Anomalies in the drag electron dislocation force in a phase transition of order 2–1/2

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Anomalies of the electron dislocation drag force are investigated for various types of phase transitions of order 2–1/2 with respect to pressure P (at $P \approx P_k$). It is shown that the appearance (disappearance) of a new cavity in the Fermi surface leads to a root singularity of the dislocation drag force $\sim\sqrt{|P-P_k|}$. On rupture of the connecting neck, the nature of the anomaly of the dislocation drag force depends on the direction of motion of the dislocation. If the dislocation axis is parallel to the axis of the connecting neck, the singularity of the drag force will be the same as that on the appearance (or disappearance) of a new Fermi surface cavity. If the dislocation axis is perpendicular to the neck axis the derivative of the force will possess a logarithmic singularity. The effect of interband transitions on the anomaly of the dislocation drag force is taken into account.

The scattering of electrons by a moving dislocation leads to a specific dislocation drag force F (see Kravchenko[1]). According to present-day concepts, this force plays an essential role in the plastic properties of metals at low temperatures.

Dislocation dragging by conduction electrons becomes particularly clearly manifested by a softening of the metal as it becomes superconducting. The separation of the electronic component of the dislocation drag force is based on the fact that the transition to the superconducting state hardly affects the lattice subsystem of the metal (the structure of the defects and the phonons). Therefore the entire change in the plastic properties of the metal must be ascribed to the electronic subsystem (a detailed review of the research in this direction is contained in[2], where references to the original work can be found).

The transition to the superconducting state is not the only cause of the change in the electron energy spectrum. This spectrum can be altered in various manners: by placing the metal in a magnetic field, by subjecting it to hydrostatic pressure, etc. As shown by Kravchenko and Natsik[3,4], a sufficiently strong magnetic field can greatly influence the electron dragging of the dislocations.

Application of the sufficiently strong pressure changes all the characteristics of the metal (both lattice and electronic), and particularly its plastic properties[5-7]. It is apparently impossible to separate the role of the conduction electrons when the plastic characteristics vary smoothly with pressure. The situation should be different in a phase transition of order $2\frac{1}{2}$ (PT-$2\frac{1}{2}$, I. Lifshitz[8]), when the topology of the Fermi surface changes in a relatively narrow pressure interval without any significant change of the lattice. The change of the plastic properties (if observed at all) would have to be ascribed to a change in the electronic component of the dislocation drag force. The purpose of the present article is to calculate the anomalies of the electron drag force F in a phase transition of order $2\frac{1}{2}$. The PT-$2\frac{1}{2}$ is accompanied by anomalies in the thermodynamic and kinetic characteristics of the metals. The sound absorption coefficient should experience a significant change in PT-$2\frac{1}{2}$[9,10]. As we shall show, the calculated anomaly in F is a consequence of the anomaly of the sound-absorption coefficient.

The measure to the proximity to PT-$2\frac{1}{2}$ is the quantity[11]

$$z=\varepsilon_F-\varepsilon_K, \tag{1}$$

where ϵ_F is the Fermi energy and ϵ_c is the critical energy at which the topology of the electronic equal-energy surfaces changes (the Van-Hove singularity). The anomaly in F is manifest by the presence in F of a term δF that depends on z non-analytically. Our problem is the calculation of $\delta F(z)$.

As is well known[1-4], the procedure for calculating the drag force on a dislocation moving with velocity $\mathbf{V}$ can be reduced to a calculation of the intensity $\dot{E}$ of the dislocation-energy absorption by the electron gas (per unit dislocation length). We assume that the moving dislocation line penetrates through the entire crystal. Remaining within the framework of the macroscopic description of the dislocation, we neglect the dependence of F on the coordinate along the dislocation axis. To calculate $\dot{E}$ it is convenient to represent the energy of interaction of an electron with a moving dislocation $U(\mathbf{r}-\mathbf{V}t)$ as a superposition of plane waves

$$U(\mathbf{r}-\mathbf{V}t)=\frac{1}{L_1L_2}\sum_{\mathbf{f}}U_{\mathbf{f}}e^{-i(\omega_{\mathbf{f}}t-\mathbf{f}\mathbf{r})}, \quad \omega_{\mathbf{f}}=(\mathbf{f}\mathbf{V}), \tag{2}$$

$\mathbf{f}$ is a plane wave vector; L_1 and L_2 are the dimensions of the crystal in a plane perpendicular to the dislocation axis, $\mathbf{r}$ is a two-dimensional vector in a plane perpendicular to the dislocation axis, and the vector $\mathbf{V}$ lies in this plane.

We see that the moving dislocation generates a packet of peculiar phonons with which the conduction electrons interact. The peculiar character of the phonons (we shall call them simply phonons; this should not lead to misunderstanding, since the interactions of the dislocation with actual phonons will not be taken into account) reduces to a peculiarity of their dispersion law, namely $\omega = \mathbf{f}\cdot\mathbf{V}$ instead of $\omega = sf$ (s is the speed of sound). By considering the interaction of phonons with momentum hf with the electron gas, we can write down the drag force F in the form

$$F=-\int \hbar\omega_{\mathbf{f}}\Gamma(\mathbf{f},\omega)\,d^2f, \tag{3}$$

where

$$\Gamma(\mathbf{f},\omega)=\frac{2}{(2\pi\hbar)^4V}\int |\langle U_{\mathbf{f}}\rangle_{\mathbf{p}}|^2[n(\varepsilon_{\mathbf{p}})-n(\varepsilon_{\mathbf{p}+\hbar\mathbf{f}})]\,\delta(\varepsilon_{\mathbf{p}+\hbar\mathbf{f}}-\varepsilon_{\mathbf{p}}-\hbar\omega_{\mathbf{f}})\,d^3p, \tag{4}$$

Γ is a dimensionless phonon-absorption coefficient, which has in this case the meaning of the probability that a phonon with energy $\hbar\omega_f$ will decelerate the dislocation; $\langle U_f\rangle_p$ is the matrix element of the transition, so normalized that in the case when the wave function of the electron is plane, it is equal to U_f.

It is shown in [9,10] that the singularities of $\Gamma(f)$ in PT-$2\frac{1}{2}$ depends on the topology of the transition, namely, Γ experiences a discontinuity when a Fermi-surface cavity appears (or disappears), and has a logarithmic singularity and when a bridge is broken a connecting neck is ruptured. It must be borne in mind, however, that in [10] the results were obtained for real sound waves ($\omega = sf$; $1/l \ll f \ll 1/a$, a is the interatomic distance and l is the electron mean free path). The phonon packet (2) generated by the dislocation contains phonons with $f \lesssim f_{max} \approx 1/a$. This makes it necessary to refine the formulas for $\Gamma(f, z)$. In the calculation we shall naturally assume that $|z| \ll \epsilon_0$, where $\epsilon_0 = \hbar^2/a^2m^*$ is the characteristic electron energy and m^* is the effective mass of the electron.

According to (3) and (4), the drag force F depends in a complicated manner on the dislocation velocity V. In a normal metal, the nonlinear dependence of F on V is completely immaterial [12], since the characteristic velocity at which the nonlinear effects come into play is of the order of $v_F \approx 10^8$ cm/sec, and the dislocation cannot attain such a velocity. In superconductors it is possible to have nonlinear effects [12,13]. The characteristic velocity in this case is $v \approx \Delta/p_F \ll s$ (Δ is the gap in the electronic energy spectrum of the superconductor). In the PT-$2\frac{1}{2}$, the electron velocity $v \approx \sqrt{2m^*|z|}$ tends to zero as $z \to 0$. Therefore the nonlinear dependence on V calls for a special analysis. It appears that nevertheless this is an effect difficult to observe, since it should take place at $z \lesssim m^*V_2/2$. Since the dislocation velocity is as a rule smaller than s, the temperature and other factors can smear out the nonlinear dependence, in analogy with the manner in which the temperature, according to [10], makes it difficult to observe quantum singularities in $\Gamma(\omega)$.

We confine ourselves here to the linear approximation, i.e., we calculate the singularity of the friction coefficient B:

$$\delta F = -\delta BV; \quad B = \frac{1}{8\pi^4\hbar^2}\int f^2\beta(\mathbf{f})\cos^2\theta\, d^2f, \qquad (5)$$

$$\cos\theta = \frac{\mathbf{fV}}{fV}, \quad \beta(\mathbf{f}) = -\int |\langle U_f\rangle_p|^2 \frac{\partial n_F}{\partial \varepsilon_p}\delta(\varepsilon_p - \varepsilon_{p+\hbar f})\, d^3p.$$

Disregarding the temperature smearing of the PT-$2\frac{1}{2}$, we can assume that $-\partial n_F/\partial\epsilon_p = \delta(\epsilon_p - \epsilon_F)$. According to [10], the singularity of $\beta(\mathbf{f}, z)$ is determined by the electrons of a small region of p-space about the critical point $\mathbf{p}_c$ (we assume it to be zero). In the case when a new cavity is produced (say a sphere, for simplicity, case A), $\mathbf{p}_c$ coincides with its center; in the case of the rupture of a neck (case B), it coincides with the conical point (Fig. 1). This makes it possible to use in the calculation of δF the approximate dispersion law

$$\varepsilon = \varepsilon_c + p^2/2m^* \qquad \text{(case A)} \qquad (6)$$

$$\varepsilon = \varepsilon_c + (p_1^2+p_2^2)/2m_\perp - p_3^2/2m_\parallel \qquad \text{(case B)}. \qquad (7)$$

The notation in (6) and (7) is universal. Smallness of the integration region around $\mathbf{p}_c$ makes it possible to take the matrix element $|\langle U_f\rangle_{\mathbf{p}=\mathbf{p}_c}|^2$ outside the integral sign and assume it to be equal to $|U_f|^2$. As we shall see, in the calculation of δB, the integration with respect to f (see (5)) is limited in most cases to small values of f, thus justifying the use of the macroscopic formulas for U_f. On the other hand, in those cases when large f play an important role, the explicit form of U_f is immaterial when it comes to determining the character of the dependence of δB on z near the PT-$2\frac{1}{2}$ point.

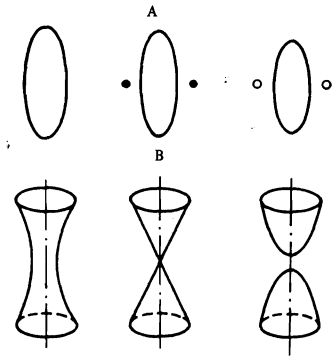

FIG. 1. Types of phase transition of order $2\frac{1}{2}$: A—formation of a new cavity of the Fermi surface; B—disruption of connecting neck.

Using the last formulas and expression (5) for $\beta(f, z)$, we obtain in case A at $z > 0$:

$$\beta(f,z) = \begin{cases} 2\pi|U_f|^2m^{*2}/\hbar f, & f < 2\sqrt{2m^*z}/\hbar, \\ 0 & f > 2\sqrt{2m^*z}/\hbar. \end{cases} \qquad (8)$$

In case B it is necessary to refine the arrangement of the dislocation relative to the neck axis. Since the dislocations are produced as a rule and move along selected crystallographic directions, there is no need to investigate an arbitrary disposition of the neck axis, of the dislocation, and of the velocity **V**. We confine ourselves to three cases (Fig. 2): the dislocation axis is parallel to the neck axis (case B—I), the dislocation axis is perpendicular to the neck axis and the velocity **V** is parallel to it (case B—II), and the dislocation axis and the velocity **V** are both perpendicular to the neck axis (case B—III).

Case B—I. Owing to the assumed isotropy of the dispersion law (7) the value of $\beta(f, z)$ depends in the plane (1, 2) on the direction of **f** only because of the matrix element U_f:

$$\beta(f,z) = \frac{2m_\perp\sqrt{m_\perp m_\parallel}|U_f|^2}{\hbar f}\ln\alpha; \qquad (9)$$

$$\alpha = \left|\frac{p_0+\sqrt{p_0^2+2m_\parallel(z-z_f)}}{p_0-\sqrt{p_0^2+2m_\parallel(z-z_f)}}\right|; \quad z_f = \frac{\hbar^2f^2}{8m_\perp}.$$

We have $\beta(\mathbf{f}, z) \neq 0$ if

$$f < f_m = \frac{2}{\hbar}\left(\frac{m_\perp}{m_\parallel}\right)^{1/4}\sqrt{p_0^2+2m_\parallel z},$$

and $\beta(\mathbf{f}, z) = 0$ if $f > f_m$. We have to restrict the integration with respect to p_3 to the limits $|p_3| < p_0 \ll \hbar/a$, in order to be able to use the approximate dispersion law (7), which is valid only in the immediate vicinity of $\mathbf{p}_c$. The final expressions either do not contain p_0 at all, or only under the logarithm sign.

Case B—II. Before we write out the final results, we note that the expression for β (see (5)) can be written in the form

$$\beta(\mathbf{f},z) = \frac{|U_f|^2}{\hbar f}\int\delta\left(\frac{p_1^2+p_2^2}{2m_\perp} - \frac{p_3^2}{2m_\parallel} - z\right)\delta\left(\tilde{s} - \frac{p_2\sin\theta}{m_\perp} + \frac{p_3\cos\theta}{m_\parallel}\right)d^3p, \qquad (10)$$

where

$$\tilde{s} = \frac{\hbar f}{2m_\parallel}R\sin^2\theta, \quad R = \operatorname{ctg}^2\theta - \operatorname{ctg}^2\theta_{cr}, \quad \operatorname{ctg}\theta_{cr} = \sqrt{\frac{m_\parallel}{m_\perp}},$$

and the angle θ is defined in (5).

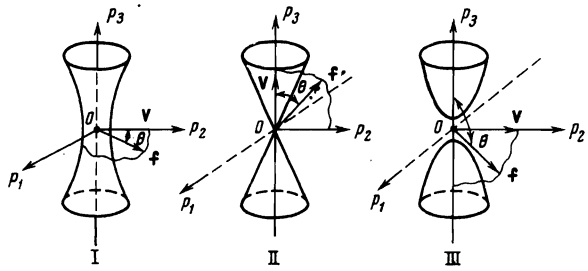

FIG. 2. Different variants of dislocation motion when the neck is broken in case B: I—dislocation axis parallel to the p_3 axis of the neck; II—dislocation axis perpendicular and the velocity V is parallel to the neck axis; III—dislocation axis and velocity perpendicular to the neck axis (the dislocation axis is shown dashed).

It is clear from (10) that we can use the results obtained in [10] (see (16) of [10]). According to (21) we have from [10] and $|\cot\theta| > \cot\theta_{cr}$:

$$\beta(\mathbf{f}, z)=\frac{2\pi m_\perp m_\parallel |U_\mathbf{f}|^2}{\hbar f\sqrt{R}\,|\sin\theta|}. \tag{11}$$

This formula is valid for $z > 0$ at arbitrary f, and at $z < 0$ it is valid only if the following inequality holds:

$$f \geqslant \frac{2\sqrt{2m_\parallel |z|}}{\hbar\sqrt{R}\,|\sin\theta|}=f_{min}(z,\theta). \tag{12}$$

If $f < f_{min}(z, \theta)$, then $\beta(\mathbf{f}, z) = 0$. When $|\cot\theta| < \cot\theta_{cr}$ we have in accordance with formula (23) of [10]

$$\beta(\mathbf{f}, z)=\frac{2m_\perp m_\parallel |U_\mathbf{f}|^2}{\hbar f\sqrt{|R|}\,|\sin\theta|}\ln\alpha, \tag{13}$$

where

$$\alpha=\alpha(\tilde z_\mathbf{f}),\quad \tilde z_\mathbf{f}=\frac{\hbar^2 f^2 |R|\sin^2\theta}{8m_\parallel},$$

with formula (13) valid if

$$f<f_{max}=\frac{2\sqrt{p_0^2+2m_\parallel z}}{\hbar\sqrt{|R|}\,|\sin\theta|}. \tag{14}$$

At $f > f_{max}$ we have $\beta(\mathbf{f}, z) = 0$.

Case B—III. The value of $\beta(\mathbf{f}, z)$ is determined by formulas (5) and (11)—(14), but in formula (5) we must put $\mathbf{f}\cdot\mathbf{V} = fV\sin\theta$. We assume that the angle θ, just as in the case B—II, is reckoned from the axis of the neck (Fig. 2, III), and then $\cos\theta = \mathbf{n}\cdot\mathbf{f}/f$, while $\mathbf{n}$ is a unit vector along this axis, the vector V being perpendicular to the vector $\mathbf{n}$.

To calculate the friction coefficient from formula (5) it is necessary to know not only $\beta(\mathbf{f}, z)$ but also $|U_\mathbf{f}|^2$.

As already mentioned, we can use macroscopic expressions for the deformation field; these expressions are valid at large distances from the core of the dislocation (small f corresponds to large r). According to [12], for a screw dislocation located along the 3-axis and moving along the 2-axis we have

$$|U_\mathbf{f}|^2=b^2(\lambda_{13}f_2-\lambda_{23}f_1)^2/f^4, \tag{15}$$

b is the Burgers vector of the dislocation and λ_{ik} are the components of the tensor characterizing the deformation potential [14]. An expression for $|U_\mathbf{f}|^2$ at a different placement of the dislocation is obtained from (15) by permuting the indices.

Case A (appearance of a new cavity). From formulas (5), (8), and (15), integrating with respect to $f < 2\sqrt{2m^* z}/\hbar$, we have

$$\delta B=\frac{m^{*2}b^2\lambda^2}{8\pi^2\hbar^4}\sqrt{2m^* z},\quad \lambda^2=3\lambda_{13}^2+\lambda_{23}^2. \tag{16}$$

In the case B—I we obtain from (5), (9), and (15)

$$B_I=\begin{cases} b^2m_\perp^2\lambda^2p_0/8\pi^2\hbar^4-b^2m_\perp^2\lambda^2\sqrt{2m_\parallel|z|}/8\pi^2\hbar^4 & z<0,\\ b^2m_\perp^2\lambda^2p_0/8\pi^2\hbar^4, & z>0,\end{cases} \tag{17}$$

from which it follows that in this case the integration of expression (9) leads not only to an anomalous part of the friction coefficient, but also to a contribution, amounting to $b^2m_\perp^2\lambda^2p_0/8\pi^2\hbar^4$, to the smooth part. That part of the friction coefficient δB which is irregular in z differs from zero in the region where the number of cavities of the Fermi surface is larger ($z < 0$):

$$\delta B_I=b^2m_\perp^2\lambda^2\sqrt{2m_\parallel|z|}/8\pi^2\hbar^4. \tag{18}$$

As follows from (9) and (17), large f contribute to the smooth part of the friction coefficient, and small f contribute to the irregular increment. Indeed, it is seen from (9) that if $f \approx f_m$, then $\beta(\mathbf{f}, z)$ is regular in z (in this case $\beta(\mathbf{f}, z) \approx m_\parallel|z|/p_0$), and if f are small ($f \approx \sqrt{m_\perp|z|/\hbar} \ll f_m$), then $\beta(\mathbf{f}, z)$ is irregular in z (in this case $\beta(\mathbf{f}, z) \approx \ln(p_0^2/m_\parallel|\Delta z|)$, $\Delta z = z - z_\mathbf{f}$, see also [10], formula (23)) and it is this which leads, after integration with respect to f and θ according to (5) and (9), to the irregular increment $\delta B_I \approx \sqrt{|z|}$, determined by formula (18).

Cases B—II and B—III. It is seen from (11)—(14) that the integration with respect to f, even in the case of small z, is carried out to arbitrarily large values of the wave vector f. If we disregard the exact dependence of $|U_\mathbf{f}|$ on f, and use the expressions for the macroscopic elasticity theory (see [12] and later), then the corresponding integrals for δB diverge, so that it is necessary either to introduce bounds on the wave vector f or, equivalently, assume that $|U_\mathbf{f}|$ tends to zero rapidly as $f \to \infty$. As will be seen below, the cutoff parameter in f enters in the expression for δB only under the logarithm sign, and it can therefore be assumed that the final expression is "stable" relative to the assumption concerning the structure of the dislocation core (the structure determines the concrete dependence of $|U_\mathbf{f}|$ on f at $f \gtrsim 1/a$). We shall henceforth use the assumption that $|U_\mathbf{f}| = 0$ at $f > f_{max}$ ($f_{max} \approx 1/a$). Then, as follows from (12) and (14), the region of integration with respect to f and θ in the calculation of $\delta B_{II,III}$ is determined by the inequalities (Fig. 3):

$$f_{min}(z,\theta)\leqslant 1/a,\quad f<f_{max}. \tag{19}$$

From (5), (11)—(15), and (19) we obtain

$$\delta B_{II,III}=-\frac{2b^2m_\perp m_\parallel\lambda_{II,III}^2}{\pi^3\hbar^4}\sqrt{2m_\parallel|z|}\ln\frac{|z|}{\varepsilon_0}, \tag{20}$$

where

$$\lambda_{II}^2=\mathrm{ctg}^3\theta_{cr}\sin^4\theta_{cr}(\lambda_{21}^2+\lambda_{31}^2\,\mathrm{tg}^2\theta_{cr})_\perp$$
$$\lambda_{III}^2=\mathrm{tg}\,\theta_{cr}(\sin^4\theta_{cr}+\cos^4\theta_{cr})(\lambda_{21}^2+\lambda_{31}^2),$$
$$\varepsilon_0=\hbar^2f_{max}^2/(m_\perp+m_\parallel).$$

From the analysis that precedes formula (20) it follows that the logarithmic singularity $\sqrt{|z|}\ln(|z|/\epsilon_0)$ stems

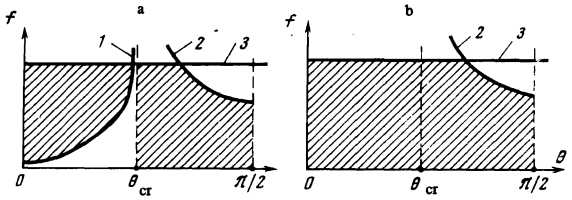

FIG. 3. Regions of integration with respect to f and θ in the calculation of B in the different cases B-II and B-III: a—at $z < 0$; b—at $z > 0$; 1—plot of $f = 2\sqrt{2m|z|/\hbar\sqrt{|R|}}\,|\sin\theta|$; 2—plot of $f = 2\sqrt{p_0^2+2m_\parallel z}/\hbar\sqrt{|R|}\,|\sin\theta|$; 3—the line $f = f_{max}$.

from the anisotropy of the contribution of phonons with different f to the friction coefficient. As follows from (12) and (14), the largest contribution to the dislocation drag is made by phonons whose wave vector increases in accordance with a square root law $f \approx \sqrt{m_{\parallel}|z|}/\hbar\sqrt{|\Delta\theta|}$ when the critical angle is approached, where $|\Delta\theta| = |\theta_{cr} - \theta|$ and $|\Delta\theta| \ll \theta_{cr}$. Consequently, as seen from (5) and (11), in the calculation of the friction coefficient B, the integrand in (5), after integration with respect to f, is proportional to $\sqrt{|z|}/|\Delta\theta|$, and the integration of the latter with respect to the angle θ with allowance for (19) leads to a singularity of the type $\sqrt{|z|}\ln(|z|/\epsilon_0)$ in the friction coefficient.

The phonon packet generated by the dislocation, as already mentioned, contains phonons with large wave vectors $f \approx 1/a$, so that it is necessary to ascertain the role of the interband transitions in the dislocation dragging. The influence of the interband transitions on the dislocation dragging will be considered with two very simple examples.

a) Let the energy spectrum of the metal consist of an electron band with a dispersion law (Fig. 4)

$$\varepsilon_{1p}=p^2/2m_1 \tag{21}$$

and let the PT-2½ give rise to a hole band with a dispersion law (Fig. 4)

$$\varepsilon_{2p}=\varepsilon_c-p^2/2m_2. \tag{22}$$

The Fermi surface consists of two spheres with centers at the point p = 0 and radii $p_F = \sqrt{2m_1\epsilon_F}$ and $p_z = \sqrt{2m_2|z|}$. From (5), (21), and (22) we find that in this case the quantity $\beta(\mathbf{f})$, which is connected with the interband transitions, is equal to

$$\beta(f)=\begin{cases}\dfrac{2\pi m_1 m_2|U_f|^2}{\hbar f}, & |f-f_{cr}|<\Delta f,\\ 0, & |f-f_{cr}|>\Delta f,\end{cases} \tag{23}$$

where $f_{cr} = p_F$, $\Delta f = \sqrt{2m_2|z|}$.

We emphasize that $\beta(\mathbf{f}) \neq 0$ only in a narrow interval of values of f, and consequently in the calculation of δB we can take $|U_f|$ outside the integral sign in this case ($|U_f| \approx |U_{f_{cr}}|$), and the character of the singularity of the drag force (i.e., the dependence on z) is not determined by the explicit form of $|U_f|$: we always have $\delta B \approx \sqrt{|z|}$.

According to (5) and (23) we have

$$\delta B = \frac{m_1 m_2 p_F^2 |U_{f_{cr}}|^2}{2\pi^2\hbar^3}\sqrt{2m_2|z|}. \tag{24}$$

The quantitative contribution of the interband transitions to the anomaly of the deceleration force is determined, naturally, by the value of $|U_f|$. At small values of f_{cr}, in particular, if $f_{cr} > f_{max}$ (see above), the contribution of the interband transitions is entirely inessential.

b) The energy spectrum of the metal consists of the electron band (22) and the PT-2½ gives rise to a new electron band with a dispersion law

$$\varepsilon_{2p}=\varepsilon_c+(p-p_0)^2/2m_2. \tag{25}$$

The Fermi surface consists of two spheres with different centers (Fig. 5). From (5), (21), and (25) we find that $\beta(\mathbf{f})$ corresponding to the interband transitions is equal to (to simplify the calculations we assume that $p_0 \gg \sqrt{2m_1\epsilon_F} \gg \sqrt{2m_2 z}$)

$$\beta(f)=\begin{cases}\dfrac{2\pi m_1 m_2|U_f|^2}{|\hbar f-p_0|}, & |f-f_{1,2}^{cr}|<\Delta f,\\ 0, & |f-f_{1,2}^{cr}|>\Delta f,\end{cases} \tag{26}$$

where $f_{1,2}^{cr} = p_0 \pm \sqrt{2m_1\epsilon_F}$, and the angle θ is small ($|\theta| \leq \sqrt{2m_1\epsilon_F}/p_0$).

The largest contribution is made by the interband transitions in this case when the vectors $\mathbf{V}$, $\mathbf{p}_0$, and $\hbar\mathbf{f}$ lie in one plane, i.e., the dislocation axis is perpendicular to the vector $\mathbf{p}_0$ (Fig. 5), and in the latter case we obtain from (5) and (26)

$$\delta B = \frac{m_1 m_2 |U|^2 p_0^2}{\pi^3\hbar^6}\sqrt{2m_2 z}, \tag{27}$$

where

$$|U|^2 = |U_1|^2+|U_2|^2, \quad |U_{1,2}| = |U_f| \text{ at } f = f_{1,2},$$

and consequently the quantitative contribution of the interband transitions to the anomaly of the drag force is determined as before by the quantity $|U_f|$ at large f. We note that if the dislocation axis is directed along the vector $\mathbf{p}_0$, and the velocity $\mathbf{V}$ is perpendicular to $\mathbf{p}_0$, then the contribution of the interband transitions to the anomaly of the drag force is equal to zero.

As noted in [10], in those cases when $\mathbf{p} = \mathbf{p}_c$ lies on the boundary of the Brillouin zone, two phase transitions of order 2½ can be located (on the z scale or pressure scale) very close to each other. One of the transitions is connected with the appearance (vanishing) of a new cavity of the Fermi surface, and the other with the breaking of a neck. Each PT-2½, naturally, is accompanied by the corresponding singularity of B (or F). It should be noted that the singularity accompanying the appearance of a new cavity should in this case depend significantly on the interband transitions, since the distances between the two cavities of the Fermi surface (new and old) are small and $U_{f_{cr}} \approx b\tilde{\lambda}/f_{cr}$ and $f_{cr} < 1/a$, $\tilde{\lambda}$ coincides in order of magnitude with the components of the tensor λ_{ik}. Since, according to (26), $\hbar f_{cr} \approx p_0$, in this case the contribution of the interband transitions is of the same order as the contribution due to the appearance of a new cavity.

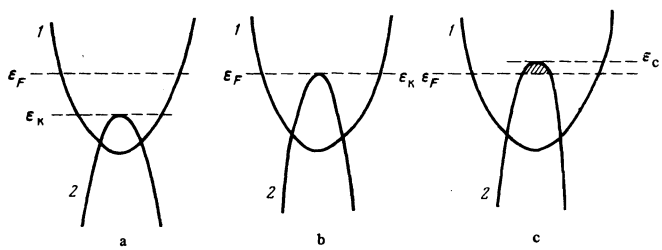

FIG. 4. Onset of hole band in a phase transition of order 2½: a) $z = \epsilon_F - \epsilon_c > 0$; b) $z = 0$; c) $z < 0$; $1 - \epsilon_{1p} = p^2/2m_1$; $2 - \epsilon_{2p} = \epsilon_c - p^2/2m_2$. The dashed lines show the hole band.

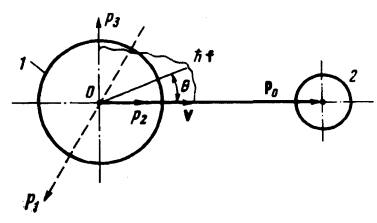

FIG. 5. Illustrating the calculation of δB for interband transitions between the anomalous and main bands: 1–the surface $\epsilon_{1p} = p^2/2m_1$; 2–the surface $\epsilon_{2p} = \epsilon_c + (p - p_0)^2/2m_2$. The dislocation axis (dashed) is perpendicular to the vector p_0.

Comparing the expressions obtained above for B with the value of the electronic component of the friction coefficient in a normal metal $B_N \approx m^2 b^2 \lambda^2 \hbar/a$ (Kravchenko[1]), we see that the anomalous increment is small to the extent that $|z| = |z_0||P - P_c|/P_c$ is small[1)]. Indeed,

$$\delta B \approx B_N \sqrt{\frac{|P-P_c|}{P_c}}, \text{ or } \delta B \approx B_N \sqrt{\frac{|P-P_c|}{P_c}} \ln \frac{|P-P_c|}{P_c}. \quad (28)$$

This increment, however, can appear in the derivative of the friction coefficient with respect to pressure, since $\partial B/\partial z \to \infty$ as $z \to 0$.

The finite temperature and the different scattering mechanisms lead to a smoothing of the singularities of the friction coefficient.

If $T \gg \hbar/\tau$ (i.e., $T[K] \gg 10^{-3}/l$ [cm]; τ and l are respectively the time and mean free path of the electrons), then in the cases A and B—I we have

$$\frac{\partial B}{\partial z} \approx \left(\frac{\partial B}{\partial \varepsilon_F}\right)_N \left(\frac{\varepsilon_F}{T}\right)^{1/2} \quad (29)$$

and in cases B—II and B—III

$$\frac{\partial B}{\partial z} \approx \left(\frac{\partial B}{\partial \varepsilon_F}\right) \left(\frac{\varepsilon_F}{T}\right)^{1/2} \ln \frac{\varepsilon_F}{T}, \quad (30)$$

where $(\partial B/\partial \epsilon_F)_N$ denotes the change in the drag force of the normal metal far from the singularity.

In the opposite limiting case at $\hbar/\tau \gg T$, the value of $\partial B/\partial z$ is given by

$$\frac{\partial B}{\partial z} \approx \begin{cases} \left(\frac{\partial B}{\partial \varepsilon_F}\right)_N \left(\frac{\varepsilon_F \tau}{\hbar}\right)^{1/2} & \text{(in cases A and B—I),} \\ \left(\frac{\partial B}{\partial \varepsilon_F}\right)_N \left(\frac{\varepsilon_F \tau}{\hbar}\right)^{1/2} \ln\left(\frac{\varepsilon_F \tau}{\hbar}\right) & \text{(in cases B—II, B—III).} \end{cases} \quad (31)$$

The quantity $\partial B/\partial z$ can be expressed in terms of the derivative with respect to the external pressure P. From (29)—(31) we see that in the case of PT-2½ the change of $\partial B/\partial P$ and $\partial F/\partial P$ at the point $P = P_c$ is appreciable

$$\frac{\partial B}{\partial P} = \left(\frac{\partial B}{\partial P}\right)_N \left(\frac{\varepsilon_F}{T^*}\right)^{1/2}, \quad T^* = T + \frac{\hbar}{\tau}, \quad (32)$$

or

$$\frac{\partial B}{\partial P} \approx \left(\frac{\partial B}{\partial P}\right)_N \left(\frac{\varepsilon_F}{T^*}\right)^{1/2} \ln \frac{\varepsilon_F}{T^*}. \quad (33)$$

In conclusion, we take the opportunity to thank V. D. Natsik and A. M. Kosevich for useful discussions.

[1)]For estimates we can use formula (13.15) of [11]:

$$|z| = \frac{|z_0||P-P_c|}{P_c},$$

where $z_0 = (\epsilon_F - \epsilon_c)_{p=0}$, P_c is the critical pressure at which the phase transition of order 2½ occurs.

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