Gravitational synchrotron radiation of a particle in Kerr and Schwarzschild fields

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A detailed analysis of the properties of gravitational synchrotron radiation (GSR) is given for a particle moving along an unstable circular geodesic trajectory in a Kerr field. Formulas are derived for the spectral-angular, spectral, and angular distributions of the intensity of GSR for various polarization states. The effect of rotation of the polarization plane of a gravitational wave propagating through a Kerr field is discussed. The differences in the properties of GSR for sources moving along geodesic and nongeodesic trajectories are discussed.

The problem of emission of gravitational waves in the general theory of relativity (GRT) has been discussed in sufficient detail in a number of original articles and monographs. However, in spite of the fact that the majority of theorists are confident that gravitational waves exist, and some of them have investigated in detail a number of possible sources of gravitational radiation[1-4], there is so far no experimental verification of the theory. The difficulties of the experiment are related to the smallness of the gravitational coupling constant that enters in the intensity of gravitational radiation, and so far there is no satisfactory model of a source of gravitational waves for which the intensity would be large enough to be detected. The last remark applies in particular to possible laboratory sources, therefore the problem of detection of gravitational waves at present is closely related to an investigation of possible astrophysical processes, in which, as is presently accepted, one can expect a noticeable yield of gravitational radiation. The interest in cosmic sources has increased in recent years in connection with the experiments of Weber[5], and although the question of what is in fact being recorded by Weber's antennas remains unclear at the present moment, it seems useful to elucidate the properties of gravitational radiation from one of the possible cosmic sources.

According to the results of GRT (cf. e.g.,[6]), nature admits the existence of so-called "black holes." Particles entering the region of strong gravitational fields surrounding these holes with relativistic energies and definite angular momentum can be captured into an unstable circular orbit. While moving on this orbit these particles must emit radiation of the synchrotron type[7]. Strictly speaking, the radiation emitted by such particles does not correspond exactly to synchrotron radiation in flat space (cf., e.g.,[8]). It is obvious that the stresses of the proper gravitational field of the particle also contribute to the gravitational radiation in the case in question. Such a model, considered by Misner et al.[1] for a scalar source and later by Breuer et al.[2] for electromagnetic and tensor sources in a Schwarzschild metric, exhibits two essential characteristics:

1) the radiation is concentrated in the equatorial plane with an opening angle $\Delta\theta \sim m^{-1/2}$ ($m \gg 1$),

2) the radiation is emitted on high harmonics of the fundamental frequency ω_p: $\omega = m\omega_p$.

It is however quite likely that the black holes have a proper angular momentum and the field produced around them is described by the Kerr metric[9]. A particle rotating around a Kerr black hole may approach the event horizon (the surface of one-way valve) much closer and consequently, if it moves along stable orbits (nonsynchrotron model), it can emit energy much more efficiently than in a Schwarzschild metric[1)]. In addition, the rotation energy of the black hole can, under certain circumstances, be transmitted to the multipole waves[11]. However, in the same manner as in the Schwarzschild field, the gravitational synchrotron radiation from particles in a Kerr field can be emitted only for motion along unstable orbits.

In the sequel we shall abbreviate the long term "gravitational synchrotron radiation" by GSR. By GSR we mean the radiation emitted by particles moving along geodesic trajectories. We shall consider the GSR from a particle revolving in the equatorial plane of a Kerr field. The metric is chosen in terms of the coordinates t, r, θ, φ (cf.[9]) in units G = c = 1 (G is the gravitational constant, c the velocity oflight). The parameters of the particle and of the central body are the following:

$$r_p = r_0(1+\Delta), \quad \Delta \ll 1$$

(r_p is the orbit radius), E is the total energy in units of the particle mass, $\epsilon_p = d\varphi/dt$ is the coordinate angular velocity of the particle on a circle of radius r_p, ϵ_0 is the velocity on a null-geodesic, r_0 is the radius of the null-geodesic, M is the mass of the central body, J = aM is its angular momentum and a is a parameter. We use the notations:

$$x_p = \frac{r_p}{M}, \quad x_0 = \frac{r_0}{M}, \quad \omega_0 = \frac{\varepsilon_0}{M}, \quad \omega_p = \frac{\varepsilon_p}{M}, \quad \alpha = \frac{a}{M},$$

i.e., all quantities are measured in terms of the mass of the central body. We consider so-called direct circular particle orbits, on which the angular momenta of the particle and of the central body have the same sign. The final formulas are valid also for the other case, if one substitutes the appropriate values of the parameters x_0, ω_0.

The calculations are carried out by means of the Newman-Penrose formalism[12] (cf. also[13]). In order to describe the gravitational field in this formalism the ten components of the Weyl tensor are replaced by five complex functions, two of which yield the "radiation parts" of the wave field

$$\psi_0 = -C_{\alpha\beta\gamma\delta} l^\alpha m^\beta l^\gamma m^\delta, \quad \psi_4 = -C_{\alpha\beta\gamma\delta} n^\alpha m^{*\beta} n^\gamma m^{*\delta},$$

where $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor l^μ, n^μ, m^μ, $m^{*\mu}$ is an isotropic tetrad[13]; $\mu, \alpha, \beta, \gamma, \delta = 0, 1, 2, 3$. These components express the energy density at spatial infinity. For waves of the type $\psi \sim e^{-i\omega t}$

$$\left(\frac{d^2E}{dt\,d\Omega}\right)_{in}=\lim_{r\to\infty}\frac{r^2}{64\pi\omega^2}|\psi_0|^2, \qquad (1)$$

$$\left(\frac{d^2E}{dt\,d\Omega}\right)_{out}=\lim_{r\to\infty}\frac{r^2}{4\pi\omega^2}|\psi_4|^2. \qquad (2)$$

Equation (1) yields the energy flux of the waves converging to the event horizon r_{hor}, and (2) yields the diverging waves.

Inhomogeneous wave equations for ψ_0 and ψ_4 have been written out explicitly in the paper of Teukolsky[13]. A complete separation of variables is possible only for the component ψ_0, so that there remains the problem of reconstructing the field ψ_4 in terms of the solution ψ_0. The complete equation for ψ_0 has the form

$$\Box\psi_0=4\pi\Sigma_1 T, \qquad (3)$$

where $\Sigma_1 = r^2 + a^2\cos^2\theta$ and $\Box$ and T are defined in[13]. The solution (3) will be sought in the form

$$\psi_0=\sum_{l\geq m}\sum_{m=-\infty}^{\infty}(\omega_p/2\pi)^{1/2}\exp(-im\omega_p t+im\varphi)R_{lm}(r)\,{}_sS_m{}^l(\theta). \qquad (4)$$

After substituting (4) into (3) we obtain the following equations for the determination of the functions R_{lm} and ${}_sS_m^l$:

$$R''(x)+(s+1)\frac{2(x-1)}{\Delta}R'(x)+\frac{m^2}{\Delta^2}\left\{U_0(x)+\frac{1}{m}(U_1(x)+iU_2(x))\right\}R(x)=\Phi(x_0), \qquad (5)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta\,{}_sS_m{}^l)+\left\{\lambda+m^2\alpha^2\omega_0^2\cos^2\theta+s-\frac{m^2}{\sin^2\theta}-2\alpha\omega_0 m\cos\theta-\frac{2sm\cos\theta}{\sin^2\theta}-s^2\operatorname{ctg}^2\theta\right\}{}_sS_m{}^l=0. \qquad (6)$$

Equations (5) and (6) can be used for the description of the scalar, electromagnetic, and gravitational radiation at s = 0, 1, 2, respectively. It is clear that Eqs. (8) and (9) below, which determine the function $\Phi(x_0)$, must be modified for the analysis of scalar and electromagnetic radiation.

In the above equations we have set $\omega_p = \omega_0$, which is valid for ultrarelativistic particles moving on circular orbits. Here

$$\begin{gathered}U_0(x)=\omega_0^2 x(x+2x_0)(x-x_+)(x-x_-),\\ U_1(x)=-[2(l-|m|)+1](1-{}^1/_2\alpha^2\omega_0^2)\Delta(x),\\ U_2(x)=2s\omega_0(x-x_0)[x^2-(3-x_0)x+{}^1/_2x_0(3-x_0)];\\ \omega_0=2/x_0^{1/2}(3+x_0),\qquad \Delta(x)=x^2-2x+\alpha^2,\end{gathered} \qquad (7)$$

and x_+ and x_- are the turning points for the radiation, related to the particle energy by:

$$x_\pm=x_0(1\pm\delta),\quad \delta=(x_0-1)/2(3x_0)^{1/2}E.$$

The right-hand side of (5), for the case of motion of the particle along circular geodesic relativistic orbits, can be transformed after cumbersome calculations to the form

$$\Phi(x_0)=-16\pi\frac{E}{M}\frac{1}{(3+x_0)(x_0-1)^2}\{a\delta(x-x_0)+b\delta'(x-x_0)+c_1\delta''(x-x_0)\}; \qquad (8)$$

$$\begin{gathered}a={}_2S_m{}'''^l(\pi/2)-i(x_0+1)x_0^{-1/2}{}_2S_m{}''^l(\pi/2)\\ -2\,{}_2S_m{}^l(\pi/2),\\ b=-ix_0^{1/2}(x_0-1)\,{}_2S_m{}'^l(\pi/2)-{}^1/_2(x_0^2-1)\,{}_2S_m{}^l(\pi/2),\\ c_1={}^1/_4x_0(x_0-1)^2\,{}_2S_m{}^l(\pi/2).\end{gathered} \qquad (9)$$

From the asymptotic behavior of (5) it follows that the solution corresponding to the real part of $\Phi(x_0)$ describes the polarization of the gravitational wave in the "direction" $(\mathbf{e}_\theta\cdot\mathbf{e}_\theta-\mathbf{e}_\varphi\cdot\mathbf{e}_\varphi)2^{-1/2}$ (the σ-component) and to the imaginary part corresponds $(\mathbf{e}_\theta\cdot\mathbf{e}_\theta+\mathbf{e}_\varphi\cdot\mathbf{e}_\theta)2^{-1/2}$ (the π-component). Here $\mathbf{e}_\varphi$ and $\mathbf{e}_\theta$ are the unit vectors of a spherical coordinate system. The solution of Eq. (5) can be written in the form

$$R(x)=\Delta^{-(s+1)/2}\rho(x); \qquad (10)$$

$$\rho^\pm(x)=\omega_0^{-1/2}\exp(-im\omega_0 x)x^{\mp s}\exp[\mp sC(x_0)],\quad x\gg1, \qquad (11)$$

$$\rho^+(x)=D_{-\nu-1}(-iz)\quad \rho^-(x)=D_\nu(z)\quad (x\sim x_0). \qquad (12)$$

Here

$$\begin{gathered}z=e^{i\pi/4}\cdot2(2m\omega_0 3^{1/2})^{1/2}(x-x_0)/(x_0-1),\\ \nu=i\{m/m_0+[2(l-|m|)+1]\mu\}-{}^1/_2,\\ \mu=(1-0{,}5\alpha^2\omega_0^2)/2x_0\omega_0\sqrt{3},\\ m_0=(x_0-1)^2\sqrt{3}/6\omega_0 x_0^2\delta^2,\\ C(3)\sim0{,}02\ (\alpha=0),\quad C(1)\approx\sqrt{3}/2\ (\alpha\to1),\end{gathered} \qquad (13)$$

where ρ^+ corresponds to the divergent wave, and D is a parabolic cylinder function[14]. The representation of ρ^+, ρ^- in the form (12) is valid for $x_0 - 1 \gg \Delta$. The wave ψ_0 has for $x \gg 1$ the following asymptotic behavior

$$\psi_0\sim e^{-im\omega_0 t}\Delta^{-(s+1)/2}(\rho^++\rho^-)\sim e^{-im\omega_0 t}\left(\frac{Ae^{-im\omega_0 x}}{x}+\frac{Be^{im\omega_0 x}}{x^5}\right). \qquad (14)$$

For our choice of sign of the exponent in the temporal factor the energy flux at infinity is given by the square of the absolute value of the wave ψ_4. For $x \gg 1$, ψ_4 has the form

$$\psi_4\sim e^{-im\omega_0 t}\left\{\frac{A_1e^{im\omega_0 x}}{x}+\frac{B_1e^{-im\omega_0 x}}{x^5}\right\}. \qquad (15)$$

The relation between the coefficients A_1 and B in ψ_4 and ψ_0 is established by comparing the solutions ρ^- and ρ^+ near x_0. Indeed, the relation between A_1 and B can be obtained considering the fields ψ_0 and ψ_4 in the near zone and determining the "potentials of the radiation reaction forces" on the source in both cases. It is clear that owing to the identity of the source the "potentials of the radiation reaction forces" must be equal in absolute value for ψ_0 and ψ_4, but shifted in phase by π.

The final formula for the calculation of the energy flux carried away by the wave ψ_4 to infinity is

$$W=\frac{C_2}{128\pi^2m^2\omega_0^2}\frac{\exp[-4C(x_0)]}{(m_0x_0\delta^2m)^{1/2}}\exp\left\{-\frac{\pi}{2}\left(\frac{m}{m_0}+[2(l-|m|)+1]\mu\right)\right\}, \qquad (16)$$

where C_2 is determined by the source and is obtained by matching the solutions at the particle. It turns out that the angular part of the wave ψ_4 must be described by the function ${}_2S_m^l(\pi-\theta)$ if the angular part of ψ_0 is proportional to ${}_2S_m(\theta)$. In order not to violate the general scheme we shall solve below the equation for ψ_0 (and ${}_sS_m^l(\theta)$) and only at the end do we go over to ψ_4. We also note that the solution of the problem can be obtained immediately by determining the function f_4 related to ψ_4 by means of the relation

$$f_4=\rho^{-4}\psi_4,\quad \rho=-i/(r-ia\cos\theta), \qquad (17)$$

for which the solution can be found directly. The final results obtained by one or the other of these methods coincide completely, of course.

The angular distribution of the radiation is determined by the functions ${}_sS_m^l$ which are the eigenfunctions of the sum of the operators

$$H_0=\frac{d^2}{d\theta^2}+\operatorname{ctg}\theta\frac{d}{d\theta}-\frac{m^2+2sm\cos\theta+s^2}{\sin^2\theta}, \quad (18)$$

$$H_1=m^2\alpha^2\omega_p^2\cos^2\theta-2s\alpha m\omega_p\cos\theta \quad (19)$$

with the eigenvalues ${}_sA_m^l(\alpha\omega)$. The eigenfunctions of the operator H_0 are described by the so-called spherical harmonics ${}_sP_m^l$ [15] 2) with the eigenvalues

$${}_sA_m^l(0)=(l-s)(l+s+1).$$

We consider the operator H_1 as a perturbation. The validity of this assumption follows from the fact that the radial parts of the wave equations contain exponential factors $\exp\{-[2(l-|m|)+1]\mu\}$ which restrict the set of eigenfunctions of the operator H_0 practically to two ${}_sP_m^m$ and ${}_sP_m^{m+1}$, and since the effective half-widths of these functions are included in a narrow interval of angles θ near the values

$$\theta=\pi/2\pm\Delta\theta,\quad \Delta\theta\sim m^{-1/2},\quad m\gg1,$$

one may consider, with a high degree of accuracy, that

$$H_1=m^2\alpha^2\omega_0^2\cos^2\theta-2m\alpha\omega_0 s\cos\theta\approx$$
$$\approx m\alpha^2\omega_0^2-2m^{1/2}\alpha\omega_0 s$$

is a perturbation of order $1/m$ to the operator H_0.

The corrections to the eigenvalues are determined in the usual manner by means of perturbation theory. The complete eigenvalues of the operator H are:

$${}_2A_m^m=m^2+m(1-{}^1/_2\alpha^2\omega_0^2), \quad (20)$$

$${}_2A_m^{m+1}=m^2+3m(1-{}^1/_2\alpha^2\omega_0^2). \quad (21)$$

In the high-harmonic approximation $m\gg1$ the functions ${}_2P_m^l$, which describe well the angular part of the radiation (not taking into consideration the rotation of the plane of polarization), can be represented in the form

$${}_2P_m^m=(-1)^m m^{1/4}\pi^{-1/4}(1-\cos\theta)^2\sin^{m-2}\theta, \quad (22)$$

$${}_2P_m^{m+1}=(-1)^m m^{-1/4}\pi^{-1/4}\sqrt{2}(1-\cos\theta)^2\sin^{m-2}\theta[(m+1)\cos\theta+s]. \quad (23)$$

It is now easy to find quantities which characterize the properties of the GSR from a particle. Thus, the spectral-angular intensity distributions of GSR corresponding to two linearly independent states of polarization (σ, π) are of the form

$$W_{\sigma,\pi}=C_{\sigma,\pi}\frac{x_0}{16\pi^{1/2}(3+x_0)^2}\frac{1}{(x_0-1)(3\omega_0^2x_0^2)^{1/4}}\frac{m_0}{m}$$
$$\times\exp\{-4C(x_0)-{}^1/_2\pi(m/m_0+[2(l-|m|)+1]\mu)\}; \quad (24)$$

$$C_\sigma=\frac{\delta_{lm}}{16\omega_0\sqrt{3}}\left[1+2\sqrt{3}x_0\omega_0\left(\mu+\frac{m}{m_0}\right)\right]^2\left|\Gamma\left(\frac{1}{4}+\frac{i}{2}\frac{m}{m_0}+i\frac{\mu}{2}\right)\right|^2|{}_2P_m^m|^2, \quad (25)$$

$$C_\pi=2x_0|\Gamma({}^3/_4+im/2m_0+i3\mu/2)|^2|{}_2P_m^{|m|+1}|^2\delta_{l,|m|+1}. \quad (26)$$

Here Γ is the gamma function.

The parameter m_0 plays the role of critical harmonic. For $m\gg m_0$ the intensity of radiation decays exponentially. In (24)–(26) only the contributions from the leading terms have been taken into account and corrections proportional to $m^{-1/2}$ have been neglected. This means that we neglect the contribution corresponding to radiation emitted under large angles and for small orders m of the harmonics. Equations (24)°(26) depend essentially only on one parameter x_0, since the quantities α and ω_0 are determined by x_0:

$$\alpha=(3-x_0)x_0^{1/4}/2,\qquad \omega_0=2/(3+x_0)x_0^{1/2}.$$

For $x_0 = 3$ these formulas describe the GSR in a Schwarzschild field. The GSR from a particle moving along a circular geodesic in a Schwarzschild field was discussed by Breuer et al.[2] The ratio C_σ/C_π in the final results of[2] differs from the one obtained here (Eqs. (25), (26) with $\alpha = 0$, $x_0 = 3$) by a numerical coefficient. Unfortunately, it is difficult to compare the results, since the formula used for calculating the power of the GSR is not given in[2]. The possible reason for the discrepancy seems to us to be that Breuer et al.[2], in calculating the energy flux of the GSR, having apparently added the absolute values squared of the functions $R_{lm}^{(e)}$ and $R_{lm}^{(m)}$, whereas according to[19] the components of the tensor of the gravitational wave are related to $R_{lm}^{(e)}$ and $R_{lm}^{(m)}$ by

$$R_{lm}^{(e)}=h_{\theta\varphi},\qquad R_{lm}^{(m)}=h_{\varphi\varphi}/2.$$

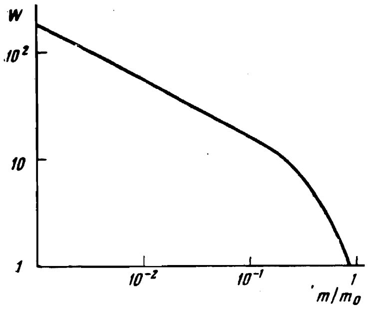

FIG. 1

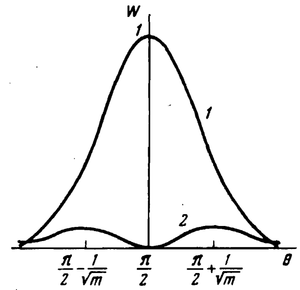

FIG. 2

FIG. 1. The intensity of GSR as a function of the order of the harmonic for given angle $\theta = \pi/2$ in relative units.

FIG. 2. The angular distribution of GSR: curve 1 represents the σ-component, curve 2 represents the π-component.

Figure 1 shows a graph of the spectral distribution of the GSR for fixed values of the angle $\theta = \pi/2$. The intensity of the radiation for $m\gg1$ is essentially concentrated in the equatorial plane and varies little within the half-width of the angle. The functions $|{}_2P_m^{|m|}|^2$ and $|{}_2P_m^{|m|+1}|^2$ have the form (cf. Fig. 2)

$$|{}_2P_m^{|m|}|^2\approx2\frac{m^{1/2}\sin^{2(m-2)}\theta}{\pi^{1/2}}(1+6\cos^2\theta+\cos^4\theta), \quad (27)$$

$$|{}_2P_m^{|m|+1}|^2\approx4\frac{m^{3/2}\sin^{2(m-2)}\theta}{\pi^{1/2}}(1+\cos^2\theta)^2(\cos^2\theta+4/m^2). \quad (28)$$

The spectral distribution of the GSR is given by the same equations (24)–(26) for $|{}_2P_m^{|m|}|^2 = 1$ and $|{}_2P_m^{|m|+1}|^2 = 1$. The particle emits essentially waves for which the total angular momentum $l = m$. The intensity of the radiation falls off as m increases;

$$W\sim m_0/m \text{ for } m\ll m_0,$$
$$W\sim\exp(-m/m_0) \text{ for } m\gg m_0. \quad (29)$$

The gravitational radiation is strongly polarized. The contributions W_σ and W_π to the global intensity

$$W=W_\sigma+W_\pi$$

depend weakly on the parameter α and equal

$$W_\sigma^{\alpha=0}/W^{\alpha=0}=89\%,\qquad W_\sigma^{\alpha\to1}/W^{\alpha\to1}=88\%.$$

The change of the ratio W_σ/W in the two extreme cases ($\alpha = 0$), ($\alpha\to1$) is 1% and cannot be guaranteed, since it is within the limits of the approximation under consideration. The graph of the spectral distribution W(m) is obtained from Fig. 1 by dividing by $m^{1/2}$. The spectral distribution of the GSR for any admissible values of the parameters α and x_0 is well approximated by the equation

$$W(m) \sim (m_0/m)\exp(-m/m_0). \qquad (30)$$

A separation by polarizations corresponds to a separation with respect to the parities of the emitted modes. Thus the fundamental contribution to W_σ comes from even modes, proportional to $(-1)^l$, and to W_π from odd modes, proportional to $(-1)^{l+1}$. The contribution of odd modes to W_σ and of even modes to W_π for high multipoles does not exceed several percent.

More interesting are the results of investigation of the polarization properties of the radiation. The dependence of the degree of polarization of the GSR

$$P = (W_\sigma(\theta) - W_\pi(\theta))/W(\theta)$$

on the angle θ for fixed values of the harmonic m vanishes (P = 0) near the angles

$$\theta_1 = \pi/2 \pm 1{,}2m^{-1/2},$$

which are within the half-width of the beam, so that an observer which receives the gravitational waves under an angle θ_1 will record radiation which is totally circularly polarized. In general, for $\theta \neq \pi/2$ the GSR is elliptically polarized, except the points θ_1 and $\pi/2$. The global power of the radiation is $W \sim E^2$ for all values $x_0 \neq 1$.

Let us discuss the question of a possible rotation of the polarization plane of the gravitational wave as it propagates from the source to observer ($r \to \infty$) in a Kerr field. The rotation of the polarization plane of an electromagnetic wave in a Lense-Thirring field, analogous to the Faraday rotation in a magnetic field, was discussed by Skrotskiĭ[16] and by Vladimirov and Iskhakov[17]. The angle of rotation of the polarization plane must be linear in the parameter α. Some estimates can be obtained by considering the functions ${}_sS_m^m$ and ${}_sS_m^{m+1}$ which describe the angular part of the gravitational wave. If the gravitational field of the central body were spherically symmetric the functions ${}_2P_m^m$ and ${}_2P_m^{m+1}$ would give a complete description of the angular part of the wave. These functions correspond to the operator H_0, which does not contain terms proportional to α. Taking into account the rotation of the central body leads to the result that angular parts of the waves are described by other functions, namely ${}_2S_m^m$ and ${}_2S_m^{m+1}$, which are eigenfunctions of the full operator H. It is also clear that after we have expanded the wave into two linearly independent polarization states proportional to ${}_2P_m^m$ and ${}_2P_m^{m+1}$ the corrections to these functions will produce mixing of the polarization components, i.e., a rotation of the polarization plane. The angular part of the σ-component of the polarization is now equal to

$${}_sS_m^m = \frac{(-1)^m}{\pi^{1/2}} m^{1/4} (1-\cos\theta)^s \sin^{m-s}\theta \left\{1 + \frac{s\alpha\omega_0}{m}[(m+1)\cos\theta + s]\right\}. \quad (31)$$

The correction for the function ${}_sS_m^{m+1}$ differs from the above only in sign. Thus, when a gravitational wave propagates in a Kerr field under angles $\theta \neq \pi/2$ its polarization plane rotates by an angle:

$$\beta = 2\alpha\omega_0\cos\theta + 4\alpha\omega_0/m.$$

We recall that the corrections have been calculated in the approximation $\cos\theta \sim m^{-1/2}$. It is convenient to represent the rotation angle of the polarization plane in the form of a sum of two terms:

$$\beta = 2(\alpha\omega_0\cos\theta + 2\alpha/\rho).$$

Here ρ is the impact parameter of the beam in the equatorial plane. The rotation effect related to the second term is due to the curvature of the curve along which the beam propagates and corresponds to the approximation of geometric optics. The contribution of the first term is analogous to the effect of Faraday rotation of the polarization plane of an electromagnetic wave in a magnetic field in flat space. It is produced by the interaction of the angular momenta of the black hole and of the wave. The effect of rotation of the polarization plane of a gravitational wave in a Kerr field can produce a partial polarization of an initially unpolarized plane gravitational wave when it is "reflected" on the black hole. Indeed, due to this effect right- and left-polarized waves will be reflected from the black hole with different amplitudes, which can lead to a partial polarization of the initial gravitational wave.

We also note that the rotation angle of the polarization plane of the gravitational wave is twice as large (due to the factor s = 2) as the corresponding angle for the electromagnetic field.

We have considered the GSR from a particle moving with relativistic velocity along a geodesic. What changes in the properties of the GSR can one expect if the source is a particle moving with a high velocity, but not near a null geodesic, i.e., when the motion is nongeodesic? In the latter case the characteristics of the radiation will be quite similar to the characteristics of synchrotron radiation in flat space. Indeed, the radiation spectrum, its polarization, etc. are essentially determined by the behavior of the potential barrier for radiation and by the amplitude of the source. The source itself is always situated in the so-called near zone inside the potential barrier, and therefore the height and width of this barrier are essential and depend on the relations between x_p and x_0. For simplicity we consider the Schwarzschild field ($\alpha = 0$). In this case we obtain in the approximation $m \gg 1$ for geodesic motion near x_0

$$U_\Gamma(x) = -{}^2/_3 m^2\{\Delta - {}^1/_9(x-x_0)^2\}, \quad \Delta \sim 1/E^2. \qquad (32)$$

For nongeodesic motion in the whole region of variation of x

$$U_H(x) = m^2\left\{\frac{\omega_p^2}{(1-2/x)^2} - \frac{1}{x(x-2)}\right\}. \qquad (33)$$

In the first case the barrier is parabolic near the source, in the second case it is a centrifugal barrier. One can use wave equations with source taking into account (32), (33) to evaluate all the basic characteristics of the radiation. Thus, frequency cut-off factors which determine the pass-band are given by the coefficients of barrier penetration of "waves" to one of the turning points ($x_2 > x_1$) and are equal to

$$D_\Gamma \sim \exp(-m/m_0), \quad m_0 \sim E^2; \qquad (34)$$

$$D_H \sim \exp(-2m/3m_0), \quad m_0 \sim (1-\beta^2)^{-1/2}. \qquad (35)$$

In (35) we have taken into account the fact that $x_p\omega_p = \beta$ ($\beta = v/c$ where v is the particle velocity). Similarly one can obtain the other characteristics, if the explicit form of the source is known.

If the particle moves along a circumference which is not geodesic the radiation emitted by it is similar in its properties to the usual gravitational radiation emitted by a mass in flat space. The spectrum, angular distribution and polarization of such radiation have been described previously.[18]

In conclusion, let us list some estimates. The time during which a particle completes a complete turn around the circumference is, by the clock of a remote observer:

$$T=2\pi/\omega_0=\pi(3+x_0)x_0^{1/2}r_g/c\sim r_g/c, \quad r_g=2GM/c^2.$$

The radiated power is

$$W=-\frac{dE}{dt}\sim\frac{G\omega_0^2}{c^5}\left(\frac{E}{\mu c^2}\right)^2(\mu c^2)^2=\bar{\alpha}\mu c^2\left(\frac{E}{\mu c^2}\right)^2, \quad \bar{\alpha}=G\omega_0^2\mu/c^3=1/\tau, \tag{36}$$

where τ is the time during which the energy of the particle decreases by a factor of e. Expression all masses in terms of the solar mass $M_\odot$ we obtain for τ:

$$\tau=1/\bar{\alpha}\sim10^{-5}n^2/k \text{ sec}, \quad n=M/M_\odot, \quad k=\mu/M_\odot.$$

If $M = 10^8 M_\odot$, $\mu \sim 10^4 M_\odot$, then for $E/\mu c^2 \sim 10^2$

$$W\sim10^{45} \text{ erg/sec}.$$

It is curious that from (36) one can obtain the estimate of the maximal power of gravitational radiation given by Zel'dovich and Novikov [10]. Indeed, for two "collapsars" we have

$$M\sim\mu\left(\frac{E}{\mu c^2}\sim1\right), \quad W\sim\frac{G\omega_0^2}{c^5}(\mu c^2)^2, \quad r_g^{\,1}\sim\frac{G\mu}{c^2}.$$

Then the power (36) depends only on c and G:

$$W\sim\frac{G\omega_0^2}{c^5}\left(\frac{c^4r_g^{\,1}}{2G}\right)^2\sim\frac{c^5}{4G}\sim10^{59} \text{ erg/sec}.$$

1) The energy of particles on critical stable orbits is $E = 0.94\,\mu c^2$ in a Schwarzschild field and $E = 0.5\,\mu c^2$ in a Kerr field in the extremal ($\alpha \to 1$) case. If the particle enters the field with parabolic velocity, the amount of radiated energy is respectively $W_{Sch} = 0.06\,\mu c^2$, $W_K = 0.42\,\mu c^2$ [10].

2) The properties of the functions ${}_sP^l_m$ are described by Vilenkin [15]. We shall not dwell here on their detailed discussion.

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