}_{1y} = 0$, and $\mathscr{H}_{1x}$ is determined by the solution inside the plate, in accord with the continuity of the tangential components of the magnetic field. For the equations of the macroscopic electrodynamics to be valid, we must put $k \ll 1/d$.

In the case of a small perturbation $\mathscr{H}_1 \ll \mathscr{H}$ we use the linearized Andreev formulas.[1] Accurate to terms quadratic in the perturbation, we can write down the expressions for the magnetic induction **B**, for the magnetic field intensity **H**, and for the electric field intensity **E** in the plate:

$$B_z = \mathscr{H} + B_{1z}, \quad B_\alpha = (\mathscr{H}/H_c) H_{1\alpha}; \quad H_\alpha = H_{1\alpha},$$
$$H_z = H_c; \quad E_x = E_{0x} + E_{1x}.$$

Here $E_{0x} = jH_c/Nec\Omega\tau$ is the electric field intensity component at $\mathscr{H}_1 = 0$, the subscript α runs through the values x and y, while the unity subscripts denote small additions that are linear in the perturbation $\mathscr{H}_1$. Under the conditions $\Omega\tau \gg 1$ and $ka \gg 1$ the general formulas for the solution of the equations of [1] for currents $J \ll J_c/ka$ become simpler, and the expressions for the small additions take the form

$$B_{1z} = \left\{1 - \frac{3i\xi}{2(1+i\xi)}\left[1 - \left(\frac{2z}{a}\right)^2\right]\right\} \mathscr{H}_1 e^{ikx},$$
$$H_{1x} = -\frac{6\xi}{1+i\xi}\frac{1}{C_n ka}\left(\frac{2z}{a}\right)\mathscr{H}_1 e^{ikx},$$
$$H_{1y} = -\frac{3i\xi^2}{1+i\xi}\frac{1}{C_n ka\Omega\tau}\left(\frac{2z}{a}\right)\left[1 - \left(\frac{2z}{a}\right)^2\right]\mathscr{H}_1 e^{ikx},$$
$$E_{1x} = \frac{1}{1+i\xi}(\Omega\tau)^2 \frac{\mathscr{H}_1}{\mathscr{H}} E_{0x} e^{ikx}, \qquad (2)$$

where the dimensionless variable $\xi = \Omega\tau kaJ/6J_c$ is used, and the concentration of the normal phase is $C_n = \mathscr{H}/H_c$. The inhomogeneous concentration increment is $C_{ni} = B_{1z}/H_c \approx (\mathscr{H}_1/\mathscr{H})C_n e^{ikx}$. We note that expressions (2) can be valid up to values $\xi > 1$, in view of the assumed large value of the factor $\Omega\tau$.

At J = 0, the tangential components of all the fields vanish, and $B_{1z} = \mathscr{H}_1 e^{ikx}$ does not depend on z. The picture of the force lines in the plate is shown in Fig. 1a. The penetration of the inhomogeneous field to an arbitrarily large depth corresponds to the existence of the zero mode of the natural oscillations in Eq. (1), and is a property peculiar to the intermediate state. It is connected with a simple circumstance, which was already noted by London,[5] namely that the force lines in the intermediate state are straight. We note that in a normal metal a surface perturbation of the form $\mathscr{H}_1 e^{ikx}$ decreases exponentially with increasing depth and has a characteristic penetration depth 1/k.

In very pure superconductors, the picture shown in Fig. 1a is not stable to small currents through the plate. At $J \approx J_c/\Omega\tau ka$, i.e., at $\xi \approx 1$, the distribution of B_{1z} in the interior of the plate is significantly altered. This is illustrated in Fig. 1b. In the central plane of the plate, the distribution of B_{1z} is shifted by a quarter-period along the x axis relative to the initial perturbation on the surface, and has a smaller amplitude.

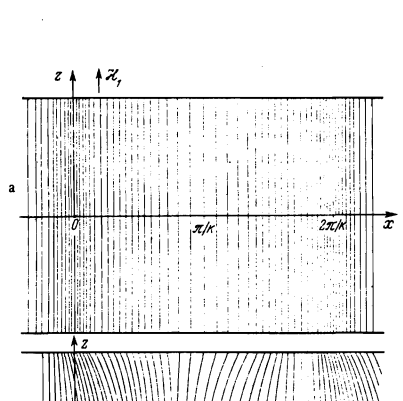

FIG. 1

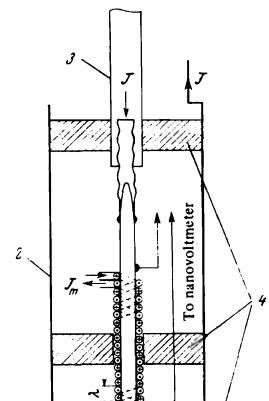

FIG. 2

FIG. 1. Picture of the force lines in a plate with inhomogeneous concentration C_n: a—at J = 0; b—at $J = 6J_c/\Omega\tau ka$. For the sake of clarity, the value of $\mathscr{H}_1/\mathscr{H}$ is exaggerated, while ka is undervalued.

FIG. 2. Diagram of apparatus: 1—sample, 2—copper vessel, 3—tube for fastening to the cover of the apparatus, 4—Plexiglas rings. The secondary details are not shown. The turns of the bifilar coil are shown not to scale.

When measuring the voltage drop along the x axis over a length $L \gg 1/k$, the oscillating quantity E_{1x} does not make a noticeable contribution, and account must be taken of the electric-field increment E_{2x} that is quadratic in the perturbation. According to Andreev,[1] the coordinate-independent E_{2x} is given by

$$E_{2x} = -\frac{1}{8\pi Nea}\,\mathrm{Re}\int_{-a/2}^{a/2} B_{1z}^{*}\left(\frac{dH_{1z}}{dz}\right)dz = \frac{(\Omega\tau)^2(\mathcal{H}_1/\mathcal{H})^2}{2(1+\xi^2)}E_{0x}.$$

This expression means that when the perturbation is turned on the plate resistance R in the intermediate state increases by the amount

$$\tilde{R} = \frac{R}{1+\xi^2}\left(\frac{\mathcal{H}_1}{H_c}\right)^2\frac{(\Omega\tau)^2}{2C_n^2}. \qquad (3)$$

Equation (3) can also be derived formally from Eqs. (29) and (30) of Andreev's paper[1] by taking the limit as $J \to 0$.

The maximum value $\tilde{R} = \tilde{R}_m$ is reached at J = 0. In perfect single crystals the factor $\Omega\tau$ can be very large, so that $\tilde{R}_m$ turns out to be larger than R even for small perturbations. The width of the zero maximum of R, determined by the condition $\tilde{R} = \tilde{R}_m/2$, is equal to $J_{1/2} = 6J_c/\Omega\tau ka$. We note that the position of the first resonance of the wave resistance, in the current scale, is $J_1 \approx J_c/ka \gg J_{1/2}$.

EXPERIMENTAL PROCEDURE

The measurements were made on cylindrical single crystals grown from ultrapure indium. The sample length was approximately 100 mm and the diameter D = 4 mm. The main results were obtained with two samples. Measurement of the damping of the free helicon oscillations, by the method described in [6], yielded $\Omega\tau = 3.4$ for sample 1 and $\Omega\tau = 3.3$ for sample 2. The samples were placed in a homogeneous magnetic field $\mathcal{H}$ perpendicular to the cylinder axis, and immersed in liquid helium whose temperature was lowered to 1.2° K.

The low-temperature part of the apparatus is shown schematically in Fig. 2. The sample was soldered with Wood's alloy to current leads made up of several thin lead wires, to avoid appreciable mechanical stresses in the sample during cooling. The current was fed to the lower end of the sample through a vessel made of thin copper foil and placed coaxially with the sample, so that no additional magnetic fields be produced in the sample by the current leads. The leads delivered a current J up to 100 A without noticeable heat release near the sample. The central part of the sample was placed inside a bifilar coil. This coil produced a homogeneous field proportional to the current J_m through the coil. The bifilar coil was made of superconducting niobium-zirconium alloy wire of 0.34 mm diameter. In the experiments we used coils wound in various manners, so that the spatial period of the field $\mathcal{H}_1$ could be varied in the range $\lambda \approx 1$ to 3 mm. The gap between the turns and the surface of the sample was usually 0.3 mm, corresponding to $\mathcal{H}_1 \approx 30$ Oe on the sample surface in the normal state at $J_m = 10$ A. The superconducting wire should distort the homogeneous external field even at $J_m = 0$. This effect appears to have been quite small and to have no noticeable influence on the results. The length of the bifilar winding was $L \approx 60$ mm.

Near the end of the winding, potential leads were soldered to the sample with Wood's alloy. The voltage U on the contacts was measured with an F-118 nanovoltmeter having a sensitivity 10^{-9} V/div. Unfortunately, the inhomogeneities of the copper wire lead to large thermoelectric powers in the measurement circuit, thus preventing us from making full use of this sensitivity. The error in the measurement of U as a result of the changes of the thermoelectric power and the instability of the null of the F-118 usually amounted to several nanovolts. The sample, the bifilar coil, and the Helmholtz system used to produce the homogeneous field were fed from stabilized sources, and all the currents were measured with accuracy ≈ 1.0%, so that the error in the measurement of the resistance sample was determined mainly by the error in the measurement of U. The nanovoltmeter readings were registered either by using an indicator with a pointer, or plotted with an automatic x-y recorder, the sweep along the abscissa axis being effected with a signal proportional to J.

The quality of the samples could be monitored directly during the course of the experiment, by observing the dependence of the sample resistance R in the intermediate state as a function of the normal-phase concentration C_n. For a long cylinder in a transverse field, we have $C_n = 2\mathcal{H}/H_c - 1$. According to the theory (see, e.g., [1]), for bulky samples with thickness much larger than the electron mean free path l we have $R = C_nR_0$, where R_0 is the resistance of the sample in the normal state. For samples whose diameter is comparable with l, this simple relation no longer holds. The point is that in the normal state an additional contribution is made to the resistance by the electron scattering from the sample surface. Guiding ourselves by Nordheim's simple rule, we can assume that R_0 increases in proportion to $1 + l/D$. In the intermediate state, the electrons move inside the normal layers and do not collide with the sample surface. By virtue of the distinctive character of the (Andreev) reflection, multiple reflection of electrons inside the n-layer makes no contribution to the electric resistance, so that R in the intermediate state is determined by the conductivity of the bulk metal. The net result is that the dependence of R on the concentration ceases to be linear.

Figure 3 shows the results of the measurement of this dependence for sample 2. The slope of the straight line at small C_n corresponds to the value of l in bulky metal. With increasing C_n, the fraction of the electrons colliding with the sample surface increases. In the normal state, the resistance corresponds to a decreased mean free path, $1/l_{eff} = 1/l + 1/D$. Starting with this relation we can estimate from the data of Fig. 3 that $l \approx 1.4$ mm, which corresponds, for medium values of the effective mass and of the electron velocity

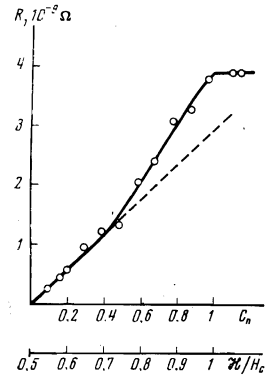

FIG. 3. Dependence of the resistance of sample 2 in the intermediate state on the concentration of the normal phase. T = 1.3°K, J ≤ 10 A. The dashed line is an extrapolation of the initial section of the experimental curve.

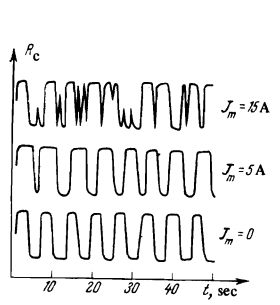

FIG. 4

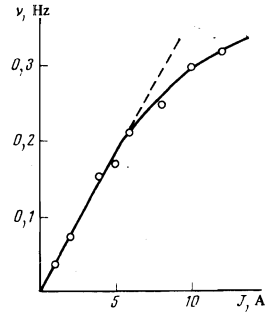

FIG. 5

FIG. 4. Plots of the resistance R_c of the microcontact (arbitrary units) against the time t. The curves are arbitrarily shifted in the vertical direction. Sample 2, T = 1.2°K, J = 5 A, C_n = 0.27, λ = 1.36 mm.

FIG. 5. Frequency ν of the oscillations of R_c as a function of the current through sample 2; J_m = 0, C_n = 0.27, T = 1.2°K.

(see [6]), to $\Omega\tau \approx 2.4$. The deviation from the data obtained in other experiments[6] using the same sample can be attributed to a deterioration of the sample quality when the latter was mounted in the apparatus.

Additional experiments were performed to check that there was no pinning of the n–s boundaries by crystal defects. To this end, a needle made of copper wire of 0.05 mm diameter was used to connect the turns at the center part of the bifilar coil to the sample surface. A special measurement circuit was used to register the change of the resistance of this microcontact during the passage of the n and s layers (for details of this procedure see [7]). A typical plot of the resistance oscillations of this microcontact when current is passed through the sample is shown in Fig. 4. These oscillations are due to the passage of the layers past the needle. The frequency ν of the oscillations is proportional to the speed of the layers. Figure 5 shows a plot of the experimentally observed ν against the current through the sample. At small current it is possible to draw through the experimental points a straight line that passes through the origin, thus indicating that there is no dry friction in the motion of the layers, i.e., the n–s boundaries move freely. Using the data (see, e.g., [7]) on the nature of the structure of the intermediate state, viz., d = 0.3 mm ($C_n \approx 0.3$) in indium cylinders (D = 4 mm) we obtain at j = 1 A/cm^2 a layer velocity v = 1.4×10^{-4} cm/sec. This is close to the theoretical v = 1.6×10^{-4} cm/sec at $N \approx 4 \times 10^{22}$ cm^{-3}.

At currents $J \gtrsim 0.03 J_c$, as already noted by Sharvin and Landau,[7] the frequency ν increases with increasing J much more slowly, apparently by virtue of the influence of the magnetic field of the current on the structure of the n and s layers.

EXPERIMENTAL RESULTS AND DISCUSSION

At a fixed current through the sample, turning on the current in the bifilar coil leads to an increase ΔU of the voltage on the sample in the intermediate state. This effect appears neither in the normal nor in the superconducting state. At the same time, in the intermediate state the effect can be large enough to cause the voltage on the sample to be several times larger than the voltage in the normal state at the same value of the current J. Figure 6 shows the measured values of the additional resistance $\tilde{R} = \Delta U/J$ on sample 2 as a function of the current through the sample. At currents J $\lesssim$ 1 A, the error in the measurement of ΔU becomes comparable with the voltage itself, so that the corresponding points on the plot of Fig. 5 have a large scatter. Nonetheless, at larger currents we can state with assurance that the resistance of the sample increases by more than 10 times when the inhomogeneous field is turned on.

Formula (3), which describes the $\tilde{R}(J)$ dependence, was derived for a plate. For a cylindrical sample one should expect the same regularity, but the expressions for the amplitude and width of the maximum should now contain numerical factors of the order of unity. The determination of these factors for the solid theoretical curve on Fig. 6 was carried out by using the two experimental points marked by the double circles.

A comparison of the experimental results with expression (3) at

$$ka = 2\pi D/\lambda \approx 20, \quad \Omega\tau \approx 3 \quad \text{and} \quad J_c = 240 \text{ A}$$

shows that both the magnitude and the width of the maximum of the experimental $\tilde{R}(J)$ plot shown in Fig. 6 are of the correct order of magnitude. The experimental points for C_n = 0.2 lie much higher than the theoretical curve at J > 10 A. This is not surprising, if account is taken of the strong dependence of the form of the $\tilde{R}(J)$ curve on the distribution of the field $\mathcal{H}_1$ along the sample axis. Under the conditions of these experiments, the distribution of the inhomogeneous field $\mathcal{H}_1$ on the surface of the sample differs from the simple sinusoidal distribution assumed in the derivation of (3). In addition, the distribution of the field H_1 on the surface should depend on the value of the current J.

In principle we can expect the resistance of the sample in the intermediate state to increase by $(\Omega\tau)^2$ times if the n and s layers are very strongly pinned by defects. If the layers that lie perpendicular to the cylinder axis are fixed somehow in this position, then the Hall component of the electric field, $E_{0y} = \Omega\tau E_{0x}$, turns out to be parallel to the n–s boundary. In order for the electric-field component tangent to the boundary of the superconducting phase to vanish, as called for by the continuity conditions, current must flow in the normal layers along the y axis, with density $j_y = -\Omega\tau j$ if the current density $j_x = j$ is assumed to be fixed. By virtue of the Hall effect, these currents will be in turn accompanied by the appearance of an electric field $E'_x = \Omega\tau E_{0y} = (\Omega\tau)^2 E_{0x}$, along the sample axis, thus corresponding to an increase of the resistance.

One should expect the layers to break away from

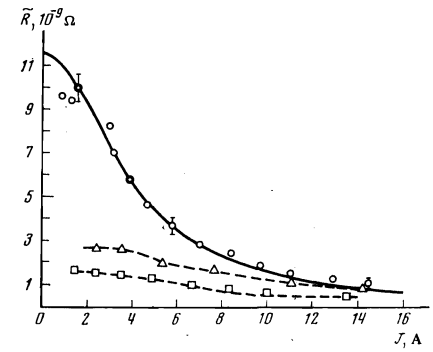

FIG. 6. Measured values of R of sample 2, T = 1.3°K, J_m = 15 A, λ = 1.36 mm; ○–C_n = 0.2; △–C_n = 0.5; □–C_n = 0.8. Solid line–theoretical curve drawn through the two points marked by the double circles.

the pinning centers with increasing current through the sample, and the resistance to return to its initial value. In practice it is difficult to expect the layer pinning to be strong enough to produce between the phases immobile boundaries that are perpendicular to the sample axis. To the contrary, as shown by experiments aimed at observing the oscillations of the microcontact resistance, when the perturbation is turned on the layers move freely along the sample axis already at currents $J \geq 1$ A. Moreover, the layer motion can be observed directly during the measurement of the additional resistance. Figure 4 shows plots of the microcontact resistance following application of the perturbation. We see that the file of the current J_m influences the structure of the intermediate state, and in particular, in the case of a strong perturbation, the structure becomes much less regular than at $J_m = 0$.

Nonetheless, in the presence of the perturbation H_1 one should expect excitation of currents along the y axis, and this is of importance for the understanding of the microscopic mechanism of the onset of the additional resistance[1)] $\tilde{R}$. Imagine that at $J = 0$ the layers are along the force lines of **B**, as shown in Fig. 1a. When the current through the sample is turned on, the layers start moving with an average velocity $v = j/Ne$. However, the velocity of the boundaries between the n and s phases will differ from the average layer velocity, by virtue of the change of the concentration C_{nl} along the sample axis. The deviation of the velocity of the n–s boundary from the value j/Ne means the appearance of a tangential component of the electric field. In order that the tangential electric field be equal to zero, as before, on the boundary with the superconting phase, currents j_y will appear in the normal phase. The Joule loss, proportional to j_y^2, causes the additional resistance $\tilde{R}$.

The stationary state is established in the metal at $\mathcal{H} < H_0$ and $J \ll J_c$ with a large time constant. At currents $J \approx 2$ A, when the perturbation $\mathcal{H}_1$ is turned on, the stationary value of ΔU is reached after a time exceeding 10 sec. When the current-voltage characteristic was recorded at constant J_m, a delay of $\Delta U(J)$ relative to its stationary value was also observed, so that the increase or decrease of R can be obtained directly from current-voltage characteristics plotted with the current decreased or increased, respectively.[2)] These time dependences offer an additional argument against the hypothesis that the resistance is increased because the layers are pinned.

The value of $\tilde{R}$ depends strongly on the applied perturbation. Figure 7 shows the experimental results concerning the dependence of $\tilde{R}$ on $\mathcal{H}_1$. At $\mathcal{H}_1 \ll \mathcal{H}$, in accord with (3), we have $\tilde{R} \sim \mathcal{H}_1^2$, and the width $J_{1/2}$ of the maximum is independent of $\mathcal{H}_1$. At $\mathcal{H}_1/\mathcal{H}C_n \approx 1$, the maximum value $\tilde{R} = \tilde{R}_m$ ceases to increase with increase of $\mathcal{H}_1$. Simultaneously with this saturation of the zero maximum, an increase is observed in its width with further increase of the perturbation.

The additional resistance $\tilde{R}$ decreases rapidly with rising temperature, and at $T \gtrsim 2.6°$ K it becomes less than the error in the measurement of the ordinary resistance $\tilde{R}$. Experiments on the temperature dependence of $\tilde{R}$ were performed on sample 1 at fixed values of the concentration (0.2 and 0.5) and of the perturbation $\mathcal{H}_1/H_c$. According to formula (3), $\tilde{R}_m$ depends under these conditions on the temperature by virtue of the change of $\Omega \sim H_c$ and of the relaxation time τ. The value of R also varies with temperature like $\sim 1/\tau$. Thus, the combination $\tilde{R}_m R/H_c^2$ of the measured quantities should not depend on the temperature. Experiments have shown that in the range $T = 1.2$ to $2.6°$ K we have $\tilde{R}_m R/H_c^2 = \text{const}$ with accuracy 10%, although $\tilde{R}_m$ changes by more than one order of magnitude. As follows from the theory, the width of the zero maximum of $\tilde{R}$ should also depend on the temperature. However, the entire temperature dependence in the expression $J_{1/2} \sim J_c/\Omega\tau$ is determined only by the factor $1/\tau$, since $J_c \sim \Omega \sim H_c$.

In the investigated temperature range, as follows from the measurements of R, the value of τ changed by approximately a factor 1.5. At the same time, owing to the rapid decrease of the signal, the error in the measurement of the width of the zero maximum of R at relatively high temperature exceeded 50%. This has prevented a detailed comparison of the results on the width of the zero maximum with the theory, but within the limits of experimental error we can state that formula (3) describes well the observed temperature dependence of $\tilde{R}$.

We turn now to the data on the dependence of $\tilde{R}$ on the concentration of the normal phase. As shown by measurements performed more accurately than in the preceding study,[4] the maximum increment to the resistance at constant J_m and constant T is given approximately by the formula $\tilde{R}_m \sim 1/C_n$ in the concentration range $C_n = 0.2 - 0.8$, in accord with formula (3), if it is assumed that $R \sim C_n$. Outside this range of C_n, a decrease of R_m takes place, probably because of the increase of the period d of the structure and because of the violation of the macroscopic-electrodynamics condition $k \ll 1/d$. It should be noted that the indicated agreement with the theory is to a certain degree accidental, inasmuch as generally speaking the condition $J_m = \text{const}$ does not ensure that $\mathcal{H}_1/Hc$ is constant when the concentration C_n is varied. Moreover, as seen from Fig. 5, an appreciable broadening of the zero maximum of $\tilde{R}$ is observed with increasing concentration, thus utterly contradicting the theory, where $J_{1/2}$ is independent of C_n. It appears that the rather artificial assumptions concerning the distribution of the perturbing field $\mathcal{H}_1$ on the surface of the sample, made to simplify the derivation of (3), cease to be

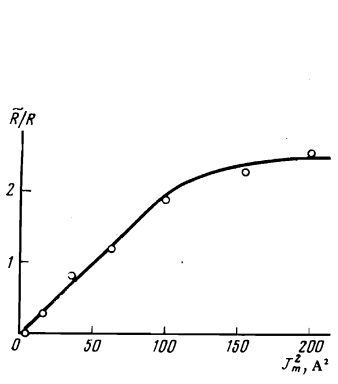

FIG. 7

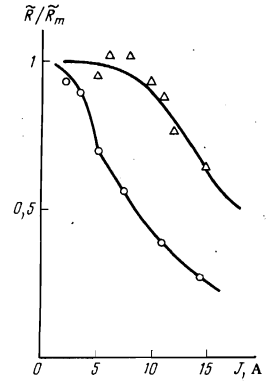

FIG. 8

FIG. 7. Dependence of $\tilde{R}(J_m)$ at $J = 3$ A. Sample 1, $T = 1.3°$K, $C_n = 0.2$.

FIG. 8. $\tilde{R}/\tilde{R}_m$ as a function of the current J for two periods of the perturbation. ○–$\lambda = 1.36$ mm, □–$\lambda = 3$ mm. Sample 2, $T = 1.3°$K, $C_n = 0.5$, $J_m = 15$ A.

justified when it comes to describing the dependence of $\tilde{R}$ on the normal-phase concentration.

At the same time, the theory describes quite well the dependence of the width of the zero maximum on the period of the perturbation. Figure 8 shows data on the variation of the ratio of $\tilde{R}$ to the maximum value $\tilde{R}_m$ under such a perturbation, before the broadening of the zero maximum due to the large value of $\mathscr{H}_1$ is observed. It is seen that the experimentally determined $J_{1/2} \sim 1/\lambda$ agrees with the theory.

Indium is an anisotropic metal with a complex Fermi surface, so that the expressions for the components of the conductivity tensor should differ from the simplified mean values assumed at the start of the article. In the intermediate state, however, as shown by various experiments, in which the averaged galvanomagnetic characteristics of indium played a role (see, e.g., [6,7]), these deviations are quite small, and the observed effects are practically isotropic. The same can be said concerning the additional resistance in the case of inhomogeneous concentration. Of course, it is impossible to compare the magnitude of the effect on two samples at different crystalline orientation of J, without monitoring the value of $\mathscr{H}_1$ on the sample surface. However, during the course of one experiment we plotted for each of the samples the function $\tilde{R}(J)$ at different orientations of the external field $\mathscr{H}$ in a plane perpendicular to the cylinder axis. $\tilde{R}$ was independent of the direction of H within the ≈5% experimental error.

CONCLUSION

In the described experiment, in the case $J \to 0$, an increase of the resistance of the superconductor in the intermediate state was observed when an inhomogeneous concentration of the normal phase was produced. The qualitative agreement with a theory based on the general equations of macroscopic electrodynamics of the intermediate state leaves no doubts concerning the nature of this effect, which is due to the Hall effect in conjunction with the hydrodynamic properties of the flow of the normal and superconducting domains. The observed phenomenon is very close in its nature to the resonant increase of the wave resistance, which was theoretically considered by Andreev, although it has no obvious wave or resonant character. During the course of our experiments we were unable to observe resonances of the wave resistance up to currents J ≈ 80 A, and probably because the natural frequencies of the helicon oscillations in the cylinder are high enough, and the positions of the resonant maxima are shifted into the region $J \approx J_c$, where the initial equations are no longer valid.

The increase of the resistance $\tilde{R}$ becomes manifest only in very pure metals, and at a controllable value of the inhomogeneity of the concentration C_n it can serve as a measure of the purity and perfection of the crystal. From the point of view of an investigation of the galvanomagnetic properties, measurements of $\tilde{R}$ can yield, in principle, the same information as measurement of the Hall angle.

An interesting possibility is uncovered by measurement of the resistance in the presence of an alternating perturbation of frequency ω, moving along the sample axis with velocity $v = \omega/k$. In this case, according to (1), the zero maximum of R should be observed at $J \neq 0$. Measurement of the current density corresponding to the maximum of R, at a known value of v, makes it possible to determine the sign and concentration of the carriers.

The author is sincerely grateful to A. F. Andreev, I. L. Landau, and Yu. V. Sharvin for a discussion of the results and for useful discussions, to L. M. Shpel'ter for preparing the high-grade samples, and to R. K. Nilolaev and A. D. Bronnikov of the Institute of Solid State Physics for supplying the ultrapure indium.

[1)]This remark is due to I. L. Landau.

[2)]An example of a current-voltage characteristic recorded too rapidly and corresponding to decreased values of $\tilde{R}$ at $J \lesssim 3$ A was given earlier. [4]

[1]A. F. Andreev, Zh. Eksp. Teor. Fiz. **52**, 1106 (1967) [Sov. Phys.-JETP **25**, 735 (1967)].
[2]Yu. V. Sharvin, ZhETF Pis. Red. **2**, 287 (1965) [JETP Lett. **2**, 183 (1965)].
[3]A. F. Andreev and Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. **53**, 1499 (1967) [Sov. Phys.-JETP **26**, 865 (1968)].
[4]I. P. Krylov, ZhETF Pis. Red. **19**, 726 (1974) [JETP Lett. **19**, 375 (1974)].
[5]F. London, Superfluids, **1**, Dover Publications, N. Y., 1961.
[6]I. P. Krylov, I. L. Bronevoi, and Yu. V. Sharvin, ZhETF Pis. Red. **19**, 588 (1974) [JETP Lett. **19**, 306 (1974)].
[7]Yu. V. Sharvin and I. L. Landau, Zh. Eksp. Teor. Fiz. **58**, 1943 (1970) [Sov. Phys.-JETP **31**, 1047 (1970)].

Translated by J. G. Adashko
170