Electron broadening of overlapping spectral lines

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The broadening of overlapping spectral lines that arise from corresponding atomic transitions is considered. Expressions are obtained for the shift and width of the sublevel, and it is found that the size of the shift is of the order of the width. The role of elastic and inelastic electron scattering by radiating atoms is investigated. It is shown that the expression for the width due to elastic processes depends weakly on the detuning if the magnitude of the detuning is less than the plasma frequency. The temperature Green function technique is used. From the spectral relations for the two-particle and one-particle temperature Green functions a general expression for the contour of the spectral line is obtained. Because of the appearance of a considerable shift, the resulting spectral line shape can differ from a Lorentzian. As compared with other work, a simpler technique, based on an analysis of the Dyson equation for the one-particle temperature Green function, is used in this paper. By a suitable choice of the form of the Hamiltonian for the interaction of an atom with the plasma electrons this integral equation reduces to an algebraic equation.

1. INTRODUCTION

The Stark profiles of the spectral lines of atoms in a plasma are formed both under the influence of the low-frequency fields of the ions and under the influence of the high-frequency fields of the electrons. The influence of the low-frequency electric fields is regarded as quasi-static, whereas that of the high-frequency fields due to the electron component of the plasma should be treated in a dynamical approximation[1,2]. The aim of this work is to construct a dynamical theory of the broadening of the spectral lines of atoms by the micro-fields of plasma electrons. The broadening due to the ions is not considered.

Apparently, the most systematic and rigorous technique is to use temperature Green functions, which make it possible to remove the divergences that arise from the long-range Coulomb interaction by summing the appropriate diagrams. The method of "truncation of integrals," used in correlation theory, suffers the defect that the truncation is performed arbitrarily, generally speaking. By virtue of the fact that the divergence that arises is logarithmic and the linewidth in correlation theory contains two parameters under the logarithm, by choosing these parameters (the Weisskopf radius and the Debye radius) it is possible to achieve satisfactory agreement between correlation theory and experiment. The procedure indicated was used in[1,2]. In the present paper the divergence of the expression for the linewidth in the limit of zero frequency-detuning, which is due to small momentum transfers from the plasma electrons to the atom being perturbed (this corresponds to large impact parameters of the perturbing electron), is removed by taking a finite momentum transfer into account. The divergence arising in correlation theory at small impact parameters, corresponding to large momentum transfers, is removed in this work by taking the recoil momentum of the plasma electron into account. Allowance for the finite momentum transfer also leads to the result that the shift of a sublevel of an atomic energy level becomes a smooth function of the frequency-detuning.

In this paper we consider the Stark profile, due to the electron component of the plasma, of a spectral line of a nonhydrogenic atom. The state of the optical electron of the atom is described in the one-electron approximation. We shall also assume that the line broadening is due to collisions, separated in time, of plasma electrons with the radiating atom. Using a diagram technique to analyze the two-particle temperature Green function, Zaidi[3] obtained a finite expression for the spectral linewidth. In Zaidi's paper[3] the shifts of the sublevels of the atomic energy level were not taken into account. The asymmetry of the line was taken into account through the dependence of the width on the frequency-detuning, whereas for narrow spectral lines, of width less than the plasma frequency, the shift and width of the sublevels depend weakly on the frequency detuning. In the given case the line asymmetry is caused by the overlap of the dispersion profiles arising from transitions from sublevels of the upper level to sublevels of the lower level of the atom. Since the magnitude of the shift of a sublevel is of the order of its width, the effect of this overlapping of the contours turns out to be important near the center of the line.

In this paper the temperature diagram technique[4] is used. From the spectral relations for the two-particle and one-particle Green functions a general expression is obtained for the profile of the overlapping spectral lines. The spectral density of the one-particle temperature Green function is found as the discontinuity of this function across the real axis[4]. Because of the fact that we are considering a sufficiently dense plasma, i.e., the thermal de Broglie wavelength of the electron is considerably less than the Debye radius, in renormalizing the potential produced by the plasma electrons we can neglect excitation of plasmons.

2. BASIC RELATIONS

Radiation by an atom should be regarded as a transition of the whole system from a certain initial state $|\nu\rangle$ to a final state $|\nu'\rangle$ with emission of a quantum with frequency ω. The radiation intensity is determined by the formula

$$J(\omega)=\tfrac{4}{3}\alpha^3\omega^4|\mathbf{r}_{\nu\nu'}|^2\delta(E_\nu-E_{\nu'}-\omega), \tag{1}$$

where α is the fine-structure constant and E_ν, $E_{\nu'}$ are the energies of the system in the initial and final states respectively. In this expression, and also in all subsequent ones, the atomic system of units is used.

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In formula (1) it is convenient to go over to the second-quantization representation [5]:

$$\mathbf{r}\to\int \chi^+(\mathbf{r})\,\mathbf{r}\chi(\mathbf{r})\,d\mathbf{r}. \tag{2}$$

The operators $\chi^+(\mathbf{r})$ and $\chi(\mathbf{r})$ creating and annihilating the optical electron of the atom at a given point in space are conveniently expanded in the wavefunctions of the unperturbed atom:

$$\chi^+(\mathbf{r})=\sum_\nu \psi_\nu^*(\mathbf{r})\,a_\nu^+,\quad \chi(\mathbf{r})=\sum_\nu \psi_\nu(\mathbf{r})\,a_\nu, \tag{3}$$

where a_ν^+ and a_ν are the creation and annihilation operators for an atomic electron in the state ν. For convenience we shall assume that the operators a_ν^+ and a_ν describe particles obeying Fermi statistics. The arbitrariness in the choice of commutation relations for the operators a_ν^+ and a_ν is justified by the fact that in a low-density plasma the population of the atomic energy levels is described by the Boltzmann formula, i.e., the occupation numbers are small.

In the expression (1) the initial state $|\nu\rangle$ and final state $|\nu'\rangle$ describe both the atom and the plasma electrons. In order to obtain the intensity of the emission from the atom in the transition from one state to another, it is necessary to average (1) over all initial states of the plasma electrons and over the sublevels of the initial state of the atom, and also to sum over the final states of the plasma electrons and over the sublevels of the final state of the atom. However, it is more convenient to calculate the function in which, in addition, averaging has been performed over all the atomic energy levels. Thus, we shall be concerned with the quantity

$$J(\omega)=\frac{4\alpha^3\omega^4}{3}Q^{-1}\sum_{\nu,\nu'}\left|\mathbf{r}_{\nu\nu'}\right|^2\exp[-\beta(E_\nu-\mu_aN_{\nu a}-\mu_eN_{\nu e})]\,\delta(E_\nu-E_{\nu'}-\omega), \tag{4}$$

where Q is the partition function of the grand canonical ensemble corresponding to the Hamiltonian

$$\mathcal{H}=H_a+H_e+V_{ae}+H_{ee}+H_{ei}, \tag{5}$$

β is the inverse temperature, $N_{\nu a}$ and $N_{\nu e}$ are the particle-number densities for the optical electrons of the atoms and the plasma electrons, and μ_a and μ_e are the chemical potentials of the optical electrons in the atoms and the plasma electrons. In formula (5) H_a is the Hamiltonian of the optical electron in the atom, H_e is the Hamiltonian of the plasma electrons, and V_{ae}, H_{ee} and H_{ei} are the interaction energies of the atom with the plasma electrons, of the plasma electrons with each other, and of the plasma electrons with the ions, respectively. If we neglect the Doppler broadening (which is small compared with the collision broadening), we can disregard the motion of the atoms.

We shall consider now the two-particle temperature Green function, with pairwise-coincident arguments, for the optical electron in the atom:

$$\mathcal{G}_2(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2)=\langle T\{\tilde\chi^+(\mathbf{r}_1,\tau_1)\tilde\chi(\mathbf{r}_1,\tau_1)\tilde\chi^+(\mathbf{r}_2,\tau_2)\tilde\chi(\mathbf{r}_2,\tau_2)\}\rangle. \tag{6}$$

Here T is the ordering operator with respect to the parameter τ, which varies in the limits $-\beta\le\tau\le\beta$; $\tilde\chi^+$ and $\tilde\chi$ are the operators (3) creating and annihilating the optical electron of the atom at a given point in space in the temperature Heisenberg picture and are defined by the usual relations for the temperature technique [4]. It is convenient to consider the Fourier components of (6), defined by the expressions [4] $\mathcal{G}_2(\mathbf{r}_1,\mathbf{r}_2;\omega_n)$, where $\omega_n=2\pi n/\beta$ and n is an integer (although the operators $\tilde\chi$ are Fermi operators, the two-particle Green function with pairwise-coincident arguments is, nevertheless, expanded in a Fourier series in even frequencies). The Green function (6) can be represented in the form of an integral of the spectral function [4]

$$J_2(\mathbf{r}_1,\mathbf{r}_2;\omega')=Q^{-1}\sum_{\nu,\nu'}[\chi^+(\mathbf{r}_1)\chi(\mathbf{r}_1)]_{\nu\nu'}[\chi^+(\mathbf{r}_2)\chi(\mathbf{r}_2)]_{\nu'\nu} \tag{7}$$
$$\times\exp[-\beta(E_\nu-\mu_aN_{\nu a}-\mu_eN_{\nu e})]\,\delta(E_\nu-E_{\nu'}-\omega')$$

in the following way:

$$\mathcal{G}_2(\mathbf{r}_1,\mathbf{r}_2;\omega_n)=\int_{-\infty}^{\infty}(e^{\beta\omega'}-1)J_2(\mathbf{r}_1,\mathbf{r}_2;\omega')\frac{d\omega'}{(i\omega_n+\omega')}. \tag{8}$$

Multiplying (7) by the quantity $4\alpha^3\omega^4\mathbf{r}_1\cdot\mathbf{r}_2/3$ and integrating over the variables $\mathbf{r}_1$ and $\mathbf{r}_2$ we obtain an expression coinciding exactly with (4). It follows from the relation (8) that the spectral function J_2 can be defined as the discontinuity of the function $\mathcal{G}_2^*(\mathbf{r}_1,\mathbf{r}_2;\omega\pm i\epsilon)$.

It can be shown that we can use the following approximation for the two-particle temperature Green function (6):

$$\mathcal{G}_2(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2)=-\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2)\mathcal{G}(\mathbf{r}_2,\mathbf{r}_1;\tau_2-\tau_1), \tag{9}$$
$$\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2;\tau_1-\tau_2)=-\langle T\{\tilde\chi(\mathbf{r}_1,\tau_1)\tilde\chi^+(\mathbf{r}_2,\tau_2)\}\rangle. \tag{10}$$

Formula (10) is the temperature Green function of the optical electron of the atom. In expression (9) we have discarded terms proportional to $\exp(\beta\mu_e)$, i.e., proportional to the plasma-electron density. In analogy with (7) and (8), we can also write spectral representations for the Fourier component of the one-particle temperature Green function (10) [4]:

$$\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2;\omega_n)=\int_{-\infty}^{\infty}J(\mathbf{r}_1,\mathbf{r}_2;\omega')\frac{d\omega'}{i\omega_n+\mu_a-\omega'},$$

$$J(\mathbf{r}_1,\mathbf{r}_2;\omega')=Q^{-1}\sum_{\nu,\nu'}[\chi(\mathbf{r}_1)]_{\nu'\nu}[\chi^+(\mathbf{r}_2)]_{\nu\nu'}$$
$$\times\exp[-\beta(E_\nu-\mu_aN_{\nu a}-\mu_eN_{\nu e})]\,\delta(\omega'-E_\nu-E_{\nu'})[1-\exp\{-\beta(\omega'-\mu_a)\}]. \tag{11}$$

Taking into account the smallness of the quantity $\exp(\beta\mu_a)$, from formulas (8)–(11) we obtain the relation

$$J_2(\mathbf{r}_1,\mathbf{r}_2;\omega)=e^{\beta\mu_a}\int_{-\infty}^{\infty}e^{-\beta\omega_2}J(\mathbf{r}_1,\mathbf{r}_2;\omega_2-\omega)J(\mathbf{r}_2,\mathbf{r}_1;\omega_2)\,d\omega_2, \tag{12}$$

which expresses the spectral density of the two-particle temperature Green function in terms of the spectral densities of the one-particle functions. As will be seen from the following, in the function J there appears a sum of only the diagonal matrix elements describing the perturbation of the atomic energy levels by the plasma electrons. In order to obtain the spectral function corresponding to the transition of the atom from the upper to the lower state, in accordance with what has been said above we must omit the summation over all the atomic energy levels, apart from the two under consideration. It is now easy to see that the approximation (9), from which formula (12) follows, means that the broadening and shift of the sublevels of the upper and lower levels occur independently of one another, i.e., in the approximation (9) we neglect the "level-mixing" effect. In fact, the contribution to the spectral density from the mixing that is induced, as mentioned above, by the perturbing influence of the plasma electrons is found to be quadratic in the quantity $\exp(\beta\mu_a)$.

3. CALCULATION OF THE TEMPERATURE GREEN FUNCTION OF THE OPTICAL ELECTRON OF THE ATOM

Assuming that the micro-field created by the plasma electrons at the position of the radiating atom is uniform over the volume of the atom, we represent the term V_{ae} appearing in the Hamiltonian (5) in the form

$$V_{ae}=-\mathbf{E}\int \chi^{+}(\mathbf{r})\mathbf{r}\chi(\mathbf{r})\,d\mathbf{r}. \tag{13}$$

The intensity $\mathbf{E}$ of the micro-field is written in the second-quantization representation in the form

$$\mathbf{E}=-\int b^{+}(\mathbf{r})\nabla u(\mathbf{r})\,b(\mathbf{r})\,d\mathbf{r}, \tag{14}$$

where $b^{+}(\mathbf{r})$ and $b(\mathbf{r})$ are operators creating and annihilating a plasma electron at the point $\mathbf{r}$ in space, and $u(\mathbf{r})$ is the renormalized potential[4] produced by the plasma electrons. The operators $b^{+}(\mathbf{r})$ and $b(\mathbf{r})$ describe electrons obeying Fermi statistics. Neglecting the exchange interaction of the plasma electrons with the optical electron of the atom, we can assume that the plasma-electron creation and annihilation operators $b^{+}(\mathbf{r})$ and $b(\mathbf{r})$ commute with the creation and annihilation operators ($\chi^{+}(\mathbf{r})$ and $\chi(\mathbf{r})$) of the optical electrons of the atoms. After the renormalization of the potential created by the plasma electrons, we can omit the terms H_{ee} and H_{ei} in the Hamiltonian (5).

The Green function of the optical electron of an isolated atom is

$$\mathcal{G}_a^{0}(\mathbf{r}_1,\mathbf{r}_2;\omega_n)=\sum_{\nu}\frac{\psi_\nu(\mathbf{r}_1)\psi_\nu^{*}(\mathbf{r}_2)}{i\omega_n+\mu_a-E_\nu}. \tag{15}$$

In formula (15) E_ν is the energy corresponding to the ν-th level, measured from the ground-state energy, and μ_a is the chemical potential of the optical electrons of the atoms, determined from the normalization condition for the Green function (15):

$$N_a=-\int \mathcal{G}_a^{0}(\mathbf{r},\mathbf{r};\tau\to-0)\,d\mathbf{r}. \tag{16}$$

The Fourier transform of the Green function of the intensities of the plasma micro-fields has the form

$$\mathcal{G}_E(\omega_n)=\int_0^{\beta}\langle T\{\mathbf{E}(\tau)\mathbf{E}(\tau')\}\rangle_c \exp[i\omega_n(\tau-\tau')]\,d(\tau-\tau'); \tag{17}$$

the subscript c means that the connected part of the diagram is taken. The contribution given by the unconnected (plasma-electron) diagrams containing the factor $\langle\mathbf{E}(\tau)\rangle$, and also by diagrams of first order in V_{ae}, is equal to zero.

We shall confine ourselves to the first correction to the self-energy of the electrons. Physically, summing such graphs means that the collisions of plasma electrons with the radiating atom are separated in time. The summation procedure requires only that the collisions be separable in time. The collisions are not divided into weak and strong in accordance with the magnitude of the impact parameter or of the momentum of the incident electron, as is done in correlation theory. It must also be noted that approximating the one-electron Green function by such graphs corresponds to a more general case than that considered in correlation theory. Summing graphs of this form means that we are considering that contribution made to the Stark profile by groups of one, two, etc., electrons, flying one after the other in such a way that the collisions of the electrons from each group with the atom are separated in time.

We write Dyson's integral equation

$$\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2;\omega_n)=\mathcal{G}_a^{0}(\mathbf{r}_1,\mathbf{r}_2;\omega_n)+\iint\mathcal{G}_a^{0}(\mathbf{r}_1,\mathbf{r};\omega_n)\cdot\sigma(\mathbf{r},\mathbf{r}';\omega_n)\mathcal{G}(\mathbf{r}',\mathbf{r}_2;\omega_n)\,d\mathbf{r}\,d\mathbf{r}' \tag{18}$$

where the mass operator $\sigma(\mathbf{r},\mathbf{r}';\omega_n)$ has the form

$$\sigma(\mathbf{r},\mathbf{r}';\omega_n)=\frac{1}{\beta}\sum_{\omega_m}\mathbf{r}\mathbf{r}'\mathcal{G}_E(\omega_m)\mathcal{G}_a^{0}(\mathbf{r},\mathbf{r}';\omega_n-\omega_m). \tag{19}$$

It is easy to see that, by virtue of the relations (15) and (19), the integral equation (18) is an equation with a degenerate kernel, the solution of which has the form

$$\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2;\omega_n)=\sum_{\nu}\frac{\psi_\nu(\mathbf{r}_1)\psi_\nu^{*}(\mathbf{r}_2)}{i\omega_n+\mu_a-E_\nu-\sigma_{\nu\nu}(\omega_n)},\qquad \sigma_{\nu\nu}(\omega_n)=\iint\psi_\nu^{*}(\mathbf{r})\sigma(\mathbf{r},\mathbf{r}';\omega_n)\psi_\nu(\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'. \tag{20}$$

The fact that the matrix elements of the mass operator are diagonal is due to the separability in time of the collisions of the plasma electrons with the atom. In other words, the contributions to the shift and broadening of the sublevels of an atomic energy level are created independently of each other by the plasma electrons. Graphically, this means that diagrams with crossing plasma-oscillation lines are not taken into account.

4. CALCULATION OF THE MASS OPERATOR

It follows from expressions (14) and (17) that

$$\mathcal{G}_E(\omega_n)=\frac{1}{12\beta\pi^{4}}\sum_{\omega_m}\int\mathcal{G}_e^{0}(\mathbf{k}+\mathbf{k}_1;\omega_n+\omega_m)\mathcal{G}_e^{0}(\mathbf{k}_1;\omega_m)\times\frac{k^2\,d\mathbf{k}\,d\mathbf{k}_1}{(k^2+d^2)^2};\quad \mathcal{G}_e^{0}(\mathbf{k}_1;\omega_m)=(i\omega_m-\varepsilon_{\mathbf{k}_1}+\mu_e)^{-1},\quad d=(8\pi\beta N_e)^{1/2}. \tag{21}$$

In this relation the energy $\epsilon_{\mathbf{k}1}$ of the plasma electrons is measured from the ground state of the atom, and their chemical potential is determined from the normalization condition for the plasma-electron Green function

$$N_e=-\mathcal{G}_e^{0}(0,\ \tau\to-0). \tag{22}$$

From (22) the relation

$$\lambda_e=\exp(\beta\mu_e)=(2\pi\beta)^{3/2}e^{I\beta}N_e, \tag{23}$$

where I is the ionization potential of the atom, follows directly. The plasma electrons are described by Boltzmann statistics, and therefore the activity λ_e is small. In calculating the sum over the frequencies in formula (21) we can confine ourselves to terms of first order in λ_e, and, by virtue of the relation

$$\lambda_a=\exp(\beta\mu_a)=\lambda_eN_e, \tag{24}$$

which follows from the normalization condition (16) and the Saha formula, we can also discard the terms proportional to λ_a as quantities quadratic in N_e.

Bearing in mind what has been said above, and also formulas (15), (19) and (21), we obtain an expression for the diagonal matrix element (20) of the mass operator:

$$\sigma_{\nu\nu}(\omega_n)=\frac{\lambda_e e^{-I\beta}}{12\pi^{4}}\sum_{\nu_1}|\mathbf{r}_{\nu\nu_1}|^2\iint k^2\exp\{-\beta\varepsilon_{\mathbf{k}_1}^{0}\}\times(i\omega_n+\varepsilon_{\mathbf{k}_1}^{0}-\varepsilon_{\mathbf{k}+\mathbf{k}_1}^{0}-E_{\nu_1}+\mu_a)^{-1}\frac{d\mathbf{k}\,d\mathbf{k}_1}{k^2+d^2}. \tag{25}$$

In this expression, in contrast to (21), $\epsilon_{\mathbf{k}1}^{0}=\mathbf{k}_1^2/2$ is the kinetic energy of a plasma electron, reckoned in such a way that its value at $\mathbf{k}_1=0$ is taken as the zero. The momenta $\mathbf{k}$ are given in atomic units ($p_0=me^2/\hbar^2$).

From the definition (11) of the quantity J as the discontinuity of the one-particle temperature Green function

(20) of the optical electron of the atom across the real axis, it follows that only the value of the integral (25) on the real axis appears in the spectral density (11). Calculating the integral over $\mathbf{k}_1$ that appears in (25), we obtain

$$\frac{1}{4\pi^4}\int \frac{\exp\{-\beta\varepsilon_{k_1}^0\}\,d\mathbf{k}_1}{b\pm i\delta+\varepsilon_{k_1}^0-\varepsilon_{k+k_1}^0} = \frac{(k/2-b/k)\exp[-1/2\beta(k/2-b/k)^2]}{2\pi^{1/2}\beta k\sqrt{z}}\,\Gamma\left(\frac{1}{2},\frac{\beta z}{2}\right),$$
$$b=\omega-E_\nu,\quad z=-(k/2-b/k)^2\pm i\delta;\quad \delta>0, \tag{26}$$

where $\Gamma(1/2, \beta z/2)$ is the incomplete gamma function. The passage to the limit $\delta \to 0$ cannot be taken directly in the expression (26). In order to take the limit, we make use of the relation [6]

$$\Gamma\left(\frac{1}{2},\frac{\beta z}{2}\right)=\sqrt{\pi}-\int_0^{\beta z/2} t^{-1/2}e^{-t}\,dt. \tag{27}$$

Substituting the expression (26) into (25), and also taking the relations (23) and (27) into account, we obtain for $\delta \to 0$

$$\sigma_{\nu\nu}(\omega\mp i\delta)=\pm i\gamma_\nu(\omega)-\Delta_\nu(\omega),$$

$$\gamma_\nu(\omega)=\frac{4}{3}\sqrt{2\pi\beta}N_e\sum_{\nu_1}|r_{\nu\nu_1}|^2\int_0^\infty\frac{k^3e^{-\nu^2}\,dk}{(k^2+d^2)^2},$$

$$\Delta_\nu(\omega)=\frac{8}{3}\sqrt{2\beta}\,N_e\sum_{\nu_1}|r_{\nu\nu_1}|^2\int_0^\infty\frac{k^3e^{-\nu^2}\,dk}{(k^2+d^2)^2}\int_0^y e^{x^2}\,dx,$$

$$y=\sqrt{\frac{\beta}{2}}\left(\frac{k}{2}-\frac{\omega-E_{\nu_1}}{k}\right). \tag{28}$$

In the expression (28) the diagonal matrix element of the mass operator σ has a discontinuity across the real axis. Because of this, the Green function of the optical electron of the atom also has a discontinuity across the real axis, and the magnitude of this discontinuity, as was pointed out above, determines the one-particle spectral density appearing in formula (12). Let ν be the set of quantum numbers describing the upper state of the atom and ν' be the set of quantum numbers corresponding to the lower state. Then the expression (12) has poles (in the zeroth approximation with respect to corrections to the energy levels E_ν and $E_{\nu'}$) at

$$\omega_1^0=E_\nu;\quad \omega_2^0=E_{\nu'}+\omega.$$

The next approximation has the form

$$\omega_2^1=E_{\nu'}+\omega-\Delta_{\nu'}(E_{\nu'}+\Delta\omega)\pm i\gamma_{\nu'}(E_{\nu'}+\Delta\omega),$$
$$\omega_1^1=E_\nu-\Delta_\nu(E_\nu+\Delta\omega)\pm i\gamma_\nu(E_\nu+\Delta\omega).$$

In these expressions the quantity $\Delta\omega$ can be regarded as the frequency detuning. For small detunings the quantities Δ_ν and γ_ν are almost constant for small variations of $\Delta\omega$. For the upper level, the quantity y appearing in (28) takes the form

$$y=\sqrt{\frac{\beta}{2}}\left(\frac{k}{2}-\frac{\omega_{\nu\nu_1}+\Delta\omega}{k}\right),\quad \omega_{\nu\nu_1}=E_\nu-E_{\nu_1}. \tag{29}$$

An analogous expression can also be written for the lower level.

In the formulas (28) there appear terms with $\nu = \nu_1$, corresponding to elastic scattering of plasma electrons by atoms, and also terms with $\nu \neq \nu_1$, describing inelastic collisions. We shall denote the contributions to the width and shift that arise from elastic scattering by γ_ν^0 and Δ_ν^0, and the inelastic part by γ_ν^1 and Δ_ν^1 respectively. For the quantities γ_ν^0 and Δ_ν^0 the transition frequency $\omega_{\nu\nu} = 0$; both the width and the shift depend only on the detuning $\Delta\omega$. If the relation

$$\sqrt{\beta}|\Delta\omega|<\sqrt{2}d, \tag{30}$$

is fulfilled, γ_ν^0 and Δ_ν^0 can be represented in the form

$$\gamma_\nu^0(\Delta\omega)=\frac{2}{3}\sqrt{2\pi\beta}\,N_e\left[\psi\left(2,1;\frac{\beta d^2}{8}\right)-\frac{\beta|\Delta\omega|^2}{2d^2}\right]\sum_{\bar\nu}|r_{\nu\bar\nu}|^2, \tag{31}$$

$$\Delta_\nu^0(\Delta\omega)=\frac{2}{3}\pi\sqrt{2\pi\beta}\,N_e\left(1+\frac{\sqrt{\beta}|\Delta\omega|}{\sqrt{2\pi}\,d}\right)\sum_{\bar\nu}|r_{\nu\bar\nu}|^2;\quad \Delta\omega<0,$$

$$\Delta_\nu^0(\Delta\omega)=\frac{2}{3}\pi\sqrt{2\pi\beta}\,N_e\left(1-\frac{\sqrt{\beta}|\Delta\omega|}{\sqrt{2\pi}\,d}\right)\sum_{\bar\nu}|r_{\nu\bar\nu}|^2;\quad \Delta\omega>0. \tag{32}$$

In (31) ψ is a confluent hypergeometric function of the second kind, and $\bar\nu$ is the set of quantum numbers describing the sublevels of the level ν.

Because the quantity d is small, the expression for the width is simplified and takes the form

$$\gamma_\nu^0(\Delta\omega)=\frac{2}{3}\sqrt{2\pi\beta}\,N_e\left(0.7216-\ln\frac{\beta d^2}{8}-\frac{\beta|\Delta\omega|^2}{2d^2}\right)\sum_{\bar\nu}|r_{\nu\bar\nu}|^2. \tag{33}$$

The quantity $\beta d^2/8$ appearing in (33) can be represented in the form

$$\beta d^2/8=\frac{1}{4}(\lambda\!\!\!^{-}/\varkappa)^2,$$

where $\lambda = \hbar/\sqrt{mkT}$ is the thermal de Broglie wavelength of the plasma electron, $\varkappa = (kT/4\pi n_e e^2)^{1/2}$ is the Debye radius, and m and n_e are the plasma-electron mass and concentration, expressed in grams and cm^{-3} respectively. On the other hand, we can write

$$\sqrt{\beta}|\Delta\omega|/\sqrt{2}\,d=|\widetilde{\Delta\omega}|/2\omega_p, \tag{34}$$

where $\omega_p = (4\pi n_e e^2/m)^{1/2}$ is the plasma frequency, and $\widetilde{\Delta\omega}$ is the frequency-detuning, expressed in sec^{-1}. It follows from (32)–(34) that the width and shift depend very weakly on the frequency-detuning if the linewidth is less than the plasma frequency. Thus, formula (33), which has been obtained in correlation theory, is valid for the indicated frequency detunings. It should be noted that allowance for the recoil momentum of the plasma electron in formula (25) removes the divergence of the width in the limit of zero frequency detuning. For elastic scattering the recoil momentum is of the order of the thermal momentum of the plasma electron and, consequently, the minimum impact parameter is of the order of the thermal de Broglie wavelength $\lambda\!\!\!^{-}$, as follows directly from formula (33).

When considering the contribution of inelastic collisions to the width and shift we can neglect the frequency detuning $\Delta\omega$ in comparison with the transition frequency $\omega_{\nu\nu_1}$. By virtue of the inequality

$$\beta|\omega_{\nu\nu_1}|>1$$

the quantities γ_ν^1 and Δ_ν^1 can be represented in the form

$$\gamma_\nu^1=\frac{4\sqrt{2}\,\pi}{3}N_e\sum_{\nu_1\neq\nu}|r_{\nu\nu_1}|^2\frac{\exp\{-\beta|\omega_{\nu\nu_1}|\}}{|\omega_{\nu\nu_1}|^{1/2}};\quad \omega_{\nu\nu_1}<0 \tag{35}$$

$$\gamma^1=\frac{4\sqrt{2}\,\pi}{3}N_e\sum_{\nu_1\neq\nu}\frac{|r_{\nu\nu_1}|^2}{|\omega_{\nu\nu_1}|^{1/2}};\quad \omega_{\nu\nu_1}>0,$$

$$\Delta_\nu^1=\frac{4\sqrt{2}\,\pi}{3}N_e\sum_{\nu_1\neq\nu}\frac{|r_{\nu\nu_1}|^2}{|\omega_{\nu\nu_1}|^{1/2}};\quad \omega_{\nu\nu_1}<0$$

$$\Delta_\nu^1=0;\ \omega_{\nu\nu_1}>0. \tag{36}$$

Formulas (35) and (36) enable us to estimate the contribution of inelastic collisions to the width and shift. If the broadening and shift of the ground state ν' of the atom are considered, then $\omega_{\nu'\nu_1} < 0$. From this it follows that the contribution of inelastic collisions to the width is very small, while all the atomic states make contributions to the shift. For the first excited level, the ground state makes a contribution to the width and all the atomic states except the ground state make a contribution to the shift. Thus, contributions to the width are made by states lying below the state from which the atom undergoes the radiative transition, and contributions to the shift are made by all states lying above this state.

5. CONTOUR OF THE SPECTRAL LINE

The relations (20) and (28) enable us to find the two-particle spectral density (12). According to formulas (32) and (33) the width and shift do not depend on the detuning if the detuning is less than the plasma frequency. This circumstance makes it possible to perform the integration over ω_2 in (12). Multiplying the spectral density obtained by the coefficient $4\alpha^3\omega_{\nu\nu_1}^4 \mathbf{r}_1 \cdot \mathbf{r}_2/3$, integrating over the variables $\mathbf{r}_1$ and $\mathbf{r}_2$, and omitting the summation over all the atomic energy levels apart from ν and ν', we obtain the frequency distribution of the intensity in the spectral line due to the transition $\nu \to \nu'$. Assuming that the magnitude of the energy level E_ν does not depend on the quantum numbers describing the sublevels of the given level, we write the profile of the line in the form

$$J_{\nu\nu'} = \frac{4\alpha^3\omega_{\nu\nu'}^4 N_a}{3\pi} e^{-\beta E_\nu} \sum_{\bar{\nu},\bar{\nu}'} \frac{(\gamma_\nu+\gamma_{\nu'})\,|\mathbf{r}_{\nu\nu'}|^2}{(\Delta\omega+\Delta_\nu-\Delta_{\nu'})^2+(\gamma_\nu+\gamma_{\nu'})^2}, \qquad (37)$$

where $\Delta\omega = \omega - \omega_{\nu\nu'}$, N_a is the density of atoms, and $\bar{\nu}$, $\bar{\nu}'$ are the sets of quantum numbers corresponding to the sublevels of the upper state ν and lower state ν' of the atom. Let the quantities ν_l^*, l and m_ν denote, respectively the effective principal quantum number, the orbital quantum number and the magnetic quantum number of the optical electron of the atom for the state ν. Then $\bar{\nu}$ denotes the pair of quantum numbers l and m_ν. In the formulas (32), (33), (35), (36) and (37) there appears the quantity $\Sigma_{mj}|\mathbf{r}_{\nu j}|^2$, which can be represented in the form [7]

$$\sum_{m_j} |\mathbf{r}_{\nu_j}|^2 = \begin{cases} \dfrac{l+1}{2l+1}[R(\nu_l^*, l; j_{l+1}^*, l+1)]^2; & l \to l+1, \\ \dfrac{l}{2l+1}[R(j_{l-1}^*, l-1; \nu_l^*, l)]^2; & l \to l-1. \end{cases} \qquad (38)$$

To calculate the radial integral R we can make use of the Bates-Damgaard tables, in which the following values are given [7]:

$$R(j_{l-1}^*, l-1; \nu_l^*, l) = \frac{3\nu_l^*}{2}\sqrt{\nu_l^{*2}-l^2}\, I(j_{l-1}^*\nu_l^* l),$$
$$\nu_l^* = (E_\nu)^{-1/2}, \quad j_{l-1}^* = (E_j)^{-1/2}, \qquad (39)$$

where E_ν and E_j are the experimental term values, expressed in Rydbergs.

As follows from formulas (32), (33), (35), (36) and (38), the shift and width of a sublevel depend only on the orbital quantum number and do not depend on the magnetic quantum number. In other words, although in the given scheme we are considering transitions between magnetic sublevels, these, nevertheless, do not experience a shift or broadening. Therefore, in the numerator of expression (37) we can perform the summation over the magnetic sublevels of the lower state (in accordance with (38)) and then sum over the magnetic sublevels of the upper level. Thus, in (39) there remains the summation over l and l'. The resulting Stark profile due to the electron component of the plasma is formed by superimposing the dispersion profiles corresponding to the transitions $\nu^* l \to \nu^{*\prime} l'$ with all possible values of l and l'. It follows from formulas (32) and (33) that, for a plasma with an electron concentration of the order of 10^{15}—10^{16} cm^{-3} and a temperature of (5—10) $\times 10^3$ K, the magnitude of the shift of the l-th sublevel of a given level is of the order of its width. It follows from (37) that, by virtue of the dependence of the shift on the orbital quantum number, the spectral-line profile due to the electron component of the plasma is asymmetric. In the case of a resonance line, for detunings less than the plasma frequency the line profile has no asymmetry.

6. CONCLUSION

Zaidi [3] obtained for the width due to elastic scattering of the perturbing plasma electrons an expression that coincides in form with formula (31) of this paper in the limit of zero frequency detuning. The difference is that the argument $\beta d^2/8$ in the expression (31) is twice the corresponding argument in the expression (11) given in [3]. This difference is connected with the fact that in the present work we have taken into account not only electron-electron correlations, as in [3], but also electron-ion correlations. An important difference between our and Zaidi's results [3] is that we study the shift of the l-th sublevel; this shift was found to be of the order of the width, and this, as pointed out above, leads for overlapping lines to asymmetry of the contour. We have also obtained formulas describing the contribution made by inelastic scattering of the perturbing plasma electrons by atoms to the width and shift of the sublevel. It turns out that these quantities depend only on the plasma-electron concentration and do not depend on the temperature. In addition, we have established that the formulas of correlation theory are valid at frequency-detunings smaller than the plasma frequency.

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