Kinetic theory of generation of a strong field in semiconductor lasers

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A set of equations describing the generation of a strong ($\lambda\tau \gg 1$) electromagnetic field by a semiconductor laser is investigated. The set utilizes kinetic equations for quasiparticles that represent electrons interacting with the field being generated. The dependence of the field on the pumping current is derived. It is shown that with an increase in pumping this dependence becomes nonlinear and there exists an upper limit on the generator field strength.

In a previous paper by the authors[1] an investigation was made of the theory of a semiconductor laser for the case of weak field when $\lambda\tau \ll 1$ ($\lambda = dE$ is the frequency of interband transitions of electrons under the action of an electromagnetic field **E**, **d** is the dipole moment for the transition, $1/\tau$ is the frequency of collisions of electrons with phonons, electrons etc.). It was shown there that the electron distribution function has as a result of the saturation effect a minimum (a dip) at the spot corresponding to a transition at the frequency being generated. Although the magnitude of the dip is not great in a weak field, it alters the qualitative picture by enabling the distribution function to change above the threshold with increased pumping.

The results of[1] have been obtained by means of linearizing the system of equations describing the generation. As the pumping current becomes larger this approximation becomes inapplicable. In nonlinear equations the effect of saturation manifests itself more strongly, but the field continues to increase as the pumping is increased. For certain values of pumping the quantity λ which characterizes the field attains the value $1/\tau$ and with further increase begins to satisfy the condition[1)]

$$\lambda\tau \gg 1. \tag{1}$$

In this region, as has been shown in[3], the electron transitions between bands occur during times less than the time of interaction with phonons, and the use as a zero-order approximation of particles which do not interact with the field loses its meaning. The taking of interband resonance transitions into account is accomplished by introducing quasiparticles related to electrons by a uv transformation. The kinetic equations for electrons must here be replaced by kinetic equations for the quasiparticles. The new physical consideration corresponding to this scheme consists of the fact that the direct transitions are saturated and the processes of emission and absorption of quanta of electromagnetic field occur as a result of indirect transitions involving emission of phonons.

In the present paper we investigate the kinetic theory of a semiconductor laser in the strong field domain[2)]. The stationary case of single-mode generation is investigated. The distribution function for the quasiparticles is obtained and also the dependence of the laser field on the pumping current. It is shown that as the pumping increases this dependence becomes nonlinear, and there exists an upper limit on the field of the generator

$$2\lambda_u \leq \omega_{ph}, \tag{2}$$

where ω_{ph} is the maximum phonon frequency[3)].

1. SATURATION OF STATIONARY GENERATION BY A SEMICONDUCTOR LASER

In the case of a strong field the quasiparticles α_p, β_p represent a superposition of electrons and holes a_p, b_p[3]:

$$\alpha_\mathbf{p} = u_\mathbf{p} a_\mathbf{p} - v_\mathbf{p} b_\mathbf{p}^+, \quad \beta_\mathbf{p} = u_\mathbf{p} b_\mathbf{p} + v_\mathbf{p} a_\mathbf{p}^+,$$
$$u_\mathbf{p}^2, v_\mathbf{p}^2 = \frac{1}{2}\left(1 \pm \frac{\xi_\mathbf{p}}{\varepsilon_\mathbf{p}}\right), \quad \varepsilon_\mathbf{p} = (\xi_\mathbf{p}^2 + \lambda^2)^{1/2}, \tag{3}$$

$$\xi_\mathbf{p} = \frac{\mathbf{p}^2}{2m} - \frac{p_0^2}{2m}, \quad \frac{p_0^2}{m} = \omega - E_g, \quad \lambda = \frac{e\mathbf{v}_{cv}\mathbf{E}}{2\omega}, \quad \hbar = c = 1. \tag{4}$$

Here ω is the frequency of the electromagnetic field **E** generated by the laser, E_g is the width of the forbidden band in a semiconductor with symmetric bands and with identical effective masses of electrons and holes m, v_{cv} is the matrix element for the transition between a valence band and a conduction band.

The kinetic equations for the distribution functions for the quasiparticles n_p have the form (cf.,[5])

$$n_\mathbf{p} = n_\mathbf{p}^\alpha = n_\mathbf{p}^\beta, \quad n_\mathbf{p}^\alpha = \langle \alpha_\mathbf{p}^+ \alpha_\mathbf{p} \rangle, \quad n_\mathbf{p}^\beta = \langle \beta_\mathbf{p}^+ \beta_\mathbf{p} \rangle,$$
$$\frac{\partial}{\partial t} n_\mathbf{p} = (1 - n_\mathbf{p}) S_\mathbf{p}^+ - n_\mathbf{p} S_\mathbf{p}^- - n_\mathbf{p} S_\mathbf{p}^A + (1 - n_\mathbf{p}) J_\mathbf{p}, \tag{5}$$

$$\begin{pmatrix} S_\mathbf{p}^+ \\ S_\mathbf{p}^- \\ S_\mathbf{p}^A \end{pmatrix} = 2\pi \sum_{\mathbf{p}'\mathbf{q}} g^2(\mathbf{q})\, \delta(\mathbf{p}' + \mathbf{q} - \mathbf{p}) \begin{pmatrix} n'\delta(\varepsilon - \varepsilon' + \omega_q)(uu' + vv')^2 \\ (1 - n')\delta(\varepsilon' - \varepsilon + \omega_q)(uu' + vv')^2 \\ n'\delta(\varepsilon + \varepsilon' - \omega_q)(uv' - u'v)^2 \end{pmatrix},$$
$$n_{\mathbf{p}'}, u_{\mathbf{p}'}, \varepsilon_{\mathbf{p}'} \equiv n', u', \varepsilon', \tag{6}$$

where $g^2(\mathbf{q})$ is the matrix element of the electron-phonon interaction, $\omega_q = sq$ is the phonon frequency, s is the speed of sound.

The first two terms in (5) describe the scattering of quasiparticles by phonons, where S^+ corresponds to incoming and S^- to outgoing particles, while the third term S^A describes annihilation of quasiparticles accompanied by emission of a phonon. The last term in (5) corresponds to a source of quasiparticles localizes when $\xi = \xi_0 > 0$.

If one integrates Eq. (5) over the momentum p, then in the stationary case one obtains the equation

$$\sum_\mathbf{p} n_\mathbf{p} S_\mathbf{p}^A = \sum_\mathbf{p} (1 - n_\mathbf{p}) J_\mathbf{p}, \tag{7}$$

which relates the rate of annihilation to the rate of production of quasiparticles by the source.

To Eqs. (5)–(7) one must also add the equation for the number N of photons of frequency ω:

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau_0}\right)N=Q, \quad N=\frac{\mathbf{E}^2}{4\pi}V,$$
$$Q=\sum_{\mathbf{p}}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}[n_{\mathbf{p}}S_{\mathbf{p}}{}^{A}+n_{\mathbf{p}}S_{\mathbf{p}}{}^{-}-(1-n_{\mathbf{p}})S_{\mathbf{p}}{}^{+}]. \tag{8}$$

Here τ_0 is the lifetime of a photon in the resonator, V is the volume which we subsequently set equal to unity, Q has the meaning of the number of photons created or absorbed in the semiconductor per unit time. The expression for Q has been obtained earlier (cf.,[5]) in a somewhat different form.

Equations (4)–(8) represent a closed system for a semiconductor laser. We note that the equations have been written for T = 0 and in them recombination terms have been omitted which in the strong field domain do not produce any significant effect. In future we shall investigate the system (5)–(8) for the stationary case.

One of the important results that follow from the system (5)–(8) consists of the fact that there exists a certain limiting value of the field generated by the laser. One can verify this most simply in the situation when the pumping source is localized for $\xi = \xi_0 \gg \lambda$ (such a situation is usually realized if optical pumping is utilized). Indeed, multiplying (5) by $v_{\mathbf{p}}^2$ and integrating over p we obtain the relation

$$\sum_{\mathbf{p}}\{n_{\mathbf{p}}S_{\mathbf{p}}{}^{A}+n_{\mathbf{p}}S_{\mathbf{p}}{}^{-}-(1-n_{\mathbf{p}})S_{\mathbf{p}}{}^{+}\}v_{\mathbf{p}}{}^2=\frac{\lambda^2}{\xi_0{}^2}[1-n(\xi_0)]J(\xi_0)\frac{m(p_0{}^2+2m\xi_0)^{1/2}}{2\pi^2}\approx 0$$

$(\xi_0 \gg \lambda)$, with the aid of which in place of (8) we obtain

$$N/\tau_0 \approx \sum_{\mathbf{p}} n_{\mathbf{p}}S_{\mathbf{p}}{}^{A}, \tag{9}$$

having in mind the fact that $2v^2 = 1 - \xi/\epsilon$.

As can be easily seen (cf., (6)) the integral of the function $n_{\mathbf{p}}S_{\mathbf{p}}^{A}$ is a bounded quantity which attains its maximum value for $n_{\mathbf{p}} = 1$. Thus, it follows from (9) that the field tends to a limiting value. The value of the limit will be calculated below. We first discuss the physical meaning of the result obtained.

From (9) it can be seen that in the stationary regime the generation is determined by the annihilation of quasiparticles. Annihilation is equivalent to a transition of electrons from the conduction band to the valence band with the emission of a photon and a phonon, so that generation is occurring as a result of indirect transitions. The latter circumstance is associated with the fact that under the condition of a strong field (1) the direct transitions turn out to be saturated. The laws of conservation of quasiparticles in the case of annihilation impose restrictions on the range of energies from which generation can occur

$$|\xi|<\omega_{\mathrm{ph}}(1-2\lambda/\omega_{\mathrm{ph}})^{1/2}, \quad \omega_{\mathrm{ph}}=2p_0 s. \tag{10}$$

The interval indicated above reduces to zero if the field attains the value

$$2\lambda_{\mathrm{u}}=\omega_{\mathrm{ph}}. \tag{11}$$

In this case annihilation of quasiparticles, and, consequently, also the generation of photons, are impossible ($S^A = 0$), so that λ_u is the upper limit on the field generated by the laser. This limiting value, as we shall see below, is attained under the definite condition:

$$\eta=\frac{\pi\tau_0 r}{2\tau_{\mathrm{ph}}}\gg 1, \quad r=\frac{e^2 v_{cv}{}^2 m}{2\pi\omega^2 s}, \tag{12}$$

to which we shall refer as the high-Q condition (τ_{ph} is the relaxation time for the momentum in the case of scattering by phonons as defined below).

In the low-Q regime, which is realized under the condition $\eta \ll 1$, the field attains its lower limiting value which is equal to

$$\lambda_l \approx \omega_{\mathrm{ph}}\eta. \tag{13}$$

In this regime generation is saturated when the distribution function for the quasiparticles attains values of approximately unity over the whole interval $|\xi| \leq \omega_{\mathrm{ph}}$. Its further increase when $|\xi| \geq \omega_{\mathrm{ph}}$ does not lead to additional generation of photons.

2. THE LIMITING VALUE OF THE FIELD IN THE HIGH-Q REGIME

The simplest task is to find the limiting value of the field for $\eta \gg 1$. Substituting the expression for S^A into (9) and integrating over q and the angles

$$\frac{N}{\tau_0}=\frac{m^2}{4\pi^3 s^3}\int n(\xi)\,d\xi\int d\xi' n(\xi')(u'v-uv')^2(\varepsilon+\varepsilon')g^2(\varepsilon+\varepsilon'), \tag{14}$$
$$0\leqslant\varepsilon+\varepsilon'\leqslant\omega_{\mathrm{ph}}.$$

In the high-Q regime the value of 2λ tends to ω_{ph}. For such λ the annihilation term is close to zero and is small in comparison with S^+, S^-. This means that the probability of annihilation of quasiparticles is much smaller than the probability of their being scattered. Therefore, an approximate solution of the kinetic equation (5) will be the Fermi function with the chemical potential μ determined from the condition (7). At T = 0 the distribution function for the quasiparticles is equal to unity in the interval $\epsilon \leq \mu$. Since the contribution to the integral (14) in the energy interval $(\epsilon - \lambda)/\lambda \approx (\omega_{\mathrm{ph}} - 2\lambda)/\lambda \ll 1$ is significant, then $n_{\mathbf{p}}$ in (14) can be set equal to unity and one can omit the terms odd in ξ in the coherent factor:

$$(uv'-u'v)^2=1-\xi\xi'/\varepsilon\varepsilon'-\lambda^2/\varepsilon\varepsilon'.$$

Then (14) can be represented in the form

$$\frac{\lambda^2}{\tau_0}=\frac{4r}{\tau_{\mathrm{ph}}}\int_{\lambda}^{\omega_{\mathrm{ph}}-\lambda}\frac{\varepsilon\,d\varepsilon}{(\varepsilon^2-\lambda^2)^{1/2}}\int_{\lambda}^{\omega_{\mathrm{ph}}-\varepsilon}\frac{\varepsilon'\,d\varepsilon'}{(\varepsilon'^2-\lambda^2)^{1/2}}\left(1-\frac{\lambda^2}{\varepsilon\varepsilon'}\right)$$
$$1/\tau_{\mathrm{ph}}=g^2 m/\pi. \tag{15}$$

Here for the sake of simplicity we have set $g^2(q) = g^2/q$, having in mind that this assumption does not affect the limiting value of λ_u. After integration we obtain the following equation for λ:

$$\lambda^2/\omega_{\mathrm{ph}}{}^2=2\eta(1-2\lambda/\omega_{\mathrm{ph}})^2. \tag{16}$$

This equation has the solution

$$2\lambda/\omega_{\mathrm{ph}}=1/[1+(8\eta)^{-1/2}],$$

which, taking the condition $\eta \gg 1$ into account, leads to the limiting value

$$2\lambda/\omega_{\mathrm{ph}}\approx 1-(8\eta)^{-1/2}, \tag{17}$$

which coincides with (11). We obtain the same value for λ for $\eta \gg 1$ if we use the matrix element $g^2(q)$ with a different dependence on q.

An estimate of the limiting field for $\omega_{\mathrm{ph}} = 10^{13}\ \mathrm{sec}^{-1}$ gives $\lambda_u = 0.5 \times 10^{13}\ \mathrm{sec}^{-1}$ and $P = 10^7\ \mathrm{W/cm}^2$. It is of interest to note that in the experiments known to us the stationary value of the field in a single-mode regime is limited by approximately this value.

3. A SEMICONDUCTOR LASER IN THE LOW-Q REGIME

We investigate in greater detail generation in the low-Q regime when the condition $\eta \ll 1$ is satisfied. The kinetic equation (5) after integration over q and the angle between p and p′ assumes the form

$$\begin{pmatrix} S_{\mathbf{p}}^{+} \\ S_{\mathbf{p}}^{-} \\ S_{\mathbf{p}}^{A} \end{pmatrix} = \frac{1}{\tau_{\mathrm{ph}}\omega_{\mathrm{ph}}} \begin{pmatrix} \int d\xi' n(\xi')(uu'+vv')^2, & 0 \leqslant \varepsilon'-\varepsilon \leqslant \omega_{\mathrm{ph}} \\ \int d\xi'(1-n(\xi'))(uu'+vv')^2, & 0 \leqslant \varepsilon-\varepsilon' \leqslant \omega_{\mathrm{ph}} \\ \int d\xi' n(\xi')(uv'-u'v)^2, & 0 \leqslant \varepsilon+\varepsilon' \leqslant \omega_{\mathrm{ph}} \end{pmatrix}, \tag{18}$$

if we set $g^2(q) = g^2/q$.

In the low-Q regime the ratio $\lambda/\omega_{\mathrm{ph}}$ is small (cf., (13)), while the distribution function $n_{\mathbf{p}}$ is approximately equal to unity in the range $0 \le \xi \le \bar{\xi}(\lambda \ll \bar{\xi} \ll \omega_{\mathrm{ph}})$. Therefore the principal contribution to S^+ and S^- is made by large values of $\xi' \gg \lambda$. This circumstance gives us the possibility of simplifying the equation for large ξ (but smaller than ω_{ph}!), neglecting terms of the order of $\lambda/\omega_{\mathrm{ph}}$. Moreover, the annihilation term S^A in Eq. (5) can be omitted for large values of ξ, since it is different from zero in the range $|\xi| \le \lambda$. Annihilation is taken into account with the aid of condition (7). This approximation reflects the fact that annihilation has a small effect on the distribution of quasiparticles for large ξ, but determines the number of quasiparticles. As a result of simplifications we obtain the following equation:

$$(1-n(x))\int_x^1 dx'\, n(x') - n(x)\int_0^x dx'(1-n(x')) = 0, \quad x = \xi/\omega_{\mathrm{ph}} < 1. \tag{19}$$

Introducing the function

$$\chi(x) = \int_0^x (1-n(x'))\,dx',$$

we transform (19) into the differential equation

$$\frac{d\chi}{dx}(a-x-2\chi)-\chi=0, \tag{20}$$

where the quantity

$$a=\int_0^1 n(x')\,dx' \tag{21}$$

is determined from the condition (7).

The solutions for χ and n have the form

$$\chi=\frac{x-a}{2}+\left[\left(\frac{x-a}{2}\right)^2+|c|\right]^{1/2},$$
$$n(x)=\frac{1}{2}\left[1-\frac{x-a}{((x-a)^2+4|c|)^{1/2}}\right]$$

We determine the constant $|c|$ from the condition $\chi(0) = 0$, $|c| = 0$. Thus, in this approximation the distribution function has the form of the step function:

$$n(x)=\begin{cases}1, & 0<x<a \\ 0, & a<x\end{cases}.$$

The vanishing of the constant $|c|$ is associated with the neglect in equation (19) of the function n for $x > 1$. Taking it into account gives a value for $|c|$ of the order of $\lambda/\omega_{\mathrm{ph}}$. Knowledge of the behavior of $n(\xi)$ for large values of ξ gives us the possibility of finding $n(\xi)$ for small ξ for any arbitrary value of $g^2(q)$. We express n from Eq. (5) in the following form:

$$n(\xi)=\frac{S_{\mathbf{p}}^{+}}{S_{\mathbf{p}}^{+}+S_{\mathbf{p}}^{-}+S_{\mathbf{p}}^{A}}. \tag{22}$$

In the region of small $\xi \lesssim \lambda S^-$ is small, while S^+ and S^A (having in mind that the main contribution to S^+, S^A is made by large values of $\xi' \gg \lambda$) can be represented in the form

$$S_{\mathbf{p}}^{+}\approx u_{\mathbf{p}}^2 S_{\mathbf{p}}, \quad S_{\mathbf{p}}^{A}\approx v_{\mathbf{p}}^2 S_{\mathbf{p}},$$
$$S_{\mathbf{p}}=2\pi\sum_{\mathbf{p}'} g^2(\mathbf{p}-\mathbf{p}')\,\delta(\xi'-\omega_{\mathbf{p}-\mathbf{p}'})\,n'u'^2. \tag{23}$$

Substituting (23) into (22) we obtain

$$n(\xi)\approx u_{\mathbf{p}}^2 S_{\mathbf{p}}/(u_{\mathbf{p}}^2 S_{\mathbf{p}}+v_{\mathbf{p}}^2 S_{\mathbf{p}})=u_{\mathbf{p}}^2.$$

Thus, the distribution function for the quasiparticles has the form

$$n(\xi)=\begin{cases}u_{\mathbf{p}}^2, & -\infty<\xi\le a\omega_{\mathrm{ph}} \\ 0, & a\omega_{\mathrm{ph}}<\xi\end{cases}. \tag{24}$$

It is of interest to analyse the distribution function for electrons (holes). It is described by the formula

$$f(\xi)=\langle a_{\mathbf{p}}^{+}a_{\mathbf{p}}\rangle\approx u_{\mathbf{p}}^2 n_{\mathbf{p}}+v_{\mathbf{p}}^2(1-n_{\mathbf{p}}) \tag{25}$$

and is represented in Fig. 1. From (25) and Fig. 1 it can be seen that the function $f(\xi)$ at $\xi = 0$ is equal to 1/2, i.e., in the electron distribution function a hole is "burned out" equal in height to 1/2 and in width to λ. For $\xi > \lambda$ an additional "hump" of the function arises whose width $\sim a\omega_{\mathrm{ph}}$ increases with increasing pumping current.

We obtain the dependence of the quantity a and of the field being generated on the pumping current. Substituting the function $n_{\mathbf{p}}$ from (24) into equations (7) and (5) we obtain after some calculation

$$a\approx\left(\frac{2}{\pi\eta}J\right)^{1/2}, \quad J=\frac{\pi^2\tau_{\mathrm{ph}}}{p_0^2 ms}\left(\sum_{\mathbf{p}} J_{\mathbf{p}}\right), \tag{26}$$

$$\frac{\lambda}{\omega_{\mathrm{ph}}}\approx a\eta, \quad \eta=\frac{\pi\tau_0}{2\tau_{\mathrm{ph}}}r. \tag{27}$$

These equations are valid until a reaches unity, i.e., i.e., until the pumping current exceeds a certain critical value

$$J_{\mathrm{cr}}=\pi\eta/2. \tag{28}$$

In this region the dependence of the number of photons on pumping remains linear. Indeed, substituting (26) into (27) we obtain

$$N=\tau_0\left(\sum_{\mathbf{p}} J_{\mathbf{p}}\right), \quad J\ll J_{\mathrm{cr}} \tag{29}$$

But when the pumping current approaches the critical value the dependence becomes nonlinear and for $\tilde{J}=\tilde{J}_{\mathrm{cr}}$ the field tends to a limiting value obtained from (27) if in it we set a = 1 (cf., (13)).

And what will happen for $\tilde{J} < \tilde{J}_{\mathrm{cr}}$? Since for $\tilde{J} > \tilde{J}_{\mathrm{cr}}$, $\lambda < \lambda_{\mathrm{u}}$ the field generated by the laser does not increase, an accumulation of particles occurs, so that the distribution function becomes equal to approximately unity $(1-O(\lambda/\omega_{\mathrm{ph}}))$ right up to the point $\xi = \xi_0$ at which the source acts. Because of the factor $(1-n_{\mathbf{p}})$ in the source (5) (arising as a result of taking the Pauli principle into account) the increase in the function at the point of ac-

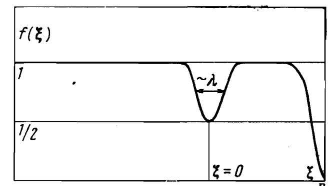

FIG. 1

tion of the source diminishes the number of particles created by the source. Therefore as pumping increases the flux of particles does not increase and the field does not grow.

It should be noted that taking two- and more phonon transitions into account leads, generally speaking, to a further increase in the field for $\tilde{J} > \tilde{J}_{cr}$. However, in view of the comparative smallness of these processes the rate of growth of the field will be significantly smaller.

In conclusion we direct attention to the following. The sharp increase in the distribution function for the particles for $\tilde{J} > \tilde{J}_{cr}$ significantly alters the amplification coefficient for the laser. This can lead to a disruption of the single-mode regime of generation considered in the present paper. As a result additional modes or a nonstationary regime of generation can arise, i.e., phenomena often observed experimentally. An analysis of these phenomena will be carried out later.

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1)In recent years it has been established experimentally [2] that in semiconductor lasers (particularly in "narrow diodes") large values of the field intensity are attained (10^5-10^7 w/cm^2) which correspond to condition (1) being satisfied.

2)Attention was drawn in [4] to the possibility of realization of the strong field regime in semiconductor lasers.

3)We have obtained a solution of the problem of generation by a semiconductor laser in the case of emission both of acoustic and of optical phonons. The results agree qualitatively. One needs only in (2) in place of ω_{ph} to substitute the frequency of the optical phonons ω_0. It should also be noted that in the low-Q regime (cf., below) the optical phonons cannot participate in generation, since the energies of the electrons (holes), concentrated in a narrow region above (below) the Fermi level (and it is specifically they that participate in generation) are insufficient to emit an optical phonon.

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