Stark profiles of hydrogen spectral lines in a plasma with Langmuir turbulence

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We consider Stark profiles of hydrogen spectral lines in a plasma, with account taken of the nonadiabatic action of the Langmuir noise. It is shown that the decrease of the lifetime of the atom in an individual Stark state under the influence of Langmuir noise leads to two principal effects: the appearance of characteristic "reliefs" on the quasistatic profiles of the sideband components, and broadening of the central component. Both stochastic and regular plasma oscillations are considered, and a comparison is made with the existing experimental results. It is shown that measurement of the "reliefs" on the line profile makes it possible to determine such turbulent-plasma parameters as the noise energy density and the characteristic frequency of the nonlinear processes.

1. INTRODUCTION

The theory of Stark broadening of spectral lines is based on separate allowance for the low-frequency (LF) and high-frequency (HF) components of the electric microfield [1,2]. Accordingly, one of us [3] has formulated a theory of Stark broadening of hydrogen lines in a turbulent plasma, in which account is taken of the quasistationary character of the action of the LF (ion-sound) and of the non-adiabaticity of the action of the HF (Langmuir) noise on the radiating atom.

The quasistatic theory of the action of LF noise was confirmed in a number of experiments with different methods of turbulence excitation (see, e.g., [4]). This theory was subsequently developed by us in [5]. The non-adiabatic character of the broadening by HF noise has also found confirmation in laboratory [6] and computer [7] experiments. In many theoretical papers [8-10], however, the nonadiabaticity of the action of the HF noise on the hydrogen atom was ignored. Within the framework of the adiabatic approximation, the action of the HF noise leads to a single effect, namely to the appearance of "satellites," the search for which was the subject of a number of experimental studies [11,12].

The experiments have indeed verified the existence of several singularities on the profile of the hydrogen line near the "satellite" frequencies, but the structure of these singularities differed from that calculated on the basis of the adiabatic model. It is natural to assume that these deviations are due to the inadequacy of the adiabatic approach for Langmuir noise.

In the present paper we analyzed the contour of the hydrogen line with allowance for the simultaneous action of a low-frequency quasistatic field and a high-frequency nonadiabatic field of the oscillations, and principal attention is paid to those singularities on the quasistatic profile which are due to the resonant change of the lifetime of the atom on the Stark sublevel in the presence of a high level of stochastic Langmuir noise. As will be shown later on, this approach makes it possible to interpret the experimental results more completely and incontrovertibly than in the case of the adiabatic approximation.

In [8-10], no attention was paid to the difference between the "stochastic" and "stationary" random fields. It is known from the theory of random processes (see, e.g., [13]) that if the evolution has a stochastic character, then the state of a system at the instant of time t cannot be, generally speaking, uniquely determined by its states in preceding instants $s < t$, and is described by probability laws. E. V. Lifshitz [8] has represented the electric field of a one-dimensional spectrum of Langmuir noise in the form

$$E_l(t)=\sum_{j=1}^{J} E_j\cos(\Omega_j t+\varphi_j),$$

where the amplitude E_j and the frequency Ω_j are given functions of the wave number k_j; the phase shift φ_j is a random quantity uniformly distributed over the interval $(0, 2\pi)$; j is the number of the wave packet[1)]. This function $E_l(t)$ corresponds to a stationary random process which, in spite of the terminology of [8], is not stochastic. Indeed, in each term the phase shift φ_j, once chosen (at the instant s_0), does not vary in time, and the field $E_l(t)$ is uniquely determined for any $t > s_0$. The corresponding correlation function oscillates with a frequency close to the plasma frequency Ω_l. A physical consequence of this model is the absence of additional damping of the atomic oscillator and the appearance in its spectrum of satellites at frequencies that are multiples of Ω_l.

The inconsistency in the terminology was preserved also in later papers of Yakovlev and Dolginov [9,10]. Thus, Yakovlev [9] cites the results of the paper of E. V. Lifshitz [8] as a calculation of the line profile in stochastic fields. Moreover, he makes in [9] the curious statement that according to [8] the profile of the Stark component has a Lorentz shape with a half-width

$$\gamma=2\alpha^2 s^2\int_0^\infty \langle E(0)E(t)\rangle\, dt.$$

It is easy to verify, however, that no profile obtained by E. V. Lifshitz has or can have a Lorentz shape, inasmuch as in principle the formulation of the problem does not include the damping connected with a finite phase memory. As to the foregoing integral of the oscillating correlation function with respect to time, it is simply equal to zero for the field considered by E. V. Lifshitz.

Yakovlev and Dolginov [9,10] have proposed to designate the plasma oscillation field as stochastic at $T_0\Omega_{HF} \lesssim 1$ (T_0 is the smallest time scale of the plasma turbulence), when the oscillations are transformed into an aperiodic process, and to call the field regular at $T_0\Omega_{HF}$

$\gg 1.$[2)] However, such a field is ipso facto stochastic in both cases, since it is strictly determined only on finite time intervals (smaller than T_0). A reasonable classification of fields into regular and stochastic can be carried out only when account is taken of the reaction of the atom to the field. To this end it is necessary to compare T_0 not with the period of the field $T_{HF} = 2\pi\Omega_{HF}^{-1}$, but with the time t_0 characteristic of the atom. In a plasma, t_0 is determined by the frequency of the strong collisions γ_S:

$$t_0^{-1} \sim \gamma_s \sim n^2\hbar^2 N / m^2 V_{Te}$$

(n is the principal quantum number, N is the density of the plasma, and m and V_{Te} is the mass and thermal velocity of the electrons), which lead to a "destruction" of this Stark state with a probability close to unity [1].

The difference between the "stochastic" and "stationary" random fields is not significant in the calculation of the instantaneous characteristics, but is decisive for the temporal correlation functions, which enter in the expressions for the line profile. As a rule, the nonlinear processes in the plasma lead to a stochastic character of the oscillations. For two-stream instability, for example, the Langmuir noise consists of trains of harmonic oscillations with average duration $T_0 \approx$ (3 to 10)$T_{HF} \ll \gamma_S^{-1}$ [14]. Nonetheless, experimental conditions are possible in which $T_0 \gg \gamma_S^{-1}$. In this study we investigate both stochastic and regular action of HF noise, and demonstrate the decisive role played by nonadiabatic effects in the formation of the line profile.

2. FORMULATION OF PROBLEM

In a plasma, the hydrogen atom is situated in an electric field that can be regarded as a superposition of a quasistatic field $\mathbf{F}$ produced by the ions and by the LF noise, and an HF field $\mathbf{E}(t)$ produced by the strong ($\mathbf{E}_S(t)$) and weak ($\mathbf{E}_W(t)$) collisions during individual passages of the electron inside the Debye sphere, and by the Langmuir noise $E_l(t)$. The stochastic field of the Langmuir noise can be represented in the form

$$\mathbf{E}_l(t) = \sum_{j=1}^{J} \mathbf{E}_j(t) \cos\{\Omega_j t + \varphi_j(t)\}, \qquad (1)$$

where the phase shift $\varphi_j(t)$ and the amplitude $E_j(t)$ change their values with each change of the state of a certain Poisson process with average counting rate γ_l; between the changes of the state, the $\varphi_j(t)$ and the components $E_j^\sigma(t)$ are constant and assume random values with a definite distribution law. In particular, the phase shift φ_j is uniformly distributed over the interval $(0, 2\pi)$ with a density $1/2\pi$. The frequency $\gamma_l \lesssim \boldsymbol{\Omega}_l$ is the largest of the characteristic frequencies of the nonlinear processes (generation of oscillations, induced scattering by particles, etc.). The stochastic function $\mathbf{E}_l(t)$ in (1) is a realization of a jumplike (purely discontinuous) homogeneous Markov stationary random process.

We confine ourselves to the case when the average amplitude $E_0 \equiv (\langle|\mathbf{E}_l(t)|^2\rangle)^{1/2}$ of the HF noise is small in comparison with the average value F_0 of the LF field[3)]. Then the inequality $|\mathbf{E}_l(t)| \ll F$ holds for most atoms and for the greater part of the time. In this case the field $\mathbf{F}$ determines the quantization of the atom in the upper (a) and lower (b) states and the precession frequencies $\omega_F^{a,b} = 3n_{a,b}ea_0F/2\hbar$ of its dipole moment $\mathbf{d}$, while the HF field can be regarded as a perturbation.

The field $\mathbf{E}_S(t)$ of the strong collisions constitutes short-time bursts (of duration $\sim n\hbar/T_e$) of large amplitude ($E_S^{max} \gg (F, E_0)$), the appearance of which is a Poisson process with average counting rate γ_S. A strong collision disrupts the initial (α) and final (β) states of the atom and ceases the process of emission of a given Stark component.

The additive description in [2] of the strong and weak collisions in the impact-broadening operator does not reflect the situation adequately. Even in the initial expression for the profile of the component $S_{\alpha\beta}$ it is necessary to take explicit account of the fact that the autocorrelation function of the light-wave amplitude differs from zero only in the time interval $\Delta\tau$ between the strong collisions, which is a random quantity with a distribution density $\gamma_S \exp(-\gamma_S\Delta\tau)$:

$$S_{\alpha\beta}(\omega, F) = \mathrm{Re}\int_0^\infty d(\Delta\tau)\gamma_s e^{-\gamma_s\Delta\tau}\int_0^{\Delta\tau} d\tau\, e^{i(\omega-\omega_{\alpha\beta})\tau} \times \sum_{\alpha',\beta'} (\mathbf{d}_{\beta\alpha}\mathbf{d}_{\alpha'\beta'})\{[T(\tau,0)]_{\alpha\alpha'}[T^*(\tau,0)]_{\beta\beta'}\}_{cp}, \qquad (2)$$

where ω is the observable frequency reckoned from the unperturbed position ω_0; $\omega_{\alpha\beta}(F) = (d_{\alpha\alpha} - d_{\beta\beta})F/\hbar$.

The evolution operator is

$$T(\tau,\tau') = \exp\left\{\frac{i}{\hbar}H_0(\tau-\tau')\right\}\exp\left\{-\frac{i}{\hbar}\left[H_0(\tau-\tau') + \int_{\tau}^{\tau'} dt(V_w + V_l)\right]\right\},$$

where H_0 is the Hamiltonian of the hydrogen atom in the static field $\mathbf{F}$, $V_W = \mathbf{d}\cdot\mathbf{E}_W(t)$, and $V_l = \mathbf{d}\cdot\mathbf{E}_l(t)$. Since the random functions $\mathbf{E}_W(t)$ and $\mathbf{E}_l(t)$ are not correlated, the action of weak collisions in the HF noise can be calculated independently. In particular, the effect of weak collisions reduces to a replacement in $[T_a(\tau, 0)T_b^*(\tau, 0)]$ of the operator

$$-\frac{i}{\hbar}\int_0^\tau dt(\mathbf{d}_a - \mathbf{d}_b^*)\mathbf{E}_w(t)$$

by $\Phi_{ab}^e\tau$, where Φ_{ab}^e is the operator of impact electron broadening [2] (after subtracting the term describing the strong collisions).

For the case of isolated lines ($\omega_F^{(a)} - \omega_F^{(b)} \gg \gamma_S$), only the diagonal matrix elements of the evolution operator are significant [15]. In this case

$$[T_a(\tau,0)T_b^*(\tau,0)]_{\alpha\alpha\beta\beta} \approx \approx \exp\{[\Phi_{ab}^e]_{\alpha\alpha\beta\beta}\tau\}\left[P\exp\left(-\frac{i}{\hbar}\int_0^\tau dt(V_{la} - V_{lb}^*)'\right)\right]_{\alpha\alpha\beta\beta}, \qquad (3)$$

where the prime denotes the interaction representation and P is the chronological operator [16]. The result for a simultaneous perturbation of the levels a and b can be obtained in rather simple manner from the corresponding results for one level [1,15]. Thus, our problem reduces to a calculation of the expression

$$[T_l(\tau,0)]_{\alpha\alpha} = P\exp\left(-\frac{i}{\hbar}\int_0^\tau dt\, \mathbf{d}\mathbf{E}_l(t)\right). \qquad (4)$$

3. EVOLUTION OPERATOR IN THE STOCHASTIC CASE

As indicated in the introduction, the atom senses the HF noise as stochastic when $\gamma_l \gg \gamma_S$. This means that in the time interval $\Delta\tau \sim \gamma_S^{-1}$ between the strong collisions there occur many ($\sim\gamma_l\Delta\tau$) sudden changes of the phase and of the amplitude of the wave packets, so that each of the quantities $\varphi_j(t)$ and $\mathbf{E}_j(t)$ of (1), say $\varphi_j(t)$, can be represented in the form

$$\varphi_j(t)=\sum_{q=0}^{Q(t)}\varphi_{jq}\theta(t-T_{jq})\theta(T_{jq+1}-t),\qquad \varphi_{jq}=\text{const.}$$

Here T_{jq} is the instant of the q-th collapse of the phase and the amplitude of the j-th wave packet; Q(t) is the number of such collapses within a time t ($Q(t)\approx\gamma_l t$ at $t\gg\gamma_l^{-1}$).

In first-order perturbation theory we have $[T_l(\tau,0)-1]^{(1)}_{\alpha\alpha}=0$.

In second-order we have

$$[T_l(\tau,0)-1]^{(2)}_{\alpha\alpha}=-\frac{1}{2\hbar^2}\sum_{\alpha'}\sum_{j,j'=1}^{J}\sum_{q=0}^{\gamma_l\tau}\int_{T_{jq}}^{T_{jq+1}}dt\times\sum_{q'=0}^{q}\int_{T_{j'q'}}^{T_{j'q'+1}}dt'(\mathbf{d}_{\alpha\alpha'}\mathbf{E}_{jq})(\mathbf{d}_{\alpha'\alpha}\mathbf{E}_{j'q'})\exp\{i(\omega_{\alpha\alpha'}t+\omega_{\alpha'\alpha}t')\}$$
$$\times[\cos(\Omega_jt-\Omega_{j'}t'+\varphi_{jq}-\varphi_{j'q'})+\cos(\Omega_jt+\Omega_{j'}t'+\varphi_{jq}+\varphi_{j'q'})].\tag{5}$$

When summing over q′, the second term in the square brackets vanishes, and a nonzero contribution remains in the first term only from the terms with j′ = j and q′ = q. Recognizing that $\omega_{\alpha'\alpha}=-\omega_{\alpha\alpha'}$ and $(\mathbf{d}_{\alpha\alpha'}\mathbf{E}_{jq})(\mathbf{d}_{\alpha'\alpha}\mathbf{E}_{jq})=|\mathbf{d}_{\alpha\alpha'}|^2E_{jq}^2/3$, we obtain[4)]

$$[T_l(\tau,0)-1]^{(2)}_{\alpha\alpha}=-\frac{1}{12\hbar^2}\sum_{\alpha'}\sum_{j=1}^{J}\sum_{q=0}^{\gamma_l\tau}|\mathbf{d}_{\alpha\alpha'}|^2E_{jq}^2\Bigg[\frac{i\Delta T_{jq}}{\omega_{\alpha\alpha'}-\Omega_j}$$
$$+\frac{1-\exp\{i(\omega_{\alpha\alpha'}-\Omega_j)\Delta T_{jq}\}}{(\omega_{\alpha\alpha'}-\Omega_j)^2}+\frac{i\Delta T_{jq}}{\omega_{\alpha\alpha'}+\Omega_j}+\frac{1-\exp\{i(\omega_{\alpha\alpha'}+\Omega_j)\Delta T_{jq}\}}{(\omega_{\alpha\alpha'}+\Omega_j)^2}\Bigg].\tag{6}$$

The time interval ΔT_{jq} between the phase collapses is a random quantity distributed with a density $\gamma_l\exp(-\gamma_l\Delta T_{jq})$. The amplitudes H_{jq} have a Rayleigh distribution $W_R(E_{jq})$ [5,17] with a mean-squared value E_j^2.

Introducing the form of the spectrum of the Langmuir noise $P_T(\Omega)$ due to the thermal motion, in accordance with the relation

$$\sum_{j=j'}^{j''}E_j^2=E_0^2\int_{\Omega_{j'}}^{\Omega_{j''}}d\Omega P_T(\Omega),\qquad E_0^2=\sum_{j=1}^{J}E_j^2,\tag{7}$$

we obtain

$$[T_l(\tau,0)-1]^{(2)}_{\alpha\alpha}=\Bigg\{-\frac{E_0^2}{12\hbar^2}\sum_{\alpha'}|\mathbf{d}_{\alpha\alpha'}|^2\int_{\Omega_1}^{\Omega_2}d\Omega P_T(\Omega)$$
$$\times\Bigg[\frac{1}{\gamma_l-i(\omega_{\alpha\alpha'}-\Omega)}+\frac{1}{\gamma_l+i(\omega_{\alpha\alpha'}+\Omega)}\Bigg]\Bigg\}\tau.\tag{8}$$

Inasmuch as $[T_l(\tau+\Delta\tau,\tau)-1]^{(2)}_{\alpha\alpha}=[T_l(\Delta\tau,0)-1]^{(2)}_{\alpha\alpha}$ ll $\Delta\tau$, it follows that by using the impact-approximation formalism [1,2] we obtain the equation

$$\frac{d}{d\tau}[T_l(\tau,0)]_{\alpha\alpha}=\Phi_{\alpha\alpha}[T_l(\tau,0)]_{\alpha\alpha},$$

where $\Phi^c_{\alpha\alpha}$ is the expression in the curly brackets of (8) and depends on the quasistatic field F.[5)]

Consequently

$$[T_l(\tau,0)]_{\alpha\alpha}=\exp[\Phi_{\alpha\alpha}{}^c(F)\tau].\tag{9}$$

Thus, the action of the stochastic HF noise leads to the appearance of a collective width $\Gamma^c_\alpha=-\mathrm{Re}\,\Phi^c_{\alpha\alpha}$ and a collective shift $D^c_\alpha=-\mathrm{Im}\,\Phi^c_{\alpha\alpha}$; these depend in resonant manner on the quasistatic field[6)].

Considering for the sake of argument the case when γ_l greatly exceeds the characteristic width $(\Delta\Omega)_T$ of the spectrum $P_T(\Omega)$, we obtain

$$\Gamma_\alpha{}^c=\frac{E_0^2\gamma_l}{12\hbar^2}\Bigg[\frac{2\mathbf{d}_{\alpha\alpha}^2}{\gamma_l^2+\Omega_l^2}+(|\mathbf{d}_{\alpha\alpha-1}|^2+|\mathbf{d}_{\alpha\alpha+1}|^2)\times\Bigg(\frac{1}{\gamma_l^2+(\omega_F^{(a)}-\Omega_l)^2}+\frac{1}{\gamma_l^2+(\omega_F^{(a)}+\Omega_l)^2}\Bigg)\Bigg],\tag{10}$$

$$D_\alpha{}^c=\frac{E_0^2\gamma_l}{12\hbar^2}(|\mathbf{d}_{\alpha\alpha-1}|^2-|\mathbf{d}_{\alpha\alpha+1}|^2)\Bigg[\frac{\omega_F^{(a)}-\Omega_l}{\gamma_l^2+(\omega_F^{(a)}-\Omega_l)^2}+\frac{\omega_F^{(a)}+\Omega_l}{\gamma_l^2+(\omega_F^{(a)}+\Omega_l)^2}\Bigg],$$

where

$$\mathbf{d}_{\alpha\alpha}^2=\frac{9}{4}e^2a_0^2n_a^2(n_1-n_2)_\alpha^2,\qquad|\mathbf{d}_{\alpha\alpha-1}|^2-|\mathbf{d}_{\alpha\alpha+1}|^2=\frac{\mathbf{d}_{\alpha\alpha}^2}{(n_1-n_2)_\alpha},$$
$$|\mathbf{d}_{\alpha\alpha-1}|^2+|\mathbf{d}_{\alpha\alpha+1}|^2=\frac{\mathbf{d}_{\alpha\alpha}^2[n^2-(n_1-n_2)-m^2-1]_\alpha}{2(n_1-n_2)_\alpha^2}.\tag{11}$$

The formulas for the simultaneous perturbation of the upper and lower levels can be obtained from (10) by replacing $|(\mathbf{d}_a)_{\alpha\alpha'}|^2$ by

$$|(\mathbf{d}_a-\mathbf{d}_b)_{\alpha\alpha'\beta\beta'}|^2=|(\mathbf{d}_a)_{\alpha\alpha'}|^2\delta_{\beta\beta'}+|(\mathbf{d}_b)_{\beta\beta'}|^2\delta_{\alpha\alpha'}-2(\mathbf{d}_a)_{\alpha\alpha}(\mathbf{d}_b)_{\beta\beta}.$$

As a result we have

$$\Gamma_{\alpha\beta}{}^c=\Gamma_\alpha{}^c+\Gamma_\beta{}^c-\frac{(d_a)_{\alpha\alpha}(d_b)_{\beta\beta}E_0^2\gamma_l}{3\hbar^2(\gamma_l^2+\Omega_l^2)},\qquad D_{\alpha\beta}{}^c=D_\alpha{}^c-D_\beta{}^c.\tag{12}$$

4. LINE PROFILE IN THE STOCHASTIC CASE

1. We consider first the profile of the central component $S_0(\omega)$, for which $\omega_{\alpha\beta}=0$. In this case the collective shift $D^c_{\alpha\beta}$, as well as the electron impact shift $D^e_{\alpha\beta}=-\mathrm{Im}\,\Phi^e_{\alpha\beta}$ (see [15]), is equal to zero. The total profile of the component is obtained by averaging over the distribution $W_{st}(F)$ of the LF fields:

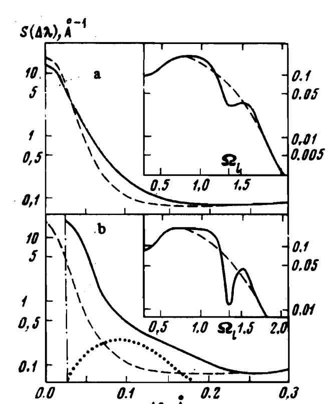
$$S_0(\omega)=|\mathbf{d}_{\alpha\beta}|^2\int_0^\infty dF\,W_{st}(F)\frac{\Gamma_{\alpha\beta}{}^e+\Gamma_{\alpha\beta}{}^c(F)}{[\Gamma_{\alpha\beta}{}^e+\Gamma_{\alpha\beta}{}^c(F)]^2+\omega^2},\tag{13}$$

where $\Gamma^e_{\alpha\beta}=-\mathrm{Re}\,\Phi^e_{\alpha\beta}$ was calculated in [15]. Under typical experimental conditions we have $\omega^{(a)}_{F_0}\ll\Omega_l$. In this case we have for the values of F that are significant in the integration

$$\Gamma_{\alpha\beta}{}^c(F)\approx\Gamma_{\alpha\beta}(0)=\frac{2\gamma_l(\varepsilon_{1\alpha}^2+\varepsilon_{1\beta}^2)}{\Omega_l^2},\qquad\varepsilon_{1\nu}^2=\frac{(|\mathbf{d}_{\nu\nu-1}|^2+|\mathbf{d}_{\nu\nu+1}|^2)E_0^2}{12\hbar^2},\qquad\nu=\alpha,\beta.\tag{14}$$

Consequently, the central component has a Lorentz profile with half-width $(\Delta\omega)_{1/2}\approx\Gamma^e_{\alpha\beta}+\Gamma^c_{\alpha\beta}(0)$ (see Fig. a).

At a sufficiently high level of the LF noise, the relation $\omega^{(b)}_{F_0}\gg\Omega_l$ may be satisfied. In this case



Profile of Ly-α hydrogen line in a turbulent plasma with $N\approx10^{17}$ cm^{-3}, $T_e\approx100$ eV, and $T_a\approx0.1$ eV following excitation of strong HF ($F_0\approx500$ kV/cm) and Langmuir ($E_0\approx300$ kV/cm) noise: a) the HF noise is stochastic ($\gamma_l\approx2\times10^{12}\,\text{sec}^{-1}\gg\gamma_s\approx3\times10^9\,\text{sec}^{-1}$); b) the HF noise is regular ($\gamma_l\ll3\times10^9$ sec^{-1}); the dash-dot curve shows the red shift of the maximum; the dotted curve shows the profile of the nonadiabatic "satellite" of the central component. The dashed line shows the profile in the absence of HF noise.

$$\Gamma_{\alpha\beta}{}^c(F)\approx\Gamma_{\alpha\beta}{}^c(F_0)\approx 2\gamma_l[(\varepsilon_{1\alpha}/\omega_F)^2+(\varepsilon_{1\beta}/\omega_F^{(b)})^2]. \quad (15)$$

2. We now consider the singularities of the sideband profiles. It follows from (13) that an atom executing a transition $\alpha\to\beta$ in a field F emits a Lorentz line of width $\Gamma^e_{\alpha\beta}+\Gamma^c_{\alpha\beta}(F)$ with a center at the frequency $\omega_{\alpha\beta}(F)+D^e_{\alpha\beta}+D^c_{\alpha\beta}(F)$. Since different atoms experience, generally speaking, the action of different fields F, the observed profile is the envelope of Lorentz contours radiated at different values of F. As a rule, we have

$$\omega_{F_0}^{(a,b)}\gg\Gamma_{\alpha\beta}{}^e+\Gamma_{\alpha\beta}{}^c(F_0).$$

In this case the envelope is for the most part proportional to $W_{st}(F)$[3,5]. The deviations from the quasistatic profile come from radiators for which the precession frequency $\omega_F^{(a,b)}$ for the upper or lower level is at resonance with the frequency Ω_l of the HF noise. On the sideband component contour, this corresponds to the frequencies[7)]

$$\omega_1=\left[(n_1-n_2)_\alpha-\frac{n_b}{n_a}(n_1-n_2)_\beta\right]\Omega_l,$$
$$\omega_2=\left[\frac{n_a}{n_b}(n_1-n_2)_\alpha-(n_1-n_2)_\beta\right]\Omega_l=\frac{n_a}{n_b}\omega_1. \quad (16)$$

Let us investigate first the case when $n_1-n_2=0$ for the upper or lower level (for the sake of argument we put $(n_1-n_2)_\alpha=0$). Then the collective shift is $D^c_\alpha=D^e_\alpha=0$, and for the frequencies ω near ω_1 the profile of the component can be represented in the form

$$S_{\alpha\beta}(\omega)\approx|\mathbf{d}_{\alpha\beta}|^2\int_0^\infty dFW_{st}(F)\frac{\Gamma_\alpha{}^e+\Gamma_\alpha{}^c(F)}{[\Gamma_\alpha{}^e+\Gamma_\alpha{}^c(F)]^2+(\omega+d_{\beta\beta}F/\hbar)^2}. \quad (17)$$

The qualitative character of the singularities of the contour at the frequency ω_1 is clear directly from (17). Atoms situated in a field $F^*_a=\hbar\Omega_l/|d_{\alpha\alpha}-d_{\alpha-1\,\alpha-1}|$ radiate a broader Lorentz profile (in comparison with other atoms), which is consequently less intense in its central part and more intense in the wings. Therefore the envelope passes lower than its "quasistatic" position near the frequency $\omega=-d_{\beta\beta}F^*_a/\hbar=\omega_1$ and higher near the frequencies $\omega_1\pm\Delta\omega$ ($\Delta\omega\sim(\epsilon^2_{1\alpha}\gamma_l)^{1/3}$. Such a "relief", which consists of a "valley" and two "hills," will be clearly pronounced in the case $\epsilon^2_{1\alpha}/\gamma_l\gg(\Gamma^e_\alpha,\gamma_l)$. Calculation shows that under this condition the half-width of the valley is

$$(\Delta\omega_r)_{1/2}=[\varepsilon_{1\alpha}{}^2\gamma_l(n_1-n_2)_\beta{}^2n_b{}^2/n_a{}^2]^{1/3},$$

and its depth amounts to 2/3 of the unperturbed value of the intensity.

3. Let us investigate finally the joint action of the collective width and the collective shift. It follows from (11) and (15) that if, for example, $(n_1-n_2)_\alpha>0$, then $D^c_\alpha>0$ at $\omega_F^{(a)}>\Omega_l$ and $D^c_\alpha<0$ at $\omega_F^{(a)}<\Omega_l$, with $|D^c_\alpha|$ reaching a maximum at $|\omega_F^{(a)}-\Omega_l|=\gamma_l$. This means that the maximum of the radiation intensity shifts in the red direction for atoms in a field $F<F^*_a$, and in the blue direction for atoms in a field $F>F^*_a$. One should therefore expect simultaneous action of the collective width and collective shift to cause the "relief" near the frequency ω_1 to become more clearly pronounced (see Fig. a).[8)]

For the frequency ω_2, the result depends on the ratio of the signs of $(n_1-n_2)_\beta$ and ω_2. If the signs are opposite, then the result coincides with that described above. If the signs are the same[9)], then the effects of the width and of the shift are oppositely directed. Indeed, $D^c_{\alpha\beta}=D^c_\alpha-D^c_\beta$, and in the case, for example, when $(n_1-n_2)_\beta>0$, the maximum of the radiation intensity shifts toward the blue at $F<F^*_b$ and towards the red at $F>F^*_b$, thus leading to the "relief smearing" connected with the collective width $\Gamma^c_\beta(F)$.

Calculations shows that at $\omega_F^{(a)}\gg\epsilon_{2\alpha}\gg\gamma_l$ in the "relief" at the frequency ω_1 the half-width of the valley is $(\Delta\omega_r)_{1/2}=\epsilon_{2\alpha}\sqrt{\omega_1/\Omega_l}$, and its depth is $\sim[1-(\epsilon^2_{1\alpha}\gamma_l/2\epsilon^3_{2\alpha})\times\sqrt{\omega_1/\Omega_l}]$, where $\epsilon^2_{2\alpha}\equiv|(\mathbf{d}_{\alpha\alpha-1}|^2-|\mathbf{d}_{\alpha\alpha+1}|^2)E_0^2/12\hbar^2$. For the frequency ω_2, in the case $(n_1-n_2)_\beta\omega_2>0$, under analogous conditions, the "relief" turns out to be shallow (the depth of the valley is $(\epsilon_{2\alpha}\gamma_l/2\epsilon^2_{1\alpha})\sqrt{\Omega_l/\omega_2}\ll 1$).

5. CASE OF REGULAR ACTION OF HF NOISE

1. At $\gamma_S\gg\gamma_l$, the phase and the amplitude of the wave packet in (1) can be regarded in the time interval $\Delta\tau\sim\gamma_S^{-1}$ as random quantities that do not vary with time.

In second order, calculations analogous to (5) and (6) yield[10)]

$$[T_l(\tau,0)-1]^{(2)}_{\alpha\alpha}=-\frac{1}{12\hbar^2}\sum_{\alpha'}\sum_{j=1}^{J}|\mathbf{d}_{\alpha\alpha'}|^2E_j{}^2\left[\frac{i\tau}{\omega_{\alpha\alpha'}-\Omega_j}+\frac{1-\exp\{i(\omega_{\alpha\alpha'}-\Omega_j)\tau\}}{(\omega_{\alpha\alpha'}-\Omega_j)^2}+\frac{i\tau}{\omega_{\alpha\alpha'}+\Omega_j}+\frac{1-\exp\{i(\omega_{\alpha\alpha'}+\Omega_j)\tau\}}{(\omega_{\alpha\alpha'}+\Omega_j)^2}\right]. \quad (18)$$

Let us write out separately the "elastic" part $[T_l-1]_{el}$, i.e., the terms with $\alpha'=\alpha$:

$$[T_l(\tau,0)_{\alpha\alpha}-1]^{(2)}_{el}=-\frac{d_{\alpha\alpha}{}^2}{6\hbar^2}\sum_{j=1}^{J}\frac{E_j{}^2}{\Omega_j{}^2}(1-\cos\Omega_j\tau). \quad (19)$$

In higher even orders, the calculations can be carried through to conclusion in the general form only for the "elastic" part of the evolution operator. It turns out here that

$$[(T_l(\tau,0)_{\alpha\alpha}]_{el}=\exp\left\{-\frac{d_{\alpha\alpha}{}^2}{6\hbar^2}\sum_{j=1}^{J}\frac{E_j{}^2}{\Omega_j{}^2}(1-\cos\Omega_j\tau)\right\} \quad (20)$$

corresponds to the correlation function obtained by E. V. Lifshitz[8]. The neglect, in that reference, of the "inelastic" part of the evolution operator is justified only in the adiabatic case, when $|\mathbf{d}_{\alpha\alpha'}|E_0/\hbar\Omega_l\gg 1$. The envelope of the satellites that appear at frequencies that are multiples of Ω_l is then proportional to the Rayleigh distribution $W_R(E)$ of the field amplitudes. This result corresponds to the quasistatic approximation described in[3,5]. In the nonadiabatic case ($|\mathbf{d}_{\alpha\alpha'}|E_0/\hbar\Omega_l\ll 1$) formula (20) leads to $[(T_l(\tau,0)_{\alpha\alpha}]_{el}\approx 1$, and we can expect effects connected with the HF noise to become manifest precisely in the "inelastic" part of the evolution operator.

We calculate first $[T_l(\tau,0)_{\alpha\alpha}]_{inel}$ for one harmonic Ω_j of the HF field. In the case $|\omega_F^{(a)}-\Omega_j|\gg\epsilon_{1\alpha j}$, as can be easily verified, we have

$$[T_l(\tau,0)_{\alpha\alpha}]_{inel}\approx\exp\left\{-i\tau\varepsilon^2_{1\alpha j}\frac{2\omega_F^{(a)}}{(\omega_F^{(a)})^2-\Omega_j{}^2}\right\}. \quad (21)$$

On the other hand, in the case of resonance ($|\omega_F^{(a)}-\Omega_j|\ll\epsilon_{1\alpha j}$) it turns out that[11)]

$$[T_l(\tau,0)_{\alpha\alpha}]_{inel}\approx\cos(\varepsilon_{1\alpha j}\tau). \quad (22)$$

Formulas (21) and (22) mean that the regular action of the HF field does not lead to an additional damping of the atomic oscillator, but only changes the radiation frequency. The Stark component emitted in the field F, for which $|\omega_F - \Omega_j| \gg \epsilon_{1j}$, is shifted by an amount $D^c(F) = 2\epsilon_{1j}^2\omega_F/(\omega_F^2 - \Omega_j^2)$; on the other hand, if F is such that $|\omega_F - \Omega_j| \ll \epsilon_{1j}$, then a splitting into two components of equal intensity takes place, separated in frequency by $2\epsilon_{1j}$. The splitting is noticeable if the width of each component is $\Gamma^e \ll \epsilon_{1j}$.

2. The considered effect is most strongly reflected in the profile of the sideband components. Reasoning analogous to that given in Sec. 4, leads to the following conclusion: on the profile of each sideband component there appear, at the frequencies ω_1 and ω_2 given by (16), "reliefs" whose peaks are spaced $4\epsilon_{1\nu j}\omega_{1,2}/\Omega_l$ apart ($\nu = \alpha, \beta$; see Fig. b). The conclusion drawn for a single harmonic Ω_j remains in force for the case of a narrow spectrum of HF noise with width $(\Delta\Omega)_T \ll \epsilon_{1\nu}$ (with $\epsilon_{1\nu j}$ replaced in all formulas by $\epsilon_{1\nu}$).

The profile of the central component, under the typical experimental conditions $\omega_{F_0} \ll \Omega_l$, also changes under the influence of the regular HF field; the component undergoes (for most emitters) a "red" shift that is linear in the static field. According to (21), the maximum emission of the lines with the central component shifts in the "red" direction by an amount $D^c(F_0) = 2\omega_{F_0}\epsilon_1^2/\Omega_l^2$ (see Fig. b). On the other hand, if $\omega_{F_0} \gg \Omega_l$, then the shift becomes "blue," and decreases to $\sim 2\epsilon_1^2/\omega_{F_0}$. Under the influence of the regular HF field, the nonadiabatic transitions become manifest, furthermore, in the form of two "satellites" whose frequencies ω_s are not connected with the frequency of the regular field, but are determined by its amplitude, $\omega_s \approx \epsilon_{1\nu}$. At $\Omega_l \gg \epsilon_{1\nu} \gg (\Delta\omega)_{1/2}$, the profiles of the "satellites" take the form

$$S_s(\omega) = [F_0 W_{st}(\Omega_l F_0/\omega_{F_0})][E_0 W_R(\omega E_0/\varepsilon_{1\nu})]\omega/\omega_{F_0}\varepsilon_{1\nu}.$$

Naturally, to observe these "satellites" it is necessary to satisfy the inequality

$$[F_0 W_{st}(\Omega_l F_0/\omega_{F_0})]\varepsilon_{1\nu}^2/\omega_{F_0}(\Delta\omega)_{1/2} \gg 1.$$

6. DISCUSSION

1. The foregoing analysis shows that the nonadiabatic action of HF noise of not too large an amplitude $E_0 \ll (F_0, F^*)$ leads to the appearance, on the line profile, of a characteristic "relief" near the frequencies $\omega_1(\Omega_l)$ and $\omega_2(\Omega_l)$ determined by formulas (16). The intensity oscillations constitute an appreciable fraction of the fundamental quasistatic profile $S_{ab}(\omega_{1,2})$ if $E_0 \gg |d_{\alpha\alpha'}|^{-1}\hbar \max(\gamma_l, \gamma_s)$. The conditions for observing the "relief" are most favorable when $F_0 \lesssim F^*$, for in this case the frequencies $\omega_{1,2}$ corresponds to the line wings (see the figure). On the other hand, if $F_0 \gg F^*$, then $\omega_{1,2}$ correspond to the central part of the profile, and the "relief" can be observed only for lines without a central component, especially if the line is formed by transitions from highly-excited states.

In the case $F_0 \gg E_0 \gg F^*$, the action of the HF noise is adiabatic and the line profile is proportional for the most part to the convolution $W_R(E)*W_{st}(F)$, a fact corresponding to the quasistatic approximation [3,5].

2. When the HF field amplitude increases to $E_0 \gg F_0$, the conditions for the applicability of perturbation theory are violated. In this case the direction of the quasistatic field **F** can no longer be used as the quantization axis. To determine the frequency of the nonadiabatic transitions γ_{nonad} it is natural to use a coordinate system with OZ axis along the resultant field **E**(t) + **F**, and to estimate the time interval $\Delta\tau$ during which one can speak of conservation of the quantum numbers n_1, n_2, and m. Nonadiabatic transitions to other Stark states will be caused by "magnetic" interaction [18], so that the transition probability $W_{\alpha'\alpha}$ can be represented in the form

$$W_{\alpha'\alpha}(\Delta\tau) = \hbar^{-2}\left|(L_x)_{\alpha\alpha'}\int_0^{\Delta\tau} dt\,\dot\psi(t)\exp\left[i\int_0^t \omega^{(\alpha)}_{|E+F|}(t')\,dt'\right]\right|^2,$$

where ψ is the angle of rotation of the resultant field (**E**(t) + **F**) from the initial position along **E**(0).

Without loss of generality, we can turn to the analysis of a simple model in which **E** and **F** are mutually perpendicular. In this case

$$\dot\psi = \Omega_l \frac{EF\sin\Omega_l t}{F^2 + E^2\cos^2\Omega_l t},$$

and it is easy to verify that at $F \ll \sqrt{F^*E}$ the dipole moment of the atom is rotated together with the resultant field through an angle $\overline{\psi} \sim 1$ ($\overline{\psi} < \pi/2$); in this case we have $W_{\alpha'\alpha}(\Delta\tau) \to 1$. Near $\overline{\psi}$, the adiabaticity is strongly violated ($W_{\alpha'\alpha}(\Delta\tau) \to 1$), so that the atom ceases to "follow" the direction of the electric field. Thus, in this case an important role is played by the spatial rotation of the dipole moment of the atom through the angle $\widetilde{\psi} \sim 1$, as a result of which the initial orientation is completely forgotten. The frequency γ_{nonad} which determines the half-width $(\Delta\omega)_{1/2}$ of lines having a strong central component reaches in this case its maximum value $\gamma_{nonad} \sim \Omega_l$. For lines without central components, the value of $(\Delta\omega)_{1/2}$ depends on the ratio of $d_{\alpha\alpha}E_0/\hbar$ and Ω_l. At $d_{\alpha\alpha}E_0/\hbar \ll \Omega_l$, the Stark components coalesce and $(\Delta\omega)_{1/2} \sim \Omega_l$. On the other hand, if $d_{\alpha\alpha}E_0/\hbar \gg \Omega_l$, then $(\Delta\omega)_{1/2}$ is determined by the adiabatic splitting, namely $(\Delta\omega)_{1/2} \sim d_{\alpha\alpha}E_0/\hbar$.

A different physical situation arises in the case $E_0 \gg F_0 \gg \sqrt{E_0F^*}$, namely, the adiabaticity in the rotating coordinate system is violated already at $\psi \ll 1$. Analysis based on the sudden perturbation method, in a coordinate system with OZ axis along **E**(0), shows that when the field direction is reversed the atom goes from the state $(n_1\, n_2\, m)$ into the state $(n_2\, n_1 - m)$. Thus, one can speak of conservation of the quantization axis in the immobile coordinate system. In this case, the main cause of the line broadening is the stochasticity of the HF noise. Consequently, for lines with central component we have $(\Delta\omega)_{1/2} \sim \gamma_l$, and for lines without the central component we have $(\Delta\omega)_{1/2} \sim \max(d_{\alpha\alpha}E_0/\hbar, \gamma_l)$.

3. In the analysis of the experimental data, particular attention should be paid to the wings of the hydrogen lines near the frequencies ω_1 and ω_2 of (16). The appearance of a "relief" is evidence of the presence of HF noise. However, for quantitative conclusions it is necessary to determine the manner in which they are sensed by the atom. This can be done by using the profile of a line with a central component whose half-width increases only in the stochastic case.

We note that in the regular case, as follows from Sec. 5, the satellites and "reliefs" connected respectively with the adiabatic and nonadiabatic effects can exist simultaneously at $\epsilon_1 \ll (\omega_{F_0}, \Omega_l)$. The maximum intensity of the relief is $S_r \sim S(\Omega_l) \gtrsim \omega_{F_0}^{3/2}/\Omega_l^{5/2}$ at $\Omega_l \gg \omega_{F_0}$.[12] The maximum of the satellite intensity is

$$S_s \sim (\varepsilon_1/\Omega_l)^2[S(\omega)]_{max} \sim \varepsilon_1^2/\Omega_l^2\omega_{F_0},$$

consequently, $S_r \gg S_s$ at $\epsilon_1 \ll \omega_{F_0}^{5/4}/\Omega_l^{1/4}$, a limitation which is only slightly less stringent than the initial $\epsilon_1 \ll \omega_{F_0}$ ($E_0 \ll F_0$). We can therefore expect the profile singularities at frequencies that are multiples of Ω_l to be due almost always to nonadiabatic effects.

In a number of presently known experiments [11,12] with turbulent plasma, characteristic "reliefs" located $\omega \sim \Omega_l$ away from the line center were observed on the spectral-line profiles. The results of [11,12] can apparently be interpreted as nonadiabatic effects of HF noise. Gallagher and Levine [11] observed a clearly pronounced "relief" on the H_β profile. They treated the two "peaks" as adiabatic satellites of frequency Ω_l and $2\Omega_l$. The appearance of two satellites of approximately equal intensity is possible only in an HF field $E_0 \sim \hbar\Omega_l/nea_0 \sim 200$ kV/cm. However, under the conditions of [11] ($N \sim 3 \times 10^{15}$ cm^{-3}, $T_e \sim 10$ eV), such strong fields are not very probable, since the corresponding noise level is $E_0^2/4\pi NT_e \sim 1$. Moreover, the profile measured in [11] revealed at a frequency $\omega \sim \Omega_l$ a characteristic "valley," a lowering of the profile in comparison with the quasistatic variation of the intensity; this cannot be explained at all within the framework of the adiabatic theory. According to the theory developed in this paper, the position of this "valley" can be identified with the frequency $\omega_1^{4\sigma} = \Omega_l$ of the most intense component 4σ. The shallower "valley" at the frequency $\omega_1^{2\sigma} = \Omega_l/2$ from the weak component 2σ ($I_{2\sigma}/I_{4\sigma} \approx 0.16$) can be seen in the the central part of the line profile in [11]. The next "reliefs" could appear at the frequencies $3\Omega_l/2$, $2\Omega_l$, etc., corresponding to the far wing outside the range investigated in [11].

An estimate based on the formulas of Secs. 4 and 5 yields $E_0 \approx 18$ kV/cm (for the profile on Fig. 2a of [11]). The LF field intensity calculated from the half-width of the line is $F_0 \approx 42$ kV/cm, so that the condition $E_0 < F_0$ for the existence of the "relief" is satisfied. Unfortunately, the experimental data of Gallagher and Levine are insufficient for an unambiguous determination of the manner in which the atom senses the HF noise, since they did not investigate a line with a central component[13].

A similar explanation can be proposed also for the results of the experiments in [12], where a pronounced lowering of the profile was observed in comparison with the quasistatic variation of the intensity (plasma parameters $N \approx 3 \times 10^{14}$ cm^{-3} and $T_e \sim 10^2$ eV). The corresponding estimates yield $E_0 \approx 5$ and $F_0 \approx 10$ kV/cm.

Thus, the theory developed above explains the singularities of the profiles of the hydrogen lines observed in a number of experiments with turbulent plasma as being due to nonadiabatic effects of the HF noise. It is important to note that a simultaneous detailed study of the profiles of different hydrogen lines (for example H_α and H_β) makes it possible to determine not only the energy density of the HF noise, but also one other essential characteristic of stochastic oscillations, namely the frequency γ_l of the nonlinear processes. Experiments in which such a measurement program is completely realized have not yet been performed to date. But only such a realization of the experiment can provide an unambiguous answer to the question of the stochasticity of the noise and of the noise level.

1) In [8], the number of the packet and the wave number were designated by a single index.

2) It is precisely the latter case which is investigated by Yakovlev and Dolginov quantitatively, and furthermore in the adiabatic approximation. The result, naturally, coincides with that obtained by E. V. Lifshitz at $|d_{n_1n_2m}|\sqrt{\langle E^2\rangle}/\hbar\Omega \ll 1$. This circumstance was not noted by the authors of [9,10] probably because they used a notation different from that in [8].

3) This limitation is discussed in Sec. 6.

4) The distribution of the HF noise is assumed isotropic.

5) When $\alpha' \neq \alpha$ the quantity $|d_{\alpha\alpha'}|$ differs from zero only for transitions between neighboring Stark sublevels $\alpha' = \alpha \pm 1$, for which $|\omega_{\alpha\alpha'}| = \omega_F^{(a)}$. Here and elsewhere we use in addition to α' also the notation $\alpha + u$, where u is the energy difference between the sublevels α' and α in units of $\hbar\omega_F^{(a)}$.

6) The result (9) was obtained in second order perturbation theory. It is easy to verify, however, that at $E_0 \ll F_0$ and $(\gamma_l, \omega_F) \gg \gamma_s$, the perturbation-theory series can be summed in all orders, and expression (9) remains unchanged.

7) In the case of Lyman lines, the singularities appear only at the frequencies ω_1, since the ground state is not degenerate.

8) It is assumed that ω_1 has the same sign as $(n_1 - n_2)_\alpha$. This assumption is valid for intense Stark components, and in the case of the H_α and H_β lines it is valid for all the components with $(n_1 - n_2)_\alpha \neq 0$.

9) For the lines H_α and H_β this assumption is valid for all the intense components with $(n_1 - n_2)_\alpha \neq 0$, with the exception of $2\pi H_\alpha$ and $2\sigma H_\beta$.

10) The quantity $[T_l(\tau, 0) - 1]$ vanishes in all odd orders of perturbation theory.

11) In the case of resonance, the terms proportional to $|d_{\alpha\pm1\alpha\pm2}|^2$, $|d_{\alpha\pm2\alpha\pm3}|^2$ etc. introduce into the evolution operator a contribution that is generally speaking of the same order as that of the terms proportional to $|d_{\alpha\alpha\pm1}|^2$. Calculation for the Ly-α line shows that the deviation from (22) reduces merely to division of the argument of the cosine by $\sqrt{2}$. Of course, formula (22) is exact for the central component of the Ly-α line.

12) We take into account here the fact that the distribution of the quasistatic fields approaches a Holtsmark distribution in the wing even at a high level of the LF noise.

13) The observations of satellites of the forbidden 6632 Å line of HeI can be treated as evidence in favor of the regularity observed in [11].

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106