Electron-electron collisions in an ultra-quantizing magnetic field

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In an ultra-quantizing magnetic field, when almost all electrons are in the lowest Landau level, the rate of redistribution of energy between the electrons in binary collisions is negligible. In these conditions the energy redistribution occurs in more complicated processes than binary collisions (in three-body collisions of electrons and in binary collisions in the field of an impurity). The collision integral associated with electron-electron scattering in the field of charged impurities is calculated. In the limit of strong magnetic fields the characteristic time of the energy redistribution between the electrons does not depend on the magnetic field and is equal in order of magnitude to $\tau_{ee}(H) \approx \tau_{ee}(0)(n/N_i)(\kappa T/n^{1/3}e^2)^{3/2}$, where $\tau_{ee}(0)$ is the energy-redistribution time in the absence of a magnetic field, and n and N_i are the concentrations of electrons and charged impurities. An estimate of $\tau_{ee}(H)$ for *n*–InSb at liquid-helium temperatures shows that $\tau_{ee}(H)$ is considerably shorter than the time for scattering of energy by phonons. This agrees with the experimentally observed fact that in *n*–InSb electrons situated in a quantizing magnetic field and a "warming" electric field has an effective temperature, and removes the previous contradiction between theory and experiment.

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1. INTRODUCTION

It is well-known (cf., e.g., [1]) that if the electron-electron collision time τ_{ee} is shorter than the time τ_e for scattering of energy by the lattice vibrations, the energy distribution function $f_0(\epsilon)$ of electrons in a strong electric field is close, to within terms of order τ_{ee}/τ_e, to an equilibrium (Maxwellian or Fermi) function with a certain effective temperature T. On the form of $f_0(\epsilon)$, i.e., on whether or not the electron-temperature approximation is valid, depends, in particular, the onset of the superheating instability [1-3].

To estimate the rate of redistribution of energy in an electron gas as a result of the electron-electron interaction it is usually sufficient to calculate the collision integral in the approximation of binary collisions between free electrons. In an ultra-quantizing magnetic field, however, in this approximation there is practically no exchange of energy between the electrons, since, owing to the energy and momentum conservation laws, the collision integral for collisions between electrons situated in the lowest Landau level vanishes identically. If the electron temperature is not equal to zero, energy exchange between electrons in binary collisions turns out to be possible only in collisions of electrons situated in different Landau levels N; in this case, however, the collision integral for binary collisions in an ultra-quantizing magnetic field is proportional to the concentration of electrons in the level N = 1, i.e., contains an exponentially small factor $\exp(-\hbar\omega_H/T) \ll 1$, where ω_H is the cyclotron frequency [4,5]. This means that to estimate the rate of energy redistribution in such an electron gas it is necessary to calculate the collision integral with an accuracy greater than that provided by the ordinary binary-collision approximation.

As is well-known, refinement of the collision integral leads to the successive consideration of scattering processes of higher and higher order: three-particle collisions, four-particle collisions, etc. It is easy to convince oneself that in a strong magnetic field the conservation laws permit redistribution of energy between electrons with the same N, e.g., in the collision of three electrons or in the simultaneous scattering of electrons by each other and by a charged impurity center. In addition, scattering processes which make no contribution to the energy exchange in the free-electron approximation turn out to be important if we take into account the quasi-particle character of the excitations in the electron gas. Below it will be shown that when the damping is taken into account binary collisions of quasi-particles and scattering of quasi-particles by impurities play as important a role as three-particle collisions in the energy redistribution.

In Sec. 2 we obtain the collision integral for electron-electron collisions in an ultra-quantizing magnetic field with allowance for the interaction of the electrons with charged impurities (i.e., with three particles taking part), and in the last Section the characteristic time of the redistribution of energy between the electrons is calculated. As regards triple collisions between electrons, it follows from simple physical considerations that, firstly, for equal concentrations of electrons (n) and charged impurities (N_i) their contribution to the rate of energy redistribution is of the same order as that of the processes considered. Secondly, for strong compensation ($N_i \gg n$) the contribution of triple collisions should be smaller.

At the present time, experimental facts are known [6] on the basis of which we can conclude that in n-InSb at liquid-helium temperatures and in magnetic fields ~ 10 kOe, when $\exp(-\hbar\omega_H/T) \sim 10^{-10}$, an effective electron temperature is established in the electron gas with concentration $n \sim 10^{14}\ \text{cm}^{-3}$. This means that the time of the redistribution of energy between the electrons in these conditions is considerably shorter than the time $\tau_e \sim 10^{-8}$–10^{-7} sec for scattering of energy by the phonons, although the time calculated from the formulas of Zlobin and Zyryanov [4] is $\tau_{ee} \sim 1$ sec. Below it will be shown that τ_{ee} calculated with three-particle collisions taken into account is indeed shorter than τ_e, and this enables us to explain the experimental data.

We note that an electron gas in an ultra-quantizing magnetic field is apparently the only real system for which scattering processes with three bodies taking part determine the gas kinetics.

2. DERIVATION OF THE COLLISION INTEGRAL

We shall consider a spatially-uniform gas of electrons, interacting with each other (with interaction energy $v_{ee}(\mathbf{r})$) and with charged impurities (with interaction energy $v_{ei}(\mathbf{r})$), situated in an ultra-quantizing magnetic field. To derive the collision integral we shall make use of the method of matrix Green functions, which was developed in its clearest form by Keldysh[7] (see also the review by DuBois[8]). The idea of Keldysh's method consists in the feature that the second-quantization operators in the Green functions are ordered on a double time contour consisting of two axes: one goes from $-\infty$ to $+\infty$, and the second from $+\infty$ to $-\infty$. The time on the second axis is considered to be greater (in the sense of the ordering of the operators) than any time on the first axis, and the operators on the second axis are arranged in anti-chronological order (i.e., the operator with the smallest time, closest to $-\infty$, is positioned to the left). In accordance with the fact that the times of each of the two operators in the Green function can be either on the first or on the second axis of the contour, four functions are possible, forming the matrix:

$$G^{ik}(11')=i^{-1}\langle T\psi(1_i)\psi^+(1_k')\rangle,$$

where 1 and 1′ denote the set of spatial coordinates and the time, $1 = (\mathbf{1}, t)$ (the spin is fixed by the magnetic field), and i and k are the indices of the time axes. The angular brackets denote averaging with a density matrix taken at the time t_0 at which the Heisenberg operators coincide with the Schrödinger operators, and also denote averaging over the positions of the impurity centers. We shall assume that the impurities are positioned in the crystal in an uncorrelated manner (randomly), with concentration N_i.

For the Green function the equation[7]

$$(\sigma_z)_{ij}G_0^{-1}(1)G^{jk}(11')-\Sigma^{ij}(12)G^{jk}(21')=\delta_{ik}\delta(1-1'), \tag{1}$$

is valid, where the operator $G_0^{-1}(1) = i\partial/\partial t_1 - H_0(\mathbf{1})$, $H_0(\mathbf{1})$ is the one-electron Hamiltonian, σ_z is the Pauli matrix, and $\Sigma^{ij}(12)$ is the mass operator, which, like the Green function, is a matrix.

We shall denote the eigenfunctions of H_0 in the Landau gauge, normalized to unity in the finite volume V, by $\varphi_{Np_zp_y}(\mathbf{r})$ and suppose that the system under consideration possesses axial symmetry about the magnetic-field direction. Then the Green functions in the (N, p_z, p_y)-representation are diagonal in all the quantum numbers and do not depend on p_y[9]. The distribution function is equal to

$$f(N,p_z,t)=i^{-1}\int d1\int d2\,\varphi^*_{Np_zp_y}(1)\varphi_{Np_zp_y}(2)G^{12}(1t;2t). \tag{2}$$

In the following we shall be concerned only with states in the lowest Landau level N = 0 and shall denote $f(0, p_z, t)$ by $f(p_z, t)$.

The kinetic equation for the distribution function in the semi-classical limit was derived by Keldysh from Eqs. (1) for the Green functions. Proceeding analogously, we find that in an ultra-quantizing magnetic field the collision integral in the equation for $f(p_z, t)$ is equal to

$$S(p_z)=\int\limits_{-\infty}^{+\infty}d\tau[G^{21}(p_z,t,\tau)\Sigma^{12}(p_z,t,-\tau)-G^{12}(p_z,t,-\tau)\Sigma^{21}(p_z,t,\tau)]. \tag{3}$$

In this expression we have made a change of variables from t_1 and t_2 to $t = (t_1 + t_2)/2$, $\tau = t_1 - t_2$ in the functions $G^{ik}(t_1t_2)$ and $\Sigma^{ik}(t_1t_2)$.

In order to obtain the relation between the matrices Σ and G for the case when the electrons interact with each other and with impurities, proceeding in the spirit of Kadanoff and Baym[10] and DuBois[8] we turn to the functional-derivative equations for the Green functions averaged over the positions of the impurities. The averaging method used in the derivation (see the Appendix) is due to Suris[11]:

$$\begin{aligned}&G_0^{-1}(12|U)G(21'|U)-N_i\int d^3\mathbf{R}\,v_{ei}(1\mathbf{R})G(11'|U)\\&+i\int d3\,\mathbf{v}_{ee}(13)\sigma_zG(33)G(11')-i\int d3\,\mathbf{v}_{ee}(13)\frac{\delta G(11')}{\delta U(3)}\\&-N_i\int d^3\mathbf{R}\,v_{ei}(1\mathbf{R})\sigma_z\left[\exp\left(\int d3\,v_{ei}(3\mathbf{R})\frac{\delta}{\delta U(3)}\right)-1\right]G(11')\\&+i\int d3\,\mathbf{v}_{ee}(13)\lim_{V\to U}\left\{\exp\left[N_i\int d^3\mathbf{R}\left[\exp\left(\int d4\,v_{ei}(4\mathbf{R})\frac{\delta}{\delta V(4)}\right)-1\right]\right.\right.\\&\times\left.\left.\left[\exp\left(\int d5\,v_{ei}(5\mathbf{R})\frac{\delta}{\delta U(5)}\right)-1\right]\right]-1\right\}\sigma_zG(33|V)G(11'|U)=\mathbf{I}\delta(1-1').\end{aligned} \tag{4}$$

The Green function in this equation is a functional of the classical field U, which depends on the coordinates and time and also on the time-axis index (in the final formulas we must put V = U = 0). The other symbols in (4) have the form

$$G_0^{-1}(12|U)=\sigma_z\delta(1-2)G_0^{-1}(1)-\mathcal{T}^{(0)}(12;3)U(3),$$

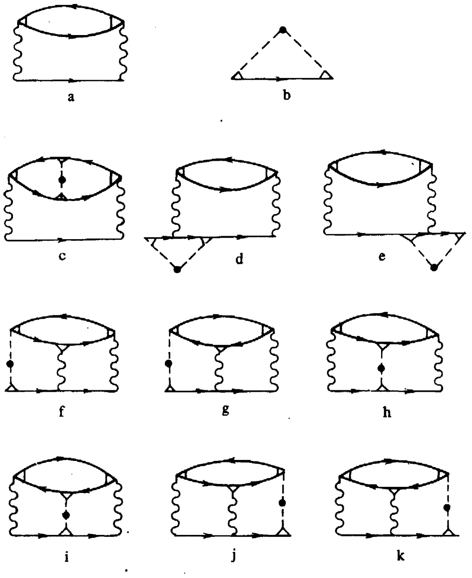
$$\mathcal{T}^{(0)}_{ijk}(12;3)=(\sigma_z)_{ij}\delta_{jk}\delta(1-2)\delta(2-3),$$

$$[\mathbf{v}_{ee}(13)]_{i_1i_3}=v_{ee}(1-3)\delta(t_1-t_3)\delta_{i_1i_3},\quad \mathbf{I}_{ij}=\delta_{ij}.$$

We shall consider a nondegenerate gas of low density. We shall iterate (4) successively, assuming v_{ee} and v_{ei} to be small, and thus obtain an expression for Σ in the form of an expansion in powers of the interactions. With each term in Σ we can associate a diagram. Those diagrams in Σ^{12} and Σ^{21} that we shall need (see below) are depicted in the figure. Each solid line is associated, as usual, with an exact Green function, each triangular vertex is associated with an elementary vertex $\mathcal{T}^{(0)}$, each wavy line corresponds to iv_{ee} and the dashed lines with a blob correspond to the integral

$$N_i\int d^3R\,v_{ei}(1\mathbf{R})v_{ei}(2\mathbf{R})$$

(here, as in (4), a summation over the types of impurity is performed). It must be borne in mind that repeated internal variables are not only integrated over, but also summed over (over the indices of the time axes).



The figure does not include diagrams corresponding to exchange processes and three-electron collisions. It can be shown that, in the problem under consideration, the former give a small contribution as compared with the graphs which we do not take into account, we do not calculate the latter in the present paper.

The diagrams for Σ can be divided into two types. The graphs a and b (see the figure), which correspond to binary collisions of quasi-particles, give a contribution to the collision integral (3) only when the quasi-particle damping due to interaction with other electrons or impurities is taken into account. The corresponding corrections to the functions $G^{12(21)}$ appearing in $\Sigma^{12(21)}$ in Figs. a and b will be found in the approximation linear in the damping by solving the equation for $G^{12(21)}(p_z, t, \tau)$ as functions of the time difference $\tau = t_1 - t_2$. This equation is obtained by subtracting from (1) the equation conjugate to it (cf. [7])

$$\left(i\frac{\partial}{\partial\tau}-\varepsilon(p_z)\right)G^{12}(p_z,t,\tau)-\frac{1}{2}\int d\tau'[\Sigma^R(\tau-\tau')+\Sigma^A(\tau-\tau')]G^{12}(\tau')$$
$$=\frac{1}{2}\int d\tau'\Sigma^{12}(\tau-\tau')[G^R(\tau')+G^A(\tau')]. \tag{5}$$

Here $\epsilon(p_z) = p_z^2/2m$, m is the effective mass, the p_z and t dependence of all the quantities is written out only in the first term, and the functions $G^{R(A)}$ and $\Sigma^{R(A)}$ are related to elements of the matrices G and Σ by the equalities [7,12]

$$G^{R(A)}=G^{11}-G^{12(21)}=-G^{22}+G^{21(12)},$$
$$\Sigma^{R(A)}=\Sigma^{11}+\Sigma^{12(21)}=-\Sigma^{22}-\Sigma^{21(12)}. \tag{6}$$

The equation for G^{21} has a form analogous to the expression (5) (with the replacement $12 \rightleftarrows 21$). We write the equation for $G^{R(A)}$ as

$$\left(i\frac{\partial}{\partial\tau}-\varepsilon(p_z)\right)G^{R(A)}(\tau)-\int d\tau'\,\Sigma^{R(A)}(\tau-\tau')G^{R(A)}(\tau')=\delta(\tau).$$

We denote by $E(p_z)$ the root of the equation

$$z-\varepsilon(p_z)-{}^1/_2[\Sigma^R(p_z,t,z)+\Sigma^A(p_z,t,z)]=0,$$

where $\Sigma^{R(A)}(z)$ is the Fourier transform of $\Sigma^{R(A)}(\tau)$. In the first approximation, $E(p_z)$ is real, and if we discard the right-hand side in (5), then, as a function of τ, $G^{12}(p_z, p, \tau)$ oscillates with frequency $E(p_z)$ without damping:

$$G^{12}(p_z,t,\tau)=G^{12}(p_z,t,0)\exp[-iE(p_z)\tau], \tag{7}$$

where $G^{12}(p_z, t, 0)$ is related directly to the distribution function $f(p_z, t)$ by the equality (2).

If in the collision integral (3) we approximate Σ by the graphs of Figs. a and b and substitute all Green functions in the approximation (7), we obtain in the collision integral terms describing, respectively, binary collisions between free electrons and ordinary scattering of electrons by impurities. The first term, as was stated above, vanishes in an ultra-quantizing magnetic field [4], while the second does not affect the redistribution of energy between the electrons.

We write out the expression for $G^{12}(\tau)$ to first order in the quantity on the right-hand side of (5):

$$G^{12}(\tau)=G^{12}(0)\exp[-iE(p_z)\tau]$$
$$+i\int_0^\tau d\tau'\exp[-iE(p_z)(\tau-\tau')]\int_{-\infty}^{+\infty}d\tau''\,\Sigma^{12}(\tau'-\tau'')(G^R+G^A)_{\tau''}Z(p_z), \tag{8}$$

where

$$Z=1-{}^1/_2\partial(\Sigma^R+\Sigma^A)/\partial\omega \quad \text{at} \quad \omega=E(p_z).$$

In the approximation we are considering for Σ^{12}, in the right-hand side of (8) it is sufficient to take the diagrams of Figs. a and b with Green functions given by the expression (7), and put Z equal to unity and $E(p_z) \approx \epsilon(p_z)$. The Fourier component of the Green function (8) is equal to

$$G^{12}(p_z,t,\omega)=2\pi f(p_z,t)\delta(\omega-\varepsilon(p_z))$$
$$+\lim_{\eta\to 0}\int d\omega'\,\Sigma^{12}(p_z,t,\omega')\frac{\delta(\omega-\omega')-\delta(\omega-\varepsilon(p_z))}{(\omega'-\varepsilon(p_z))^2+\eta^2}. \tag{9}$$

In (9) we have used the zeroth-approximation functions $G^{R(A)}$:

$$G^{R(A)}(p_z,t,\omega)=[\omega-\varepsilon(p_z)\pm i\eta]^{-1}, \quad \eta\to+0.$$

In the frequency integral in (9) the principal contribution is made by the region near $\omega' \approx \epsilon(p_z)$, and therefore the correction sought to $G^{12}(p_z, t, \omega)$ is equal to

$$\frac{d\delta(\omega-\varepsilon(p_z))}{d\omega}-\!\!\!\!\!\!\int d\omega'\,\frac{\Sigma^{12}(p_z,t,\omega')}{\omega'-\varepsilon(p_z)}. \tag{10}$$

Here we have neglected the weak variation of $\Sigma^{12}(\omega)$ in the region of $\omega \approx \epsilon(p_z)$.

We proceed now to analyze the terms arising in the collision integral, with $\Sigma^{12(21)}$ taken from Figs. a and b, when the exact (broadened) Green functions G^{12} and G^{21} are taken into account. In the linear approximation in the broadening, each Green function must be replaced in turn by the corresponding correction (10). The broadening of the Green functions must be taken into account only in Σ^{12} and Σ^{21}. The corrections to the Green functions which appear explicitly in the collision integral (3) cancel each other (from the incoming and outgoing terms). Therefore, 3 + 1 = 4 terms with corrections to G^{12} and G^{21} (in each of the incoming and outgoing terms) make a contribution to the collision integral sought.

The graphs for Σ of the second type (see Figs. c–k) describe triple collisions of quasi-particles and give a nonvanishing contribution to the collision integral when the Green functions of the noninteracting particles are substituted into (3). After ordering the time indices we obtain from these graphs 9 × 4 = 36 terms in each of the incoming and outgoing terms. A contribution to the required collision integral for electron-electron collisions in the field of an impurity is given by only 12 graphs, satisfying the following requirements: 1) the ends of the dashed impurity line lie on different time axes; this is a necessary condition for the collision to occur with transfer of momentum to the impurity; 2) two G^{12} functions and one G^{21} should be explicitly present in Σ^{12}, and two G^{21} and one G^{12} in Σ^{21}; this ensures (after Σ is multiplied by G) that the statistical factors in the collision integral (two functions f and two 1 − f) are correct.

It is necessary to bear in mind that the product $G^{11}G^{22}$ appears in the expressions for Σ^{12} and Σ^{21}. In order to obtain the contribution from this product to the required collision integral, we must replace G^{11} by G^R and G^{22} by G^A (or vice versa, but in the same manner in all terms; cf. (6)). The remaining terms that arise after the replacement of $G^{11}G^{22}$ using formulas (6) give corrections of two types: 1) to the probability of scattering by an impurity (on account of the electron-electron interaction), and 2) to the probability of binary collisions (on account of scattering by the impurity). These corrections, and also the remaining terms in the collision integral with functions Σ that are taken from the diagrams (cf. Figs. c–k) and do not satisfy the three-particle scattering criteria, make the probabilities of collisions with impurities and of binary collisions of free electrons more precise, and therefore give no contribution to the redistribution of energy between the electrons.

Thus, in the required collision integral, 4 + 12 = 16 terms appear in the incoming and outgoing terms. Each of these diverges at small momentum transfers, because of the Coulomb character of the interaction. As usual, we must take the screening into account by summing the most divergent diagrams in each order, which correspond to the same momentum transfer. The interaction potential $v_{ee}(\mathbf{q})$ is thereby replaced by $\tilde{v}_{ee}(\mathbf{q}, \omega) = v_{ee}(\mathbf{q})\epsilon^{-1}(\mathbf{q}, \omega)$, and $v_{ei}(\mathbf{q})$ by $\tilde{v}_{ei}(\mathbf{q}) = v\ \ (\mathbf{q})\epsilon^{-1}(\mathbf{q}, 0)$, where $\epsilon(\mathbf{q}, \omega)$ is the longitudinal dielectric permittivity of the electron gas, which depends on the momentum transfer and frequency. Below we only estimate the electron-electron collision time, and for this it is sufficient to approximate the dynamical screening by the static screening, as is usually done.

The collision integral for electron-electron collisions in the field of impurities for a nondegenerate gas, in the case when the quantum magnetic length $\lambda_H = (\hbar/m\omega_H)^{1/2}$ is small compared with the thermal de Broglie wavelength $\lambda_T = \hbar(2mT)^{-1/2}$, can be represented in the form

$$S(p)=\frac{N_i}{\lambda_H{}^2\hbar^4}\int dp'\int\frac{d^3\mathbf{q}}{(2\pi)^3}\int\frac{d^3\mathbf{q}'}{(2\pi)^3}[f(p+q)f(p'+q')-f(p)f(p')]\\ \times M_3{}^2(p, p'; \mathbf{q}, \mathbf{q}')\delta(\omega_{p+q}+\omega_{p'+q'}-\omega_p-\omega_{p'}) \tag{11}$$

where the "amplitude"[1)] is given by

$$M_3(p, p'; \mathbf{q}, \mathbf{q}')=\tilde{v}_{ei}(\mathbf{q}+\mathbf{q}')\{\tilde{v}_{ee}(\mathbf{q})[(\omega_{p'+q+q'}-\omega_{p'})^{-1}\\ +(\omega_{p'-q}-\omega_{p'+q'})^{-1}]+\tilde{v}_{ee}(\mathbf{q}')[(\omega_{p-q'}-\omega_{p+q})^{-1}+(\omega_{p+q+q'}-\omega_p)^{-1}]\}. \tag{12}$$

Here, to simplify the notation, we have omitted the subscripts in the momenta p_z and p'_z, and also in the momenta q_z and q'_z, in quantities which depend only on these components of $\mathbf{q}$ and $\mathbf{q}'$.

In the expression for M_3^2 there are 16 terms, of which 12 (the cross terms obtained when M_3 is squared) correspond to substituting the expressions for Σ^{12} and Σ^{21} corresponding to the graphs c—k (see the Figure) into the collision integral. The energy denominators appearing in these expressions must be integrated in the principal-value sense. The remaining 4 terms in the collision integral are obtained when the corrections (10) to the Green functions are taken into account in the expressions for Σ^{12} and Σ^{21} corresponding to the diagrams a and b. The squared energy denominators (of the form $(\omega' - \epsilon_p)^{-2}$) in these terms must, in accordance with (10), be integrated with the remaining part of the integrand in (11) as the functions

$$\frac{d}{d\varepsilon(p)}\left[\mathrm{P}\frac{1}{\omega'-\varepsilon(p)}\right]=\lim_{\eta\to 0}\frac{(\omega'-\varepsilon(p))^2-\eta^2}{[(\omega'-\varepsilon(p))^2+\eta^2]^2}. \tag{13}$$

Inasmuch as small $|\mathbf{q}|$, $|\mathbf{q}'| \ll \lambda_H^{-1}$ are important in the integrand of (11), $\epsilon(\mathbf{q}, 0) = \kappa(1 + q_D^2/\mathbf{q}^2)$, which is independent of λ_H[4] (where $q_D^2 = 4\pi ne^2/\kappa T$), appears in the matrix element of the effective interaction potential. The distribution function $f(p_z)$ in the Landau band with N = 0 is normalized as follows:

$$\sum_{p_y, p_z} f(p_z)=\frac{V}{4\pi^2\lambda_H{}^2}\int dp_z f(p_z)=nV. \tag{14}$$

From this it can be seen that the characteristic relaxation time of the electrons in the limiting case of strong magnetic fields does not depend on the magnetic field.

The integrand in S(p) is especially large in the region of small momentum transfers $q, q' \sim q_D \ll p_T = \lambda_T^{-1}$, and therefore, as is done in the case of an ordinary plasma, we shall make use of the diffusion approximation[14], expanding the integrand in small q_z and q'_z:

$$S(p_z)=-\frac{\partial}{\partial p_z}\left[D(p_z)\frac{\partial f}{\partial p_z}-F(p_z)f\right].$$

Here the momentum-space electron diffusion coefficient $D(p_z)$ and the dynamical-frictional force $F(p_z)$ depend on the particle distribution function $f(p_z)$. In particular,

$$D(p_z)=\frac{2^{3/2}}{\pi^{5/2}}\sum_\alpha\frac{N_{i\alpha}Z_\alpha{}^2}{n}p_T{}^7q_D\left(\frac{n^{1/3}e^2}{\varkappa T}\right)^{3/2}\frac{ne^4}{m^{1/2}\varkappa^2T^{3/2}}\\ \times\frac{1}{2\pi}\int dp'\int d^3\mathbf{q}\int d^3\mathbf{q}'\frac{f(p')}{2\pi n\lambda_H{}^2}\frac{q^2}{(q+q')^2[(\mathbf{q}+\mathbf{q}')^2+q_D{}^2]^2}\\ \times\left\{\frac{q}{(\mathbf{q}^2+q_D{}^2)(p'^2-q^2/4)}+\frac{q'}{(\mathbf{q}'^2+q_D{}^2)(p^2-q'^2/4)}\right\}^2\delta(pq+p'q'), \tag{15}$$

where $Z_\alpha e$ is the charge on an impurity of type α. It must be remembered that in formula (15) functions of the form $(p' \pm q/2)^{-1}$ are integrated in the principal-value sense, and functions $(q' + q)^{-2}$ and $(p' \pm q/2)^{-2}$ are integrated over the momenta as indicated in (13).

3. ENERGY REDISTRIBUTION TIME

The reciprocal of the time in which an electron beam that is originally monoenergetic spreads out in energy by an amount of the order of T is equal to

$$\tau_{ee}^{-1}(H)\sim D(p_T)/p_T{}^2.$$

For $p \gg p_T$ the diffusion coefficient, as follows from (15), is equal to

$$D(p)=\frac{\pi}{2^{1/2}}\left(\sum_\alpha\frac{N_{i\alpha}Z_\alpha{}^2}{n}\right)\frac{p_T{}^6}{p^4}\frac{ne^4}{\varkappa^2m^{1/2}T^{1/2}}\left(\frac{n^{1/3}e^2}{\varkappa T}\right)^{-1/2}\ln\left(\frac{4p_T{}^2}{q_D{}^2}\right).$$

Hence it follows that, for $p \approx p_T$,

$$D(p_T)\approx\sum_\alpha\frac{N_{i\alpha}Z_\alpha{}^2}{n}p_T{}^2\frac{ne^4}{\varkappa^2m^{1/2}T^{3/2}}\left(\frac{n^{1/3}e^2}{\varkappa T}\right)^{3/2}\ln\left(\frac{4p_T{}^2}{q_D{}^2}\right).$$

Bearing in mind that, in the absence of a magnetic field[14],

$$\tau_{ee}^{-1}(0)\approx\frac{ne^4}{\varkappa^2m^{1/2}T^{3/2}}\ln\left(\frac{4p_T{}^2}{q_D{}^2}\right),$$

we find that the reciprocal time for redistribution of energy between electrons in an ultra-quantizing magnetic field is equal to

$$\tau_{ee}^{-1}(H)\approx\tau_{ee}^{-1}(0)\left(\frac{n^{1/3}e^2}{\varkappa T}\right)^{3/2}\sum_\alpha\frac{N_{i\alpha}Z_\alpha{}^2}{n}. \tag{16}$$

in order of magnitude.

Conditions in which almost all the electrons are in the lowest Landau level are easily realized in n-type indium antimonide. In the experiments of Miyazawa[6] the minimum electron concentration in the n-InSb samples was $10^{14}\,\mathrm{cm}^{-3}$. For such a concentration and at temperature 4.2 K the parameter $n^{1/3}e^2/\kappa T \sim 1$. Above, confining ourselves to triple collisions, we assumed that $n^{1/3}e^2/\kappa T \ll 1$. We must expect, however, that the estimate (16) that we have obtained also gives the correct order of magnitude of $\tau_{ee}^{-1}(H)$ in the case when the gas parameter is of order unity. But this means that in the experiments (cf.[6]) $\tau_{ee}^{-1}(H)$ is not much smaller than $\tau_{ee}^{-1}(0)$. Consequently, in n-InSb at liquid-helium temperatures, owing to triple collisions of electrons and binary electron collisions with the participation of charged impurities, an effective electron temperature is established even in an ultra-quantizing magnetic field.

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APPENDIX

Derivation of Eq. (4)

The functional derivative equation for the Green functions with only the electron-electron interaction taken into account has the form (cf.[10])

$$(\sigma_z)_{ik}G_0^{-1}(1|U)G_e^{kj}(12|U)+i(\sigma_z)_{ik}\int d3\,v_{ee}(13)G_e^{kk}(33|U)G_e^{kj}(12|U)$$
$$-i\int d3\,v_{ee}(13)\frac{\delta}{\delta U(3)}G_e^{ij}(12|U)=\delta_{ij}\delta(1-2). \tag{A.1}$$

Allowance for the interaction with the impurity centers reduces to adding a term $\Sigma_f\,\alpha(f)v_{ei}(\mathbf{1f})$ to the variational potential U(1) in $G_0^{-1}(1|u)$, where $\alpha(f) = 1$ or 0 depending on whether or not the lattice site f is occupied by a charged impurity. The Green function averaged over the positions of the impurities is equal to

$$G^{ij}(12|U)=\mathrm{Sp}\Big[\rho G_e^{ij}\Big(12|U+\sum_f\alpha(f)v_{ei}\Big)\Big],$$

where ρ is the density matrix of the impurity centers. When these are randomly positioned over the lattice sites (in the case of one type of impurity),

$$\rho=\exp\Big[\mu\sum_f\alpha(f)\Big]\Big\{\mathrm{Sp}\Big[\exp\Big(\mu\sum_f\alpha(f)\Big)\Big]\Big\}^{-1},$$

where the "chemical potential" μ is related to the atomic concentration of impurity centers: $c = (1 + e^{-\mu})^{-1}$.

Using the operator identity

$$G_e^{ij}(12|U+\bar{U})=\exp\Big[\int d3\,\bar{U}(3)\frac{\delta}{\delta U(3)}\Big]G_e^{ij}(12|U),$$

we obtain that, in the case of low impurity concentration $c \ll 1$,

$$G^{ij}(12|U)=\exp\Big\{W\Big(\frac{\delta}{\delta U}\Big)\Big\}G_e^{ij}(12|U),$$
$$W\Big(\frac{\delta}{\delta U}\Big)=c\sum_f\Big[\exp\int d3\,v_{ei}(3\mathrm{f})\frac{\delta}{\delta U(3)}-1\Big].$$

In the case when there are impurities of several types (it is convenient to denote them by the index α), the operator W is a sum over all types of impurities (the concentration of each is c_α). When the summation over the sites is transformed to an integration over the whole volume of the crystal, W takes the form

$$W\Big(\frac{\delta}{\delta U}\Big)=\sum_\alpha N_{i\alpha}\int d\mathbf{R}\Big[\exp\Big(\int d3\,v_{ei}(3\mathbf{R})\frac{\delta}{\delta U(3)}\Big)-1\Big].$$

We apply the operator e^W term-by-term to all terms in Eq. (A.1). We take into account that

$$e^WG_e^{kk}(33|U)=G^{kk}(33|U),$$
$$e^WUG_e=e^WUe^{-W}G=\Big(U+\frac{\delta W}{\delta(\delta/\delta U)}\Big)G$$
$$=UG+\sum_\alpha N_{i\alpha}\int d\mathbf{R}\,v_{ei}(1\mathbf{R})\exp\Big(\int d3\,v_{ei}(3\mathbf{R})\frac{\delta G}{\delta U(3)}\Big),$$
$$e^WG_e(33|U)e^{-W}G(12|U)=$$
$$=\lim_{V\to U}\exp\Big[\sum_\alpha N_{i\alpha}\int d\mathbf{R}\Big(\exp\Big\{\int d4\,v_{ei}(4\mathbf{R})\frac{\delta}{\delta V(4)}\Big\}-1\Big)$$
$$\times\Big(\exp\Big\{\int d5\,v_{ei}(5\mathbf{R})\frac{\delta}{\delta U(5)}\Big\}-1\Big)\Big]G(33|V)G(12|U).$$

We then arrive at Eq. (4).

[1)] An analogous expression for the triple-collision integral is obtained by Bezzerides and DuBois [13] for a gas of weakly interacting particles in the absence of a magnetic field.

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