Dynamical properties of superconducting filaments of finite length

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The dynamical properties of quasi-one-dimensional superconductors of finite length, namely, their current-voltage characteristics (CVC) and the Josephson effect, are considered within the framework of a simple generalization of the Ginzburg–Landau equations to the time-dependent case. It is shown that in the majority of cases one can use simple boundary conditions for the modulus of the order parameter in order to describe such superconducting "filaments." For large currents, the CVC of the filaments approach the straight line $\bar{V} = JR_N - \Delta V$, where the quantity ΔV does not depend on the material or the length of the filament. Furthermore, the upper frequency limit of the Josephson effect and the value of the interference component of the current, which is proportional to $(-V\cos\phi)$, are found for short filaments. The known experimental data are discussed and it is shown that a quantitative verification of the effects under consideration is possible on the recently developed bridges of variable thickness.

1. INTRODUCTION

In connection with the passage of a current J through a superconducting sample of sufficiently small transverse cross section S (the "filament"), all quantities characterizing the superconducting state depend on only a single coordinate—the coordinate along the filament. The behavior of such a quasi-one-dimensional superconductor in the time-dependent case, i.e., in a nonvanishing electric field E, is one of the most important and recently posed, but still unsolved, problems in the theory of superconductivity.

The most complete theoretical results were obtained for an infinitely long filament, when it was assumed that the dependence on the longitudinal coordinate is also absent. The critical current J_c for this case was found by Ginzburg[1] in the framework of the Ginzburg-Landau (GL) equations, and at arbitrary temperatures by Maki[2] and by Ovchinnikov.[3] Kulik[4] considered the influence on J_c of a microwave current through the filament within the framework of a simple time-dependent generalization of the GL equations. In[5] it was shown in this same approximation that, the normal state of an infinite filament, which arises upon the destruction of its superconductivity by a constant current, is stable with respect to small thermal fluctuations. These fluctuations only lead to small deviations of the filament's current-voltage characteristics (CVC) from ohmic.

Experimental investigations of quasi-one-dimensional superconductors are practically always carried out on samples of finite length L. As will be shown below, the influence of the filament's boundaries (contacts) with the "electrodes" conducting currents to it may be significant even for relatively long lengths L. Furthermore, taking account of the finite length of the filament is fundamental in connection with an investigation of the question of the upper limits of the Josephson effect in such a structure with regard to the parameters L and J.

In[6] we analyzed the steady-state properties of quasi-one-dimensional normal and superconducting samples of finite length. The basic dynamical properties of such filaments—the form of the CVC and the time-dependent Josephson effect—are investigated in the present work within the framework of a simple time-dependent generalization of the GL equations.

2. BOUNDARY CONDITIONS FOR THE ORDER PARAMETER

For the case when the electrodes have the same transverse cross section as the filament, one can immediately use the boundary conditions derived from the microscopic theory of Zaĭtsev[7] for the analysis in the GL approximation. However, in an experiment it is considerably easier to achieve connections to a quasi-one-dimensional superconductor (whisker,[8] narrow thin film,[9-11] etc.) with the aid of superconducting electrodes ("banks") with a considerably larger transverse cross section. Let us show that the following boundary condition, corresponding to the modulus of the order parameter on the filament–shore boundaries being equal to its equilibrium value Δ_B[1)] in the "banks," is satisfied for such systems with a high degree of accuracy:

$$|\Delta(0)| = |\Delta(L)| = \Delta_B. \tag{1}$$

The change (decrease) of $|\Delta|$ on the boundaries may occur for two reasons: the effect of proximity of the filament to the bank, and the suppression of $|\Delta|$ by the current in the "banks." Let us consider these effects separately.

For the analysis of the proximity effect we shall assume the current J equal to zero, and the filament long ($L \gg \xi(T)$) and superconducting ($T < T_c$). It is easy to show that the final result (formula (5)) remains unchanged in other cases.

According to[7], $|\Delta|$ must be continuous, and according to the GL equations the quantity $|\Delta|$ varies in the filament from the equilibrium value Δ_0 to the unknown (as yet) value Δ_p on the boundary over a distance of order ξ, and in the "bank"—it varies from Δ_p to Δ_B over a distance of order ξ_B. The increase of the GL free energy due to the proximity effect is given by

$$F \approx \frac{H_c^2}{8\pi} V\left(\frac{\Delta_p-\Delta_0}{\Delta_0}\right)^2 + \frac{H_{cB}^2}{8\pi} V_B\left(\frac{\Delta_B-\Delta_p}{\Delta_B}\right)^2, \tag{2}$$

where the volume of the boundary region of the filament, in which $|\Delta|$ is not equal to the equilibrium value, is given by $V \approx S\xi$. The volume V_B of the boundary region in the "bank" depends on the shape of the filament's cross section. For a sample with short transverse dimensions $\sim D \ll \xi$, for example, for a round whisker of diameter D, V_B is of order ξ_B^3, and for a filament with a planar transverse cross section, for example, a film of thickness $d \ll \xi_B$ and width $w \gg d, \xi_B$, the volume $V_B \approx \xi_B^2 w$.

Minimizing $\tilde{F}$ with respect to Δ_p and expressing H_c in terms of ξ and the magnetic field penetration depth δ, we find that the deviation of the boundary value Δ_p and Δ_B is small in comparison with $|\Delta_B - \Delta_0|$ under the conditions

$$D \ll (\xi\xi_B)^{1/2} \frac{\delta}{\delta_B} \left| \frac{\Delta_0}{\Delta_B} \right|, \quad d \ll \xi \frac{\delta}{\delta_B} \left| \frac{\Delta_0}{\Delta_B} \right| \tag{3}$$

respectively for "round" and "planar" filaments.

The second reason–the suppression of $|\Delta|$ by the current in the "banks"–takes place at distances of the order of $(J/j_{cB})^{1/2}$ and $J/j_{cB}w$, respectively, from the filament–bank boundary. Here j_{cB} denotes the critical current of the "bank" material. Being interested in currents J of the order of the critical filament pair-breaking current[1], we find that these distances are much smaller than the distances of interest to us of order ξ for

$$D \ll (\xi\xi_B)^{1/2} \frac{\delta}{\delta_B}, \quad d \ll \xi \frac{\xi}{\xi_B} \frac{\delta}{\delta_B}. \tag{4}$$

Since we assume the "bank" material to be, in any case, not a "worse" superconductor than the filament material ($\Delta_B \gtrsim \Delta_0$, $\delta_B \lesssim \delta$), the inequalities

$$D \ll (\xi\xi_B)^{1/2}, \quad d \ll \xi, \ \xi^2/\xi_B. \tag{5}$$

would be sufficient for the fulfilment of conditions (3) and (4). Thus, the boundary condition (1) is always satisfied for sufficiently thin filaments, especially if it is taken into consideration that the quantity $\xi^{1/2}\delta$ appearing on the right hand sides of inequalities (3) and (4) varies like $(D, d)^{1/2}$ upon predominance of surface scattering of the electrons.

3. TIME-DEPENDENT EQUATIONS

The time-dependent, one-dimensional GL equations[5,12] can be written in the form

$$u\left(\frac{\partial}{\partial t} + i\mu\right)\psi = \psi'' + (\pm 1 - |\psi|^2)\psi, \tag{6a}$$

$$J = \mathrm{Im}\,(\psi^*\psi') - \mu', \tag{6b}$$

where $\psi = \Delta/\Delta_0$, differentiation with respect to the dimensionless coordinate $x = X/\xi$ along the filament is denoted by the prime, and t is the time normalized to the relaxation time t_0 of the current. The parameter u is the relaxation time t_Δ of the order parameter, normalized to t_0. It is equal to 12 for superconductors containing a high concentration of paramagnetic impurities and is equal to $\pi^4/14\zeta(3) \approx 5.79$ for a superconductor with ordinary impurities in the dirty limit.[5]

The quantity μ is the gauge-invariant potential of the electric field, so that the dimensional voltage across the filament (between the "banks") is given by

$$V(t) = V_J \frac{-[\mu(l) - \mu(0)]}{A^2}, \quad l = \frac{L}{\xi}, \tag{7}$$

where V_J is the "Josephson" value of the voltage:[13,14]

$$V_J = \frac{\pi}{4}\frac{\Delta_B^2}{eT} \approx 635[\mu\mathrm{V-K^{-1}}](T_{cB} - T)[\mathrm{K}], \tag{8}$$

which does not depend on the filament parameters,

$$A^2 = \Delta_B^2/\Delta_0^2 = (T_{cB} - T)/|T_c - T|. \tag{9}$$

The current is normalized to the characteristic value $Sc\Phi_0/8\pi^2\delta^2\xi$, so that the critical depairing current is simply $J_c = 2/\sqrt{27}$. The two signs in Eq. (6a) correspond to the cases ($T < T_c$) (superconducting filament) and $T > T_c$ (normal filament). For the latter case the quantities ξ, δ, and Δ_0 are formally determined:

$$\xi, \delta, \Delta_0^{-1/2}(T - T_c) = \xi, \delta, \Delta_0^{-1/2}(T_c - T) \infty |T_c - T|^{1/2} \tag{10}$$

According to Eq. (1), the boundary conditions for Eqs. (6) can be written in the form

$$\psi(0) = A, \ \psi(l) = Ae^{i\varphi}, \ \mu(0) = 0, \ \mu(l) = -\partial\varphi/\partial t. \tag{11}$$

For the analysis of Eqs. (6) with the boundary conditions (11), together with the analytical methods which are feasible in certain limiting cases, we also applied numerical integration to them. For the latter it is convenient to change to the Cartesian variables $\psi = R + iI$, in terms of which Eqs. (6) and (11) are rewritten in the form

$$u\frac{\partial R}{\partial t} = R'' + (\pm 1 - R^2 - I^2)R - u\mu I,$$
$$u\frac{\partial I}{\partial t} = I'' + (\pm 1 - R^2 - I^2)I + u\mu R,$$
$$\mu = \int_0^x (R'I - I'R)\,dx - Jx, \quad R(0) = A, \quad I(0) = 0, \tag{12}$$
$$R(l) + iI(l) = A\exp\{i\varphi\},$$

which is convenient for solution on an electronic computer by the standard methods of finite differences. The Cartesian representation for ψ differs advantageously from the usual representation in polar variables, $\psi = f\exp\{i\chi\}$, by the fact that for Eqs. (12) the point $R = I = |\psi| = 0$ is not singular and the process of its intersection by the phase trajectory $\{R, I\}$ can be counted regularly. Thereby the introduction of artificial and inexact "discontinuity conditions" is avoided.[12] The CVC of superconducting and normal filaments, calculated in this manner for three different values of the length, are shown in Fig. 1.

4. THE PROBLEM OF THE "EXCESS CURRENT"

The most characteristic feature of the filaments' CVC is their approach to the straight line $-\mu(l) = Jl - 2\epsilon A^2$ as $J \to \infty$ or, in dimensional units,

$$\bar{V} = JR_N - \Delta V, \quad \Delta V = 2\epsilon V_J, \tag{13}$$

where R_N is the resistance of the filament in the normal

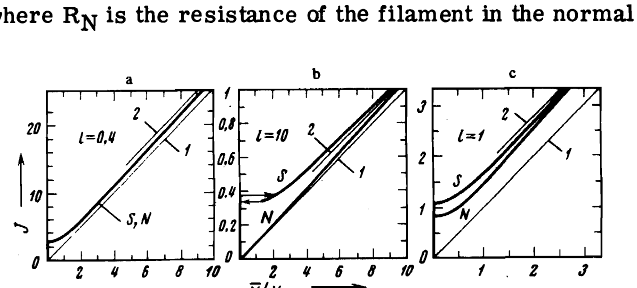

FIG. 1. CVC of filaments for different values of their lengths. The symbols S and N denote the cases $T < T_c$ and $T > T_c$, respectively. The parameter A is equal to unity, which corresponds to $T_c = T_{cB}$ for S-filaments and $T = (T_c + T_{cB})/2$ for N-filaments. The straight line 1 corresponds to $\bar{V} = IR_N$, and the straight line 2 corresponds to $\bar{V} = IR_N - 2\epsilon V_J$.

state, and where the quantity ϵ does not depend on the filament parameters l and A: $\epsilon \approx 0.375$. Thus, ΔV is a function of only the order parameter in the "banks" at the given temperature:

$$\Delta V \approx 0.749 V_J \approx 475[\mu V - K^{-1}] \cdot (T_{cB} - T)[K]. \tag{14}$$

A similar phenomenon has been repeatedly observed in the CVC of weak superconducting links; in this connection one usually talks about the existence of an "excess current" $\Delta J = \Delta V/R_N$ which is independent of the voltage.

In[12] they proposed for this effect an explanation that actually reduces to the following. If the parameter u is set equal to zero, Eq. (5) reduces to the following simple equation for the phase difference of the order parameter:

$$l^{-1}\frac{\partial\varphi}{\partial t} + J_s(\varphi) = J, \tag{15}$$

where $J_S(\varphi)$ is the time-independent dependence of the superconducting current on the phase. Since for $T < T_c$, for sufficiently long lengths[2)] $l > l_c$ the dependence $J_S(\varphi)$ is not single-valued, Eq. (15) must be supplemented by a discontinuity condition that calls for continuity of $\varphi(t)$. In this case ($u = 0$, $T < T_c$, and $l > l_c$) the CVC also has an excess current of the order of the critical current.

However, such a description is only suitable for $u(\partial\varphi/\partial t) \ll 1$. Since the characteristic value of $\partial\varphi/\partial t$ for $J > J_c$ is given by

$$\frac{\partial\varphi}{\partial t} \approx \overline{\frac{\partial\varphi}{\partial t}} \gtrsim \begin{cases} A^2, & l \lesssim A^{-1}, \\ l, & l \gg 1, \end{cases} \tag{16}$$

i.e., not less than unity, it is necessary to assume $u \ll 1$ for the validity of the explanation proposed in[12]. At the same time, as already mentioned above, the actual values of u are, in any case, not less than unity.[3)]

Let us show that in an actual case of finite values of u, the existence of "excess current" is exclusively related to the proximity effect in the immediate neighborhoods of the filament's boundaries ($x = 0, l$). Let the current in the filament be so large that the superconductivity in it is practically destroyed. Then $|\psi|$ is equal to zero everywhere, except small neighborhoods of the points $x = 0, l$ with dimensions of order $x_0 \ll 1, l$. In this case, as the zero-order approximation in Eqs. (6) can take

$$\mu_0 = -\frac{x}{l}\frac{\partial\varphi}{\partial t} \tag{17}$$

and seek ψ, neglecting in virtue of the smallness of x_0 all terms except the gradient term on the right hand side of Eq. (6a):

$$\psi'' = u\left(\frac{\partial}{\partial t} + i\mu_0\right)\psi. \tag{18}$$

The solution (18) can be written in the form

$$\psi = A\left[\Psi\left(\frac{x}{x_0}\right) + \Psi^*\left(\frac{l-x}{x_0}\right)\exp\{i\varphi(t)\}\right], \quad x_0 = \left(\frac{u}{l}\overline{\frac{\partial\varphi}{\partial t}}\right)^{1/3}, \tag{19}$$

which describes identical and independent processes near the boundaries. In this connection, the current does not appear explicitly in the equation for Ψ:

$$\frac{d^2\Psi(z)}{dz^2} + iz\Psi(z) = 0, \qquad \Psi(0) = 1, \qquad \Psi(\infty) = 0, \tag{20}$$

Therefore, if expression (19) is substituted in Eq. (6b), it is at once seen that the correction to the voltage on

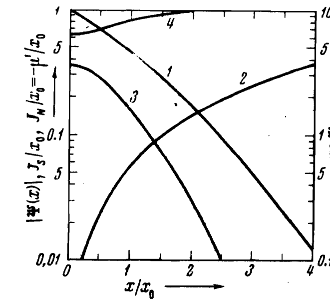

FIG. 2. Behavior of the quantities $|\psi|$ (curve 1), arg ψ (curve 2), J_S (curve 3), and J_N (curve 4) near the filament boundary $x = 0$ for sufficiently large currents (the "excess current" region).

the filament actually does not depend on the current:

$$-(\mu - \mu_0)_{x=l} = -2\varepsilon A^2, \qquad \varepsilon = -\mathrm{Im}\int_0^\infty \Psi^* \frac{d\Psi}{dz}dz = \text{const.} \tag{21}$$

The numerical value for ϵ cited above is easily obtained by the numerical solution of Eq. (20); it is in good agreement with the result obtained from a direct numerical solution of Eqs. (12) (see Fig. 1). The variation of the magnitude and phase of the order parameter, calculated from Eq. (20), and also the superconducting and normal components of the currents near the film–shore boundary are shown in Fig. 2.

The approach of the CVC to the straight line (13) occurs when the characteristic pairing distance $|\psi|$ near the boundary becomes less than half the length of the filament, that is, when the interference of the order parameters of the "banks" stops. In this connection the voltage on the bridge becomes constant in time, i.e., the Josephson effect is suppressed. Thus, the value of the current at which this happens essentially depends on the length of the filament.

5. SHORT FILAMENTS. THE TERM ($-V\cos\phi$)

As was shown in[6], for filaments of lengths $l \lesssim 1, A^{-1}$ the steady-state dependence $J_S(\varphi)$ is practically harmonic, which indicates the possibility of neglecting all terms on the right hand side of (6a) except the gradient term. As this equation shows, the effect of a finite rate of relaxation of the order parameter will be determined by the value of the parameter

$$\eta = u(lA)^2 J/J_c \approx (l/x_0)^3. \tag{22}$$

The solution of Aslamazov and Larkin,[14] which for our geometry takes the form

$$\psi_0 = A\left[\left(1 - \frac{x}{l}\right) - \frac{x}{l}e^{i\varphi}\right], \quad \mu_0 = -\frac{x}{l}\frac{\partial\varphi}{\partial t}. \tag{23}$$

is valid for $l \ll x_0$.

On the other hand, the theory developed in Sec. 4 is valid for $l \gg x_0$. Comparison of formulas (17), (19), and (23) shows that, for arbitrary values of η one can seek the solution in the form (19), (20) provided that the last boundary condition for $\Psi(z)$ is replaced by

$$\Psi(l/x_0) \equiv \Psi(\eta^{1/3}) = 0 \tag{24}$$

The solution of Eq. (20) is again easily carried out numerically; the results are shown in Fig. 3.

For $\eta \ll 1$ the solution is obtained by elementary means and leads to the following equation for the phase:

$$\frac{d\varphi}{dt}\left[\frac{1}{A^2} + \frac{ul^2}{15}(1 - \cos\varphi)\right] + \sin\varphi = \frac{J}{J_c}, \tag{25}$$

in which the second term inside the square brackets

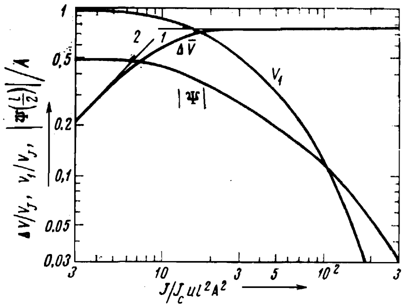

FIG. 3. $|\psi|$ at the center of the filament, the deviation $\Delta\bar{V} = \bar{V} - JR_N$, and the amplitude V_1 of the first harmonic of the Josephson voltage as functions of the average voltage $\bar{V}$ on a short filament ($l \ll 1, A^{-1}$). The straight lines 1 and 2 correspond to the asymptotes $\Delta\bar{V}/V_J = 2c$ and $\Delta R/R = ul^2A^2/15$, respectively.

gives the relaxation correction to the "pure" Josephson effect.[14] As Eq. (25) shows, an "interference" term proportional to $(-V\cos\varphi)$ appears in the superconducting component of the current through the filament; this term is analogous to that which exists in Josephson tunnel junctions[15] and has recently been experimentally observed in point contacts.[16] Although the indefiniteness of the geometry of the conducting "shorts" in point contacts excludes the possibility of a numerical comparison of expression (25) with experiment, it is possible that in these contacts the effect is caused precisely by relaxation phenomena.

Since the time-averaged value of the quantity $(d\varphi/dt)\cos\varphi$ is always equal to zero, the effect of the relaxation corrections on the CVC reduces, as is clear from Eq. (25), to a decrease in the effective normal resistance of the filament by the amount

$$\Delta R/R_N = u(lA)^2/15.$$

This change is very clearly seen in Fig. 1a ($u(lA)^2/15 \approx 0.062$).

6. UPPER FREQUENCY LIMIT OF THE JOSEPHSON EFFECT

Upon an increase of the current (and voltage) through a short ($l \ll 1, A^{-1}$) filament, when the parameter η becomes of the order of unity, i.e., $x_0 \approx l$, suppression of the magnitude of the parameter in the middle of the filament and of the Josephson generation actually occurs. As is seen from Fig. 3, such a transition from the "resistive" state of the filament to the "normal" state, notably the reduction of the amplitude of Josephson generation by a factor of two, occurs at the voltage

$$\bar{V} \approx 30\,V_J/u(lA)^2 \gg V_J/A^2.$$

The corresponding frequency,

$$\omega \approx \frac{50}{u}\frac{\Delta_0^2}{T}\left(\frac{\xi}{L}\right)^2 \approx 30\left(\frac{\xi}{L}\right)^2 t_\Delta^{-1}$$

is therefore the upper frequency limit of the Josephson effect in the approximation of Eqs. (6). Since the approach of the CVC to the straight line, "excess current" relationship (13) occurs simultaneously, this may give the possibility of measuring the parameter t_Δ in experiments at constant current under the condition of sufficiently accurate knowledge of the geometry (L).

7. LONG SUPERCONDUCTING FILAMENT

For a long ($l \gg 1$) filament, even for a current of the order of the critical current ($J_c \approx 0.385$) the characteristic size of the boundary region x_0 (defined in Eqs. (19)) is much smaller than the length of the filament. Therefore, the destruction of the superconducting state over a large portion of the filament's length occurs practically immediately upon exceeding the critical value of the current, and $|\psi| = 0$ everywhere except in segments of length $\sim x_0 \approx u^{-1/3} \approx 1 \ll l$ near the boundaries. Upon an increase of the current to values $J/J_c \gtrsim u^{1/3}$, these regions shrink to $x_0 \lesssim 1$ and the CVC approach the straight line (13) which, in the present case, is located a relatively small distance ($\sim l^{-1}$) from the straight line $\bar{V} = JR_N$. Conversely, upon a reduction of the current, the superconducting "domains" on the boundaries of the filament begin to expand and, as was shown in article[17], at a certain value of the current, $J_0(u) < J_c$, they spread from the boundaries to the center of the filament. The result is a constant current plateau on the CVC of long filaments (Fig.]b). For voltages $\bar{V} \approx J_0R_N/l \ll J_0R_N$, this plateau acquires a negative slope $d\bar{V}/dJ$ due to the attraction between neighboring boundaries of the "normal" domain.

Because of the propagation of superconducting domains from the contacts to the center, even in filaments of long length it is impossible to observe the normal section of the CVC for $J < J_c$, which was predicted in[5].

8. LONG NORMAL FILAMENT

In such a filament ($l \gg 1$) the critical current is exponentially small,[6] and the filament is found in the normal state for practically arbitrary currents, with the exception of small neighborhoods of the boundaries with the "banks." The influence of these regions on the CVC amounts to a deviation of the voltage from the "normal" value (JR_N) by a small amount, equal to $-I\Delta R$ for $J \ll u^{-1}$, where $\Delta R/R_N \approx 1/l \ll 1$, and equal to $(-\Delta V)$ given by Eq. (14) for $J \gg u^{-1}$ (see Fig. 1b).

9. COMPARISON WITH EXPERIMENT

All known experiments with superconducting weak links were carried out with superconductors without paramagnetic impurities in zero or small magnetic fields. The possibility of using the simple nonstationary Eqs. (6) for such cases remains obscure until now. Therefore, it is necessary to regard a comparison of the reresults obtained above with the experimental data as a most likely possibility for the verification of these equations.

Out of the large quantity of experimental data, it makes sense to consider only that data where the geometry of the weak link is well known and ensures a quasi-one-dimensional situation.

1) The Sn whiskers, which were investigated in[8], had diameters D of the order of 1 μ, and therefore could satisfy the condition $D \ll \xi$ only in a very narrow temperature interval near T_c. One of the consequences of this may be the fact that, in the resistive state ($\bar{V} \neq 0$) these whiskers do not behave like one-dimensional conductors, and the step structure of their CVC is explained by precisely this fact. In fact, a similar structure is observed in bridges of "variable thickness' with dimensions greater than ξ and finds there a natural explanation in terms of the motion of vortices across the bridge[9] (also see the discussion in[17]).

2) The bridges of "variable thickness," which were

investigated in[9], had large dimensions ($L, w \gg \xi$), and the destruction of the superconducting state in them occurred in a vortical (i.e., essentially two-dimensional) manner. Therefore, a comparison with the developed theory is possible only with regard to the value of ΔV given by Eq. (14). In actual fact, in the presence of large currents the normal state of the middle of the bridge imposes a uniform distribution of the current over the width and the one-dimensional equations are valid. As the experiments show, ΔV is actually proportional to[4)] $(T_c - T)$; but the coefficient of this proportionality differs markedly from sample to sample in the range from 5×10 to 5×10^3 $\mu V/°K$. Such a spread can be explained by the inexact correspondence of the shape of the edge of the bridge with the theoretically assumed shape. For the methods of fabrication employed in[9], the bridge smoothly changes into the "bank" over distances of a few tenths of a micron. In the "excess current" region, the size of the transition region x_0 is of the same order of magnitude or smaller. Therefore, the shape of the bridge–shore transitions strongly influences the behavior near the "bank," and consequently also has a strong effect on ΔV. However, we assume that the effect of the "excess current"[5)] in such bridges is actually associated with edge effects. This is in good agreement with the data in[9] explanation of the shape of the CVC at small voltages.

3) The long and narrow thin films investigated in[10] have CVC which, as already indicated in[17], are described well by the effect, following from our model, of the propagation of "domains" of superconducting phase from the ends of the sample to the middle for $J = J_0 < J_c$. Edge effects are negligible in such films.

For the shorter samples investigated in[10], a quantitative comparison with the theory is impossible–for the same reason as for the bridges investigated in[9] (see above).

4) Proximity effect bridges (see, for example,[18]) differ from the model described above because in them it is possible to have a deviation of the order parameter in the "banks" (portions of thin film, not superimposed on the normal metal) from the equilibrium value due the bridge–bank proximity effect.[19] Nevertheless, one can assume that the existence of ΔV in these bridges is also determined by effects on the boundary, and not by phase slips[12] (see Sec. 5). The recently proposed[18] modification of these structures, in which the bridge has a local tapering, should be even better described by the expounded theory. The corresponding experimental data has not yet been published.

5) A communication[11] recently appeared concerning the fabrication of "doubly scratched" microbridges with $T_c = T_{cB}$, whose geometry is very close to the model under consideration and in which relations (5) should be satisfied over a relatively wide temperature interval near T_c. Experimental data on such bridges is also for the present unknown to us.

10. CONCLUSION

Thus, the simple model (6) enables us to explain the most characteristic feature of the CVC of superconducting weak links–the "excess current" effect. However, it is still necessary to carry out a definitive test of the theory and an experimental determination of the coefficient u, by using quasi-one-dimensional structures with a smaller cross section. The recent achievements in the technology of their fabrication[11] permit us to hope that such verification will be possible in the very near future.

At the same time, on similar "filaments" it will be possible to ascertain whether the step structure of the CVC[8, 9] is in fact associated with the two-dimensionality of the resistive state, or whether it can be produced in one-dimensional structures, as conjectured in[20].

In addition, one more unsolved problem is whether two-dimensionality is necessary for the existence of such an effect as the stimulation of superconductivity by microwave radiation (the "Dayem effect"). As expression (25) and also the results of[6] show, in the present model, at least for short filaments, no terms appear in the current, which might be responsible for the appearance of this effect.

1)The subscript B on quantities will indicate that they refer to "banks."

2)For A = 1 we have $l_c \approx 3.49$. [6]

3)Failure to allow for finite values of u, and the above mentioned incorrect description of processes near the point $|\psi| = 0$, were the reason for the disparity between the majority of the results obtained in [12] and the initial system of equations (coincident with Eqs. (6)).

4)In these experiments the materials of the bridge and of the "banks" were identical, so that A = 1 and $T_c = T_{cB}$.

5)Or, according to the developed theory, preferably the effect of "insufficient voltage."

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