Stimulated Mandel'shtam–Brillouin scattering in the propagation of Alfven waves in a plasma with random inhomogeneities

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The excitation of sound pulses in the stimulated Mandel'shtam–Brillouin scattering (SMBS) of Alfven waves in a plasma with three-dimensional random inhomogeneities is considered. Equations are derived for the amplitudes of interacting waves moving along a stationary magnetic field. It is shown that random inhomogeneities result in damping of the average field and in strong phase shifts. It is found that resonance generation of sound harmonics should be made possible by the "turbulent" phase shift. The shape of the stationary nonlinear sound waves in a given field of a quasistanding Alfven wave is determined.

It is known that generation of pulses of the low-frequency waves is possible in the interaction of two high-frequency waves with low-frequency waves having a linear dispersion (see, for example,[1-4]). As applied to plasma, stimulated Mandel'shtam-Brillouin scattering (SMBS) has been considered for the isotropic case in[4], where the interaction of electromagnetic and sound waves was studied. Analysis of SMBS for propagation transverse to the external magnetic field of a high-frequency electromagnetic or magnetosonic wave in a plasma is given in[5]. It must be noted that in these experiments, the study of the interaction of the waves was carried out only for the case of homogeneous media. A series of researches have recently been reported, devoted both to nonlinear waves[6-9] and to nonlinear interaction of signals[10,11] in a medium with randomly changing parameters. It has been shown that the random inhomogeneities of the medium lead to the appearance of an effective damping of the average field, which can change considerably the picture of the nonlinear wave processes.

In the present work, the problem is considered of the generation of a nonlinear sound wave in the SMBS of Alfven waves propagating along a constant magnetic field H_0 in a nonabsorbing plasma with random inhomogeneities in the electron concentration. Contracted equations are obtained for the amplitudes of the waves, in which terms appear that correspond to damping as a result of the scattering of the average field from the fluctuations of the plasma density. The shape of the nonlinear sound wave in the given field of two opposing Alfven waves is determined. It is shown that the random inhomogeneities in a heated magnetoactive plasma lead to a strong phase shift in the interaction of the waves; because of this, resonance generation of nonsinusoidal sound wave is observed under certain conditions. The results can be useful for diagnostics of both cosmic plasma (for example, the corona or the chromosphere of the sun) and laboratory plasma, since the statistical parameters of the turbulent plasma can be determined from the character of the interaction.

1. DERIVATION OF THE FUNDAMENTAL EQUATIONS

The initial set of equations for a collision-free plasma in the approximation of magnetic hydrodynamics is of the form[12]

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}-\frac{1}{4\pi\rho}[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}]+\mathbf{G}_{\mathrm{ext}},$$
$$\frac{\partial \rho}{\partial t}+\mathrm{div}\,\rho\mathbf{v}=0,\quad \frac{\partial \mathbf{H}}{\partial t}=\mathrm{rot}[\mathbf{v}\mathbf{H}], \qquad (1)^*$$

where $p = \kappa NT$ is the pressure, N is the ion concentration (we assume the system to be quasineutral), κ is Boltzmann's constant, T is the temperature, $\mathbf{v}$ is the velocity of the ions, $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ is the magnetic field and is equal to the sum of the external magnetic field $\mathbf{H}_0$ and the wave perturbation $\mathbf{h}$, while ρ the plasma density and is determined mainly by the ions. The latter can be divided into two parts: $\rho = \rho_0 + \tilde{\rho}$, where ρ_0 is the value of the density unperturbed by the wave process, $\tilde{\rho}$ the wave perturbation, G_{ext} in (1) is the external field, which creates the stationary distribution of the plasma inhomogeneity. We shall assume that the inhomogeneities of the plasma are due to the density fluctuations $\delta\rho(\mathbf{r})$, which are random functions of the coordinates, and[1)] $\rho_0(\mathbf{r}) = \langle\rho_0\rangle + \delta\rho(\mathbf{r})$, $|\delta\rho|/\langle\rho_0\rangle \sim \mu \ll 1$, μ is a small parameter. Here and below, the angle brackets denote statistical averaging over the ensemble of inhomogeneities $\delta\rho(\mathbf{r})$.

All the wave perturbations in (1) can be represented in the form of a sum of their average values and the fluctuating departures from these averages:

$$\mathbf{h}=\langle\mathbf{h}\rangle+\mathbf{h}',\quad \tilde{\rho}=\langle\rho\rangle+\rho',\quad \mathbf{v}=\langle\mathbf{v}\rangle+\mathbf{v}'. \qquad (2)$$

Substituting (2) in (1) and carrying out the statistical averaging, we obtain the following set of equations for the average quantities:

$$\frac{\partial}{\partial t}\langle\mathbf{v}\rangle+\frac{c_s^2}{\langle\rho_0\rangle}\nabla\langle\rho\rangle+\frac{1}{4\pi\langle\rho_0\rangle}[\mathbf{H}_0\,\mathrm{rot}\langle\mathbf{h}\rangle]=\frac{c_s^2}{\langle\rho_0\rangle^2}\nabla\langle\delta\rho(\mathbf{r})\rho'(\mathbf{r},t)\rangle$$
$$-\frac{1}{4\pi\langle\rho_0\rangle}\left\langle\left(\frac{\delta\rho}{\rho_0}\right)^2\right\rangle[\mathbf{H}_0\,\mathrm{rot}\langle\mathbf{h}\rangle]+\frac{1}{4\pi\langle\rho_0\rangle^2}\langle[\mathbf{H}_0\,\mathrm{rot}\,\mathbf{h}']\delta\rho(\mathbf{r})\rangle+\mathbf{f}_1(\mathbf{r},t),$$
$$\frac{\partial}{\partial t}\langle\rho\rangle+\langle\rho_0\rangle\mathrm{div}\langle\mathbf{v}\rangle=-\mathrm{div}\langle\delta\rho(\mathbf{r})\mathbf{v}'(\mathbf{r},t)\rangle+f_2(\mathbf{r},t),$$
$$\frac{\partial}{\partial t}\langle\mathbf{h}\rangle-\mathrm{rot}[\langle\mathbf{v}\rangle\mathbf{H}_0]=\mathbf{f}_3(\mathbf{r},t), \qquad (3)$$

$$\mathbf{f}_1(\mathbf{r},t)=-(\langle\mathbf{v}\rangle\nabla)\langle\mathbf{v}\rangle+\frac{1}{2\langle\rho_0\rangle^2}\nabla\langle\rho\rangle^2$$
$$-\frac{1}{4\pi\langle\rho_0\rangle}[\langle\mathbf{h}\rangle\mathrm{rot}\langle\mathbf{h}\rangle]+\frac{\langle\rho\rangle}{4\pi\langle\rho_0\rangle^2}[\mathbf{H}_0\,\mathrm{rot}\langle\mathbf{h}\rangle],$$
$$f_2(\mathbf{r},t)=-\mathrm{div}(\langle\rho\rangle\langle\mathbf{v}\rangle),\quad \mathbf{f}_3(\mathbf{r},t)=\mathrm{rot}[\langle\mathbf{v}\rangle\langle\mathbf{h}\rangle], \qquad (4)$$

$c_S = (\kappa T/M)^{1/2}$ is the sound velocity, M the mass of the ion. In Eqs. (3) and (4), we have neglected terms of the type $\langle(\mathbf{v}'\cdot\nabla)\mathbf{v}'\rangle$, $\nabla\langle(\rho')^2\rangle$, and so on, which, in the case of weak inhomogeneities ($|\delta\rho|/\rho_0) \sim \mu \ll 1$ and small nonlinearity (expressions of the type $(\langle\mathbf{v}\rangle\nabla)\langle\mathbf{v}\rangle/c_s^2$, $\langle\rho\rangle\langle\mathbf{v}\rangle/\langle\rho_0\rangle c_s \sim \gamma^2 \ll 1$)) are small in comparison with the remaining terms. This is easy to see if we write down the set of equations for the fluctuating quantities:

$$\frac{\partial \mathbf{v}'}{\partial t}+c_s^2\frac{\nabla\rho'}{\langle\rho_0\rangle}+\frac{1}{4\pi\langle\rho_0\rangle}[\mathbf{H}_0\,\mathrm{rot}\,\mathbf{h}']=\boldsymbol{\alpha}(\mathbf{r},t),$$
$$\frac{\partial\rho'}{\partial t}+\langle\rho_0\rangle\,\mathrm{div}\,\mathbf{v}'=\beta(\mathbf{r},t), \tag{5}$$

where the functions $\alpha(\mathbf{r}, t)$ and $\beta(\mathbf{r}, t)$ are equal to

$$\boldsymbol{\alpha}(\mathbf{r},t)=\frac{c_s^2}{\langle\rho_0\rangle^2}\nabla(\delta\rho(\mathbf{r})\langle\rho\rangle)+\frac{\delta\rho(\mathbf{r})}{4\pi\langle\rho_0\rangle^2}[\mathbf{H}_0\,\mathrm{rot}\langle\mathbf{h}\rangle],$$
$$\beta(\mathbf{r},t)=-\mathrm{div}(\delta\rho(\mathbf{r})\langle\mathbf{v}\rangle). \tag{6}$$

Only the terms $\mathbf{v}'$, ρ', $\mathbf{h}' \sim \mu\gamma$ have been taken into account in Eqs. (5), (6). Components of the type $\nabla(\delta\rho\rho')$, $\delta\rho\mathbf{H}_0\times\mathrm{curl}\,\mathbf{h}'$ and so on, which were not considered in (6), are of the order of smallness of $\mu^2\gamma$ and are small in comparison with the other terms. True, we have also neglected the terms $\nabla(\rho'\langle\rho\rangle)$, $\rho'\mathbf{H}_0\times\mathrm{curl}\langle\mathbf{h}\rangle$ and so on in (7), which are also small, $\sim\mu^2\gamma$. Thus, the reduced set of equations (3) for the averaged quantities contains linear components $\sim\mu^2\gamma$ due to fluctuations of the medium and small nonlinear terms $\sim\gamma^2$. It is evident that a commensurate effect of the random inhomogeneities and of the nonlinearity on the wave propagation in the plasma should be expected in the case in which $\mu^2 \approx \gamma$ (in this connection, see also[8]). We note that in this same approximation, the set of equations (5) for the fluctuating quantities is linear. Therefore, for its solution, we can use the method of Fourier transformation. Carrying out the inverse Fourier transformation, we can calculate from the expressions $\rho'(\omega, \mathbf{k})$, $\mathbf{v}'(\omega, \mathbf{k})$, $\mathbf{h}'(\omega, \mathbf{k})$, obtained from (5), the components $\langle\delta\rho(\mathbf{r})\rho'(\mathbf{r}, t)\rangle = J_1$, $\langle\mathbf{H}_0\times\mathrm{curl}\,\mathbf{h}'\delta\rho(\mathbf{r})\rangle = J_2$ and $\langle\delta\rho(\mathbf{r})\mathbf{v}'(\mathbf{r}, t)\rangle = J_3$ that enter into the system (3). In the general case, they have a rather cumbersome form; therefore, we write down the expressions for the quantities $J_{1,2,3}$ in the special case, and at the same time, the one of practical interest, in which the interacting Alfven waves and the slow magnetoacoustic wave are propagated along $\mathbf{H}_0$ ($\mathbf{H}_0 \parallel 0x$). Here the functions of the coordinate x and time t that differ from zero will be $\langle v_x(x, t)\rangle$ and $\langle\rho(x, t)\rangle$ in the sound wave and $\langle v_y(x, t)\rangle$ and $\langle h_y(x, t)\rangle$ in the Alfven wave. The expressions $J_{1,2,3}(x, t)$ have the form

$$J_1(x,t)=\frac{1}{(2\pi)^4}\Big\{\int\Gamma(\rho)\langle v_x(x-\xi,t-\tau)\rangle \times\frac{\omega k_x(\omega^2-c_A^2k^2)}{D(\omega,k)}\exp i(\omega\tau-\mathbf{k}\rho)\,d\rho\,d\tau\,d\omega\,d\mathbf{k}$$
$$-\frac{c_s^2}{\langle\rho_0\rangle}\int\Gamma(\rho)\langle\rho(x-\xi,t-\tau)\rangle\frac{k^2(\omega^2-c_A^2k^2)}{D(\omega,k)}\exp i(\omega\tau-\mathbf{k}\rho)\,d\rho\,d\tau\,d\omega\,dk\Big\},$$
$$J_2(x,t)=-\frac{c_A^2H_0}{(2\pi)^4\langle\rho_0\rangle}\frac{\partial}{\partial x}\int\Gamma(\rho)\langle h_y(x-\xi,t-\tau)\rangle \times\left[\frac{k_x^2}{\omega^2-c_A^2k_x^2}+\frac{\omega^4k_y^2}{(\omega^2-c_A^2k_x^2)D(\omega,k)}\right]\exp i(\omega\tau-\mathbf{k}\rho)\,d\rho\,d\tau\,d\omega\,d\mathbf{k}$$
$$-\frac{iH_0^2c_s^2}{(2\pi)^4\langle\rho_0\rangle}\int\Gamma(\rho)\langle v_y(x-\xi,t-\tau)\rangle \times\frac{\omega k_yk^2}{D(\omega,k)}\exp i(\omega\tau-\mathbf{k}\rho)\,d\rho\,d\tau\,d\omega\,d\mathbf{k}, \tag{7}$$
$$J_3(x,t)=\frac{c_s^2}{(2\pi)^4\langle\rho_0\rangle}\int\Gamma(\rho)\Big[\langle\rho(x-\xi,t-\tau)\rangle\frac{\omega}{\langle\rho_0\rangle} -k_x\langle v_x(x-\xi,t-\tau)\rangle\Big]\frac{k_x(\omega^2-c_A^2k^2)}{D(\omega,k)}\exp i(\omega\tau-\mathbf{k}\rho)\,d\rho\,d\tau\,d\omega\,d\mathbf{k},$$

where $\Gamma(\rho) = \langle\delta\rho(\mathbf{r})\delta\rho(\mathbf{r}+\rho)\rangle$ as the correlation function of the fluctuations of the plasma density, $\rho = \{\xi, \eta, \zeta\}$, $D(\omega, \mathbf{k}) = \omega^4 - k^2\omega^2(c_A^2 + c_s^2) + k_x^2k^2c_s^2c_A^2$, $c_A^2 = H_0^2/4\pi\langle\rho_0\rangle$.

As is seen from (3)—(7), a set of integro-differential equations is obtained for the averaged quantities, the solution of which in the general case is very difficult. Therefore we consider its approximate solution below in Sec. 2 with the help of an asymptotic method as an example of the nonlinear interaction of two quasimonochromatic opposing Alfven waves with sound waves under conditions in which the characteristic spatial and temporal scales of the complex amplitudes of these waves are large in comparison with the scale of the inhomogeneities of the medium, as well as in comparison with its periods $T_i = 2\pi/\omega_i$ (ω_i are the frequencies of the interacting signals).

2. EQUATIONS FOR THE INTERACTING-WAVE AMPLITUDES AND THEIR INVESTIGATION

Let us consider the generation of sound waves in their interaction with two opposing Alfven waves. We seek the solution for the system (3)—(7) in the form

$$\langle\rho(x,t)\rangle=\sum_{m=1}^{3}b_m(x,t)\exp i(\Omega_mt-q_mx)+\text{c.c.}$$
$$\langle h_y(x,t)\rangle=\sum_{m=1}^{2}a_m(x,t)\exp i(\omega_mt-k_mx)+\text{c.c.} \tag{8.8}$$

where $\langle\rho(x, t)\rangle$ corresponds to sound and $\langle h_y(x, t)\rangle$ to Alfven waves, $a_m(x, t)$ and $b_m(x, t)$ correspond to slowly changing complex amplitudes of the interacting waves, the synchronism conditions for which have the form[2)]

$$(a_m=A_m\exp i\varphi_m,\quad b_m=B_m\exp i\Phi_m,\quad \varphi_m,\ \Phi_m\ \text{—wave phases})$$
$$\omega_2-\omega_1=\Omega_1=\Omega,\quad k_1+k_2=q_1=q,\quad \Omega=qc_s,\quad \Omega_2=2\Omega, \tag{9}$$
$$\Omega_3=3\Omega,\quad q_2=2q,\quad q_3=3q,\quad k_{1,2}=\omega_{1,2}/c_A.$$

We shall assume that the inequality $c_A \gg c_s$ is satisfied. Here, as is easily seen, the conditions (9) are satisfied, where

$$\omega_2-\omega_1=\Omega\ll\omega_{1,2}\approx\omega,\quad k_1\approx k_2=k,\quad q\approx2k=2\omega/c_A. \tag{10}$$

Taking (9) and (10) into account, and applying the asymptotic method of[14], we get the contracted equations for $a_m(x, t)$, $B_m(x, t)$, and Φ_m:

$$\frac{\partial a_2}{\partial t}+c_A\frac{\partial a_2}{\partial x}=-i\sigma_2a_1b_1-\nu_{2\,\mathrm{eff}}a_2+i\omega_2\left\langle\left(\frac{\delta\rho}{\rho_0}\right)^2\right\rangle a_2,$$
$$\frac{\partial a_1}{\partial t}-c_A\frac{\partial a_1}{\partial x}=-i\sigma_1a_2b_1^*-\nu_{1\,\mathrm{eff}}a_1+i\omega_1\left\langle\left(\frac{\delta\rho}{\rho_0}\right)^2\right\rangle a_1,$$
$$\frac{\partial B_1}{\partial t}+c_s\frac{\partial B_1}{\partial x}=\sigma_0A_1A_2\sin\bar\Phi+\delta B_1B_2\sin\Phi+\delta B_2B_3\sin\Phi_0-\nu_{\mathrm{eff}}^{(1)}B_1,$$
$$\frac{\partial B_2}{\partial t}+c_s\frac{\partial B_2}{\partial x}=-\delta B_1^2\sin\Phi+2\delta B_1B_3\sin\Phi_0-\nu_{\mathrm{eff}}^{(2)}B_2,$$
$$\frac{\partial B_3}{\partial t}+c_s\frac{\partial B_3}{\partial x}=-3\delta B_1B_2\sin\Phi_0-\nu_{\mathrm{eff}}^{(3)}B_3, \tag{11}$$
$$\frac{\partial\Phi_1}{\partial t}+c_s\frac{\partial\Phi_1}{\partial x}=\frac{\sigma_0A_1A_2}{B_1}\cos\Phi_1-\delta B_2\cos\Phi-\frac{\delta B_2B_3}{B_1}\cos\Phi_0-\delta_0,$$
$$\frac{\partial\Phi}{\partial t}+c_s\frac{\partial\Phi}{\partial x}=-\frac{2\sigma_0A_1A_2}{B_1}\cos\Phi_1-\frac{\delta B_1^2}{B_2}\cos\Phi+2\delta B_2\cos\Phi$$

$$-2\delta\left(\frac{B_1B_3}{B_2}-\frac{B_2B_3}{B_1}\right)\cos\Phi_0,$$

$$\frac{\partial\Phi_0}{\partial t}+c_s\frac{\partial\Phi_0}{\partial x}=-\delta\left(\frac{3B_1B_2}{B_3}-\frac{2B_1B_3}{B_2}-\frac{B_2B_3}{B_1}\right)\cos\Phi_0$$
$$+\delta\left(\frac{B_1^2}{B_2}+B_2\right)\cos\Phi-\frac{\sigma_0A_1A_2}{B_1}\cos\Phi_1,$$

where

$$\bar{\Phi}=\varphi_2-\varphi_1-\Phi_1,\quad \Phi=\Phi_2-2\Phi_1,\quad \Phi_0=\Phi_3-\Phi_2-\Phi_1.$$

$$\sigma_{1,2}\approx\omega_{1,2}/2\langle\rho_0\rangle,\quad \sigma_0=q/8\pi c_s,\quad \delta=\Omega/\langle\rho_0\rangle.$$

$$\delta_0=\begin{cases}\Omega\langle(\delta\rho/\rho_0)^2\rangle \text{ for } ql\ll 1,\\ {}^1/_2\Omega\langle(\delta\rho/\rho_0)^2\rangle \text{ for } ql\gg 1,\end{cases}\tag{12}$$

$$\nu_{1,2\,\text{eff}}=\begin{cases}{}^1/_4\omega_{1,2}(k_{1,2}l)\langle(\delta\rho/\rho_0)^2\rangle[1+4(c_s/c_A)^3(k_{1,2}l)^{-4}]\\ \text{for } c_s/c_A\ll k_{1,2}l\ll 1,\\ {}^1/_8(\sqrt{\pi}+1)\,\omega_{1,2}(k_{1,2}l)\langle(\delta\rho/\rho_0)^2\rangle \text{ for } k_{1,2}l\gg 1,\end{cases}\tag{13}$$

$$\nu_{\text{eff}}^{(1)}=\begin{cases}\Omega ql\langle(\delta\rho/\rho_0)^2\rangle & \text{for } ql\ll 1,\\ \Omega\langle(\delta\rho/\rho_0)_2\rangle/2ql & \text{for } ql\gg 1.\end{cases}\tag{14}$$

The coefficients $\nu_{\text{eff}}^{(2)}$ and $\nu_{\text{eff}}^{(3)}$ are equal to

$$\nu_{\text{eff}}^{(2)}=4\nu_{\text{eff}}^{(1)},\ \nu_{\text{eff}}^{(3)}=9\nu_{\text{eff}}^{(1)},\ \nu_{\text{eff}}^{(1)}=\nu_0 \text{ for } ql\ll 1,$$
$$\nu_{\text{eff}}^{(3)}\approx\nu_{\text{eff}}^{(2)}\approx\nu_{\text{eff}}^{(1)} \text{ for } ql\gg 1.$$

We note that the formulas for $\nu_{1,2\,\text{eff}}$ and $\nu_{\text{eff}}^{(1),(2),(3)}$ were obtained from (7) under the assumption that the fluctuation density correlation function[3] $\Gamma(r) = \langle(\delta\rho)^2\rangle \exp(-r_\perp^2/l^2 - |x|/l)$.

Thus the presence of fluctuations in the plasma leads to a certain effective damping of the average field of the interacting Alfven and sound waves, and also to the appearance of an additional phase shift. As analysis shows, for Alfven waves in the case of small-scale fluctuations ($kl \ll 1$), their damping is determined by the transfer of the energy of the average field into the fluctuation energy of the scattered Alfven (the first term in the square brackets of Eq. (13)) and sound waves. Here the scattering in the magnetic sound is not taken into account, since the effectiveness of this process is smaller than that shown by a factor of at least $(kl)^2$ as calculation shows. At the same time, the damping of the Alfven waves under the conditions of large scale fluctuations $kl \gg 1$ is determined basically by the Alfven wave and magnetic sound. For sound waves the damping (see Eq. (14)) is connected only with scattering in the same type of wave. Moreover, as is seen from (14), the contributions to the phase turn out to be much greater in comparison with the terms corresponding to the damping, for all waves in the case of small-scale fluctuations.

The study of the set of equations (14) in the general form is rather difficult; therefore, we limit ourselves below to the consideration of the special case of sound generation in the prescribed field of quasi-standing Alfven waves: $|a_1| \approx |a_2| = |a|$ and the phase shift of these waves is π. Inasmuch as the velocity of the harmonics of the sound wave is the same, we transform to moving coordinates $\xi = x - c_s t$ and t, and investigate the state of equilibrium of the resultant set of equations for $B_{1,2,3}$ and Φ, $\Phi_{0,1}$.

Inasmuch as the "turbulent" phase shift $\delta_0 \gg \nu_0$, it follows that $\Phi_0 \approx \pi + 3\nu_0/\delta_0$, and

$$\Phi=\operatorname{arctg} 2\frac{\nu_0}{\delta_0}\frac{1+(\delta^2B_1^{(0)2}/\delta_0^2)}{1-(\delta^2B_1^{(0)2}/\delta_0^2)}.\tag{15}$$

For equilibrium amplitudes $B_{1,2,3}^{(0)}$, the appearance of resonance is possible, when the denominator of the fraction in (15) is much smaller than the numerator, here

$$B_1^{(0)}\approx\delta_0/\delta,\quad B_2^{(0)}\approx B_3^{(0)}\approx\delta_0^2/10\nu_0\delta.\tag{16}$$

The phase of the wave of the fundamental $\Phi_1 \approx \pi - 1.3\,\nu_0/\delta_0$ and the amplitude of the pump is connected with $B_1^{(0)}$ by the relation

$$B_1^{(0)}\approx\delta_0/\delta\approx(10\sigma_0|a|^2/\delta)^{1/2}ql.\tag{17}$$

Here, as is not difficult to determine from (16), (17), the energy of the pump is much greater than the energy of the sound waves.

Outside the resonance region, i.e., when $|1 - (\delta^2 B_1^{(0)2}/\delta_0^2| \gg \nu_0/\delta_0$, the equilibrium amplitudes of the harmonics of the sound field are expressed in terms of the following formulas:

$$B_2^{(0)}\approx\frac{\delta}{2\delta_0}\frac{B_1^{(0)3}}{|1-(\delta^2B_1^{(0)2}/\delta_0^2)|},\quad B_3^{(0)}\approx\frac{\delta}{\delta_0}B_1^{(0)}B_2^{(0)},\quad B_1^{(0)}\approx\frac{\sigma_0|a|^2}{\eta(B_1^{(0)2})}$$
$$\eta(B_1^{(0)2})=\frac{[1-(\delta^2B_1\quad/\delta_0^2)]^2}{\delta_0[1-{}^5/_2(\delta^2B_1^{(0)2}/\delta_0^2)+{}^5/_4(\delta^4B_1^{(0)4}/\delta_0^4)]}\tag{18}$$

The expression $\eta(B_1^{(0)2})$ characterizes the transfer of energy to the second and third harmonics of the sound wave and changes from δ_0^{-1} for small $B_1^{(0)2}$ to $0.8\,\delta_0^{-1}$ for large $B_1^{(0)2}$. Here

$$\Phi_1=\operatorname{arctg}\frac{\nu_0}{\delta_0}\frac{1-\delta^2B_1^{(0)}/\delta_0^2+{}^{13}/_4\delta^4B_1^{(0)}/\delta_0^4}{1-{}^5/_2\delta^2B_1^{(0)2}/\delta_0^2+{}^5/_4\delta^4B_1^{(0)4}/\delta_0^4}\ll 1,$$

since the values of the field $B_1^{(0)2}$ at which the denominator vanishes lie outside the limits of applicability of the given study—the energy of the sound field can in this case be comparable to and greater than the energy of the Alfven wave.

We estimate the amplitude of the generated sound waves in a plasma with the following parameters: concentration $\sim 10^{12}$ cm^{-3}, T ≈ 3000°K; $H_0 \approx 10^3$ G; $\langle(\delta\rho/\rho_0)^2\rangle \approx 10^{-3}$; $l \approx 0.1$ cm; then, for a pump with amplitude $|a| \approx 2\times 10^{-4}\,H_0$ (power $\sim 1\,\mu$ W/cm^2) and frequency $\omega_{1,2} \sim 3\times 10^6$ cm^{-1} ($k \sim 5\times 10^{-2}$ cm^{-1}, $\lambda_A \sim 1$ m) in the resonance case a nonlinear sound wave is excited at a frequency $\Omega \sim 5\times 10^4$ sec^{-1} ($q \sim 0.1$ cm^{-1}, $\lambda_{ac} \sim 0.5$ m) with harmonic amplitudes $B_1^{(0)}/\langle\rho_0\rangle \approx 0.1\%$, $B_2^{(0)} \approx B_3^{(0)} \approx 10\,B_1^{(0)}$.

In conclusion, in order to establish the validity of the considerations just given, we estimate the amplitude of the fourth harmonic of the sound, which was not taken into account. By analogy with the derivation of the system (11), we can easily obtain the equations for the higher harmonics. For example, for the complex amplitude b_4, we have $\partial b_4/\partial t = -4i\delta b_3 b_1 - \nu_{\text{eff}}^{(4)} b_4$. Then, in the prescribed field $|b_{1,2,3}^0|$, the amplitude $|b_{4\,\max}| \approx (1/8)|b_{1\,\max}|$. Indeed, this quantity will be still smaller, since the departure from synchronism is important for the fourth harmonic, in view of its damping ($\nu_{\text{eff}}^{(4)} = 16\nu_0$). According to the same considerations, the amplitudes of harmonics with $n > 4$ cannot be taken into account in view of the fact that the departure from synchronism increases $\sim n^2$ while the amplitude decreases.

Thus, in a plasma with random inhomogeneities in the concentration, effective generation of nonlinear sound waves in SMBS of Alfven waves is possible and strong phase shifts are a feature of the SMBS scattering in such a randomly inhomogeneous medium. These shifts are connected with the random inhomogeneities, i.e., actually with the contribution to the real part of the effective dielectric permittivity of the medium.[15] The resonance growth of the amplitudes of the harmonics of the sound wave is also due to these phase shifts.

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*$[vH] \equiv v \times H$.

1)The plasma inhomogeneities can be assumed to be stationary if the wave frequencies are much greater than the characteristic frequencies of the turbulence spectrum and the amplitude of the sound wave $|\rho_{max}| \ll |\delta\rho|$ (see the estimates below for the small parameters μ and γ), which in the final analysis is equivalent to $c_T/c_s \ll ql$ ($c_{s,T}$ are the velocity of sound and of the pulsations, q is the wave number of the sound, and l is the scale of the inhomogeneity). For example, such a situation can be realized for a plasma the turbulence of which in the low frequency region is determined by the drift instability, [13] where $c_T/c_s \sim r_H/L \ll 1$ (r_H is the gyroradius of the ion, L the scale of the concentration gradient.

2)Only three harmonics of the sound wave were considered in (8), inasmuch as the damping, as will be shown below, in the case of small-scale inhomogeneities, is proportional to Ω^2. For large-scale inhomogeneities $ql \gg$ (l is the scale of the inhomogeneities) such a consideration should be made with account of the viscosity, which also increases with increase in Ω.

3)The integrals obtained here have been investigated fully and the final formulas are expressed in terms of the probability integral. However, in view of their complexity, we do not write them down here.

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