Attenuation of transverse sound in superconductors

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The temperature and frequency dependences of transverse ultrasonic attenuation in superconductors are studied. Two mechanisms are responsible for the attenuation of transverse sound: Joule losses due to the presence of alternating electromagnetic fields, and a mechanism associated with incomplete dragging of electrons by the moving lattice. It is shown that at temperatures near the transition point the electromagnetic contribution may be dominant. By studying this contribution one can direct information on the dispersion of the current response function in a superconductor.

It is well known (see, e.g.,[1]) that the attenuation of transverse sound in superconductors can differ significantly from that of longitudinal sound. Thus, in many cases the attenuation coefficient of transverse sound waves experiences a sharp drop just below the transition temperature of the superconductor, whereas further lowering of the temperature causes a more gradual decrease in attenuation, proportional to $\exp[-\Delta(T)/T]$, where T is the temperature and $\Delta(T)$ is the superconductor gap. A qualitative explanation adduced in[1] attributes this phenomenon to the fact that in the superconducting state the Meissner effect screens the electromagnetic fields. Therefore, in contrast to the normal state where the electromagnetic fields provide an important contribution to the sound attenuation, these fields are screened out on going to the superconducting state, and the sound attenuation coefficient decreases.

Even though the transverse sound attenuation has been studied in a large number of works, both theoretical and experimental, the role of electromagnetic fields in this process has not been fully elucidated. The point is that all the existing theoretical works were merely based on certain models and limiting cases. For example, in[2–5] the problem was treated in the free-electron approximation, ignoring the strain effects on the sound attenuation; these effects were taken into account in[6], but the analysis was restricted to the low-temperature case where the attenuation coefficient approaches zero exponentially. Some of these works[2, 4, 6], moreover, overlooked the sound attenuation associated with incomplete dragging of electrons by the lattice[7] which, as we shall see, is appreciable in superconductors[1)]. Other works[3, 5] which allow for this mechanism of attenuation, contain, in our view, a number of inaccuracies connected with the use of an incorrect kinetic equation.

Thus, a number of questions remain unresolved in the problem of transverse sound attenuation in superconductors. The most interesting of them, in our view, is the question of a possibility of observing the contribution of electromagnetic fields. Indeed, from the frequency dependence of the latter one would be able to directly assess the dispersion of the diamagnetic response of the superconductor. At the same time, the expression for the surface impedance, which serves as the usual source of information on the response function, contains the integral of this function over all wave vectors $\mathbf{q}$, Moreover, the expression for the surface impedance depends, in general, on the features of electron scattering by the sample surface.

The purpose of this work is to find out in which region the contribution of electromagnetic fields is significant, and when it can be observed and isolated from other mechanisms of sound attenuation.

We begin with the case of sufficiently pure semiconductors and the ultrasound of a moderately low frequency ω, so that the following conditions are satisfied:

$$\hbar\omega \ll \Delta(T), \tag{1}$$

$$q\xi_0 \ll 1, \tag{2}$$

$$\xi_0 \ll l. \tag{3}$$

Here $\mathbf{q}$ is the sound wave vector, l is the electron mean free path in the normal state, $\xi_0 = \hbar v_F/\Delta(0)$ is the coherence distance, and v_F is the electron Fermi velocity. In this case, one can use the kinetic equation for quasiparticle excitations to describe kinetic processes in the superconductor.[2)]

In the approximation linear in the strain tensor u_{ik}, a conduction electron moving in a self-consistent periodic field (with a potential $V_0(\mathbf{r})$) due to the lattice and other electrons, in the reference frame connected with the distorted lattice, is described by a Hamiltonian in the form[8–10]:

$$\mathscr{H}' = \frac{\hat{\mathbf{p}}^2}{2m_0} + V_0(\mathbf{r}) + V_{ik}'u_{ik} + V(r) - \frac{u_{ik}\hat{p}_i\hat{p}_k}{m_0} - \dot{\mathbf{u}}\hat{\mathbf{p}}. \tag{4}$$

Here $\hat{\mathbf{p}}$ is the electron momentum operator, m_0 is the free electron mass, $\mathbf{u}$ is the lattice displacement vector, and $V_{ik}'u_{ik}$ is the strain-induced variation of the periodic potential.

To determine the energy spectrum in the superconducting state one has to solve the Bogolyubov–de Gennes system of equations[11] with the Hamiltonian (4). Proceeding as in[12], one easily obtains the following expression for the energy spectrum $\tilde{\varepsilon}'(p', r')$ see also[6]):

$$\tilde{\varepsilon}'(\mathbf{p}',\mathbf{r}') = \varepsilon'(\mathbf{p}',\mathbf{r}') + \mathbf{p}_s\mathbf{v}' - m_0\mathbf{v}'\dot{\mathbf{u}}, \tag{5}$$

$$\varepsilon'(\mathbf{p}',\mathbf{r}') = \{[E_0(\mathbf{p}') + \lambda_{ik}(\mathbf{p}')u_{ik} - \mu]^2 + \Delta^2\}^{1/2}. \tag{6}$$

Here $E_0(p')$ is the unperturbed electron energy in the normal state, $\mathbf{v}'$ is the electron velocity, $p_s = -(e/c)\mathbf{A}$, where $\mathbf{A}$ is the vector potential of the electromagnetic field[3)], and λ_{ik} is the tensor of the deformation potential:

$$\lambda_{ik} = \langle n\mathbf{p}'| -\hat{p}_i\hat{p}_k/m_0 + V_{ik}'|n\mathbf{p}'\rangle, \tag{7}$$

where $|n\mathbf{p}'\rangle$ denotes the Bloch states of the conduction-band electron.

The sound attenuation coefficient and the electric current density are determined by the quasiparticle distribution function $n'(\mathbf{p}', \mathbf{r}')$, which can be calculated from the kinetic equation[12, 13]:

$$\frac{\partial n'}{\partial t}+\frac{\partial \tilde{\varepsilon}'}{\partial \mathbf{p}'}\frac{\partial n'}{\partial \mathbf{r}'}-\frac{\partial \tilde{\varepsilon}'}{\partial \mathbf{r}'}\frac{\partial n'}{\partial \mathbf{p}'}+\hat{I}(n')=0. \tag{8}$$

The explicit form of the collision operator $\hat{I}(n')$ was given in[12, 13]. Here, it should be emphasized that all the arguments of the δ-functions in the conservation laws in $\hat{I}$ depend on the energies $\tilde{\varepsilon}'$. Therefore, the Fermi function $n_0[\tilde{\varepsilon}(\mathbf{p}', \mathbf{r}')]$ makes the collision operator vanish.

The energy gap Δ is determined in terms of the quasiparticle distribution function by the self-consistency equation

$$1=-C\int_{\Delta}^{\omega_D} d\varepsilon \int d^3p\, \delta[\varepsilon-\varepsilon'(\mathbf{p}',\mathbf{r}')]\frac{1-n'_{\mathbf{p}'}-n'_{-\mathbf{p}'}}{\varepsilon'(\mathbf{p}',\mathbf{r}')} \tag{9}$$

where C is the effective interaction constant. Inasmuch as, according to (9), the equation for the energy gap contains the distribution function averaged over a surface of constant energy, no corrections to the gap width arise in the approximation linear in u_{ik} and p_s to which we have confined ourselves. Therefore, we can assume that Δ coincides with the thermodynamic equilibrium value Δ(T).

The electric current density $\mathbf{j}'$, according to[12, 13], is related to the quasiparticle distribution function by

$$\mathbf{j}'=e\sum_{\mathbf{p}'}\mathbf{v}'n'_{\mathbf{p}'}+eN_0\mathbf{v}(\mathbf{p}_s-m_0\dot{\mathbf{u}}), \tag{10}$$

where N_0 is the total electron concentration, and $\mathbf{v}(\mathbf{p}) \equiv \mathbf{v}_{\mathbf{p}} = \langle n\mathbf{p}|\hat{\mathbf{p}}/m_0|n\mathbf{p}\rangle$.

For simplicity we confine ourselves to the case of an isotropic energy spectrum. We can then set

$$\mathbf{v}_{\mathbf{p}}=\mathbf{p}/m,\quad \lambda_{ik}=\lambda_1 p^2\delta_{ik}/2m+\lambda p_i p_k/m,$$

where only the second term makes a contribution in the case of transverse sound waves.

Inasmuch as the collision operator vanishes for the function $n_0(\tilde{\varepsilon}')$, it is natural to seek the solution of (8) in the form

$$n'(\mathbf{p}', \mathbf{r}')=n_0[\tilde{\varepsilon}'(\mathbf{p}', \mathbf{r}')]+n^{(1)}.$$

If we introduce the operator

$$\hat{B}=i(\mathbf{q}\partial\tilde{\varepsilon}'/\partial\mathbf{p}'-\omega)+\hat{I},$$

then the formal solution of (8) can be written in the form

$$n^{(1)}=[\lambda(\mathbf{p}'\dot{\mathbf{u}})+e\mathbf{E}^{eff}\hat{B}^{-1}\mathbf{v}'-\lambda\hat{B}^{-1}\hat{I}(\mathbf{p}'\dot{\mathbf{u}})](-\partial n_0/\partial\varepsilon'),$$

$$\mathbf{E}^{eff}=-\frac{1}{c}\frac{\partial \mathbf{A}^{eff}}{\partial t},\quad \mathbf{A}^{eff}=\mathbf{A}+\frac{cm}{e}\left(\lambda+\frac{m_0}{m}\right)\dot{\mathbf{u}}. \tag{11}$$

Substituting (11) in (10), we obtain the following expression for the electric current density:

$$\mathbf{j}=-\frac{c}{4\pi}K(q)\mathbf{A}^{eff}+\lambda eN_0\dot{\mathbf{u}}\left(1-\frac{\sigma_{\eta\eta}{}^{I}(q)}{\sigma_0}\right); \tag{12}$$

$q \equiv (\mathbf{q}, \omega)$. Here K(q) is the diamagnetic response function of the superconductor (cf.[1]). If the conditions (1)–(3) are fulfilled, K(q) can be represented in the form

$$K(q)=\frac{1}{\delta_L^2}\frac{N_s}{N_0}+\frac{4\pi i\omega}{c^2}\sigma_{\eta\eta}(q), \tag{13}$$

where $\delta_L = (4\pi e^2N_0/mc^2)^{-1/2}$ is London's penetration depth at T = 0, N_s is the so-called concentration of superconducting electrons[14],

$$\frac{N_s(T)}{N_0}=1-\frac{1}{2}\int_{-\infty}^{\infty} dx\, \mathrm{ch}^{-2}\left[x^2+\left(\frac{\Delta(T)}{2T}\right)^2\right]^{1/2}, \tag{14}$$

$$\sigma_{\eta\eta}(q)=e^2\int d\gamma_{\mathbf{p}}(v_\eta\hat{B}^{-1}v_\eta)\left(-\frac{\partial n_0}{\partial\varepsilon}\right), \tag{15}$$

$$\sigma_{\eta\eta}{}^{I}(q)=e^2\tau\int d\gamma_{\mathbf{p}}(v_\eta\hat{B}^{-1}\hat{I}v_\eta)\left(-\frac{\partial n_0}{\partial\varepsilon}\right) \tag{16}$$

v_η is the projection of the electron velocity on the sound polarization vector, τ is the transport relaxation time in the normal state averaged over the Fermi surface, $\sigma_0 = N_0e^2\tau/m$ is the static conductivity in the normal state, and $d\gamma_{\mathbf{p}}$ is the volume element in p-space.

The first term in (12) has the form of the current response to an electromagnetic field with the vector potential $\mathbf{A}^{eff}$. The second term can be interpreted as a current associated with incomplete dragging of electrons by the moving periodic potential of the lattice. Indeed, in the case of frequent collisions with impurities moving along with the lattice, the average velocity of the electrons relative to the lattice must be equal to zero. It is evident from (16) that the second term in (12) satisfies this condition. In the collisionless case[6], electrons are not dragged along by the lattice, and the second term equals $\lambda eN_0\dot{\mathbf{u}}$.

The electric fields in the superconductor can be determined by substituting the expression for the total (electron and lattice) current in the laboratory frame into the equation

$$q^2\mathbf{E}=(4\pi i\omega/c^2)\mathbf{j},$$

which follows from the Maxwell equations. It is readily seen that the total current in the laboratory frame coincides with the electron current in the moving frame. We can hence use the expression (12) for the current. Taking this into account, we have

$$\mathbf{E}^{eff}=-\frac{m}{e}\ddot{\mathbf{u}}\left[\left(\lambda+\frac{m_0}{m}\right)+\frac{\lambda}{(q\delta_L)^2}\left(1-\frac{\sigma_{\eta\eta}{}^{I}(q)}{\sigma_0}\right)\right]\left[1+\frac{1}{q^2}K(q)\right]^{-1}. \tag{17}$$

The power absorbed by the system can be evaluated as the rate of the entropy production $T\dot{S}$. By using Stephen's expression for the entropy[15], one easily obtains the following expression for the absorbed power P:

$$P=T\dot{S}=-\frac{1}{2}\mathrm{Re}\int d\gamma_{\mathbf{p}'}\dot{\tilde{\varepsilon}}'(\mathbf{p}',\mathbf{r}')n^{*(1)}. \tag{18}$$

P can be easily evaluated by substituting (11) in (18). We shall consider the case of scattering by impurities with a short-range potential. Such a scattering mechanism is typical of metals at low temperatures. In this case the collision operator can be represented in the form $n^{(1)}|\xi_{\mathbf{p}}|/\tau\epsilon_{\mathbf{p}}$, where $\xi_{\mathbf{p}} = E_0(\mathbf{p}) - \mu$, the transport time being in this case identical to the leaving time. Under these assumptions we have

$$P=\frac{1}{2}\left\{|\mathbf{E}^{eff}|^2\,\mathrm{Re}\,\sigma_{\eta\eta}(q)+\frac{m^2\sigma_0|\dot{\mathbf{u}}|^2}{e^2\tau^2}2n_0(\Delta)[1-g(ql)]\right\}, \tag{19}$$

$$g(x)={}^3/_2[(x^{-3}+x^{-1})\,\mathrm{arctg}\,x-x^{-2}]. \tag{20}$$

As $x \to 0$ we set $g(x) = 1 - x^2/5$; as $x \to \infty$ we have $g(x) = 3\pi/4x$.

The first term in the braces in (19) describes the Joule losses connected with the presence of varying electromagnetic fields. The second term vanishes in the limit of frequent collisions. This term stems from the incomplete dragging of electrons by the lattice. Indeed, in the case of incomplete dragging, the average electron velocity prior to an event of scattering by an impurity is different from the lattice velocity. After the scattering, the electron velocity coincides on the average with that of the lattice. Therefore, the scattering of electrons by impurities moving with the lattice is inelastic and the energy transferred in such scattering contributes to the sound attenuation.

To evaluate the absorbed power, the quantity E^{eff} (Eq. (17)) determined from the Maxwell equations should be substituted in (19). By using the definition (16) it is easy to show that

$$\sigma_{\eta\eta}^{I}/\sigma_0=g(ql)N_n/N_0, \quad N_n=N_0-N_s. \tag{21}$$

As a consequence, we have

$$P=\frac{N_0 m|\mathbf{u}|^2}{2\tau}\left\{\left[\left(\lambda+\frac{m_0}{m}\right)+\frac{\lambda}{(q\delta_L)^2}\left(1-\frac{N_n}{N_0}g\right)\right]^2 \times\frac{\omega\tau\delta_L^2\,\mathrm{Im}\,K(q)}{|1+q^{-2}K(q)|^2}+2\lambda^2 n_0(\Delta)(1-g)\right\}. \tag{22}$$

When $\Delta\to 0$, Eq. (22) goes over to the expression for power absorbed in the normal metal (cf.[8]):

$$P_n=\frac{N_0 m|\dot{\mathbf{u}}|^2}{2\tau}\left\{(\omega\tau)^2 g\left[\left(\lambda+\frac{m_0}{m}\right)+\frac{\lambda}{(q\delta_L)^2}(1-g)\right]^2 \times\left|1+\frac{i\omega\tau}{(q\delta_L)^2}g\right|^{-2}+\lambda^2(1-g)\right\}. \tag{23}$$

The ratio Γ_s/Γ_n of the superconducting and normal attenuation coefficients equals P/P_n. Thus, as $\Delta\to 0$, the ratio $\Gamma_s/\Gamma_n\to 1$ and no jump in the attenuation coefficient occurs at the transition point T_c.[4)] In the low temperature region, the first term in the braces in (22) goes over as $\tau\to\infty$ to the expression derived by Belozorov and Kaner[6]. A simple estimate shows that in order that the second term in (22), associated with the effect of incomplete dragging of electrons, be small compared with the first one, the condition

$$\omega\tau w/v_F\gg 1 \tag{24}$$

must be fulfilled (w is the sound velocity). This condition furnishes the criterion for applicability of the results of[6]. We shall see that in the temperature region near T_c the contribution of the electromagnetic fields can be observed under much less rigid conditions that (24).

Our purpose is to find out when the first term in (22) prevails. To do this, we have to evaluate the quantity $\sigma_{\eta\eta}(g)$ in the expression for the imaginary part of the response function K(q). It is easy to see that the angular part of the integral in (15) is practically the same as the corresponding integral in the normal state, and is proportional to $g(ql)$. As to the integral with respect to ξ_p, it diverges logarithmically if one neglects small terms either of order w/v_F or of order $\hbar\omega/\Delta$. In the region of temperatures close to T_c the calculation yields

$$\sigma_{\eta\eta}(q)/\sigma_0=g(ql)f_q(T), \tag{25}$$

where for[5)] $\Delta\ll T$ and $\omega\ll\Delta/\hbar$ we have

$$f_q(T)=2n_0(\Delta)+\frac{1}{2}\frac{\Delta}{T}\begin{cases}\ln\left(\alpha\frac{v_F T}{w\Delta}\right) & \text{for } q\xi_0\frac{\Delta(0)}{\Delta(T)}\ll 1,\ \alpha\sim 1\\ \ln\frac{8\Delta}{e\hbar\omega} & \text{for } q\xi_0\frac{\Delta(0)}{\Delta(T)}\gg 1\end{cases} \tag{26}$$

The second of the expressions (26) cannot be derived from the kinetic equation which works in a lower order in the parameter $\hbar\omega/\Delta$. It was obtained by Cullen and Ferrell[4] from a quantum-mechanical expression for the response function.

In the case of low temperatures an expression for $f_q(T)$ can again be easily derived from the definition. At $\Delta\gg T$ the function $f_q(T)$ decreases exponentially with temperature. We shall not give here the formulae that are valid in the low-temperature region, since in this region the sound attenuation coefficient is exponentially small.

Two characteristic frequencies enter in the expressions (22) and (23), namely $\omega_1=w/\delta_L\sim 10^{10}-10^{11}$ sec^{-1} and ω_2 which is the frequency at which the second term in the denominator of (23) is equal to one. The frequency ω_2 satisfies the equation

$$\omega_2=\omega_1[\omega_2\tau g(\omega_2 l/w)]^{1/2}. \tag{27}$$

At $\omega_1\tau(v_F/w)^{1/2}\gg 1$, the frequency $\omega_2^0=\omega_1(3\pi w/4v_F)^{1/2}$ reaches approximately $5\times 10^{-2}\,\omega_1$. In the opposite limiting case we have

$$\omega_2^{(1)}=\omega_1^2\tau=\omega_2^0\left[\omega_1\tau\left(\frac{4v}{3\pi w}\right)^{1/2}\right]\ll\omega_2^0.$$

Since we require (3) to be fulfilled, which is equivalent to the condition $\tau\gtrsim 10^{-11}$ sec, the parameter $\omega_1\tau(v_F/w)^{1/2}\sim\omega_2^0 l/w$ greatly exceeds unity. Therefore, the requirement $ql\ll 1$ implies automatically the condition $\omega\ll\omega_2^0$, and in this case one can neglect the unities in the denominators of (22) and (23). Direct estimates show that, apart from a small neighborhood of T_c where $N_s/N_0\sim\omega\tau$, the prevailing mechanism in (22) is the one connected with incomplete dragging of electrons, whereas the contribution due to the electromagnetic fields is immaterial. Neglecting the latter we obtain the well-known expression[3]

$$\Gamma_s/\Gamma_n=2n_0(\Delta)g(ql), \tag{28}$$

according to which, as $ql\to 0$, the attenuation coefficient behaves in the same way as in the case of longitudinal sound. If this formula is applied to the case of finite values of ql, as is done in a number of works, it then turns out that the attenuation coefficient suffers a discontinuity on going to the superconducting state. At $ql\gtrsim 1$ this jump is of the order of the attenuation in the normal state. In reality, as we have seen, no jump in the attenuation coefficient occurs in the superconducting transition, since in the vicinity of T_c the contribution from electromagnetic fields becomes important, thus ensuring the continuity of the attenuation coefficient.

Now let us investigate the case $ql\gg 1$ which offers more favorable conditions for observing the contribution of the electromagnetic fields. At $\omega\ll\omega_2^0$ Eqs. (22) and (23), in view of (24), yield the following expression for the ratio of the attenuation coefficients in the superconducting and normal states:

$$\frac{\Gamma_s}{\Gamma_n}=\frac{1}{f_q(T)}\left\{\left[1+\left(\frac{N_s}{N_0}\frac{4v_F}{3\pi w f_q(T)}\right)^2\right]^{-2}+\frac{3\pi}{4ql}\frac{2n_0(\Delta)}{f_q(T)}\right\}. \tag{29}$$

Inasmuch as in the vicinity of T_c we have $2n_0(\Delta)\sim f_q(T)\sim 1$, and $N_s/N_0\sim 2\theta$, where $\theta=(T_c-T)/T_c\ll 1$, it is apparent that the contribution of electromagnetic fields can dominate in a certain region of values of θ. It follows from (29) that the width of this region is of the order of

$$\theta^{max}\sim w(ql)^{1/2}/v_F. \tag{30}$$

In a sufficiently pure metal this quantity can reach $(5-10)\times 10^{-2}$.

For frequencies $\omega\gtrsim\omega_1$ the screening of electromagnetic fields is insignificant. In this region there are two possibilities. If $(w/v_F)\omega\tau\ll 1$, then the dominant mechanism is the one connected with the incomplete dragging of electrons. In this case

$$\Gamma_s/\Gamma_n = 2n_0(\Delta). \quad (31)$$

On the other hand, if $(w/v_F)\omega\tau \gg 1$, the electromagnetic attenuation mechanism prevails in both the normal and the superconducting states. In this case

$$\Gamma_s/\Gamma_n = f_q(T). \quad (32)$$

Relation (32) is the ratio of the electric conductivities in the superconducting and normal states.

Thus, by working in the frequency region determined by the conditions $\omega \gtrsim \omega_1$ and $(w/v_F)\omega\tau \gg 1$, one can directly measure the imaginary part of the response function (more precisely, its ratio to the conductivity in the normal state). Let us emphasize one important point. According to (22) and (23), the electromagnetic contribution to attenuation in the indicated region is proportional to the square of $\lambda + m_0/m$. On the other hand, from the definition (7) of the tensor λ_{ik} it follows that the sum $\lambda + m_0/m$ vanishes in the free-electron limit. Therefore, the electromagnetic contribution to attenuation in the high-frequency region is entirely due to the presence of the periodic lattice potential, and vanishes in the free-electron limit.

In the intermediate frequency region $\omega_2^0 \ll \omega \ll \omega_1$ the imaginary part of the response function in (22) and the second term in the denominator of (23) are small compared to unity. Neglecting these quantities in the denominators of (22) and (23), we obtain the following expression for the electromagnetic contribution to the absorption coefficient in the temperature region defined by estimate (30):

$$\frac{\Gamma_s}{\Gamma_n} = f_q(T)\left(1 + \frac{N_s}{N_0}\frac{\omega_1^2}{\omega^2}\right)^2. \quad (33)$$

The contribution due to the incomplete electron-dragging mechanism in this region can be easily seen to be of the order of

$$\omega^{3/2}/\omega_1\omega_2\tau^{1/2} \quad (34)$$

and when this quantity is small, the contribution of electromagnetic fields can be observed.

Thus, we have considered the case of a sufficiently pure superconductor ($l \gg \xi_0$) and a moderate sound frequency ($q\xi_0 \ll 1$).

When conditions (1)–(3) are not complied with, the interaction between electrons and sound waves cannot be described by the kinetic equation and a quantum mechanical calculation is required. The part of the attenuation associated with the incomplete dragging of electrons can be easily expressed through the imaginary part of the polarization operator. The latter was calculated in[16] for superconductors with nonmagnetic impurities. As a result, it turns out that if condition (1) remains fulfilled, then, at $q\xi \gg 1$, the contribution due to the incomplete electron dragging effect is described by the same expression as in the case $q\xi_0 \ll 1$. However, it is well known (see[1]) that at $q\xi_0 \gg 1$, the expressions for both the imaginary and real parts of the response function undergo substantial modification. It is important that at $ql \gg 1$ the parameter ql does not enter in these expressions at all, and they can be evaluated in the collisionless approximation. In this case one can use (25) and the second of formulae (26) for the conductivity $\sigma_{\eta\eta}(q)$ near the transition point. Far from the transition point the following formula[17] should be used:

$$\frac{\sigma_{\eta\eta}(q)}{\sigma_0} = g(ql)\,n_0(\Delta)\,[1-n_0(\Delta)]\,\frac{2\hbar\omega}{T}\ln\frac{4T}{\gamma\hbar\omega} \quad (35)$$

($\hbar\omega \ll T \ll \Delta$; $\gamma = e^C = 1.78$). In doing so, the quantity N_S/N_0 should be replaced by the expression[17]

$$\frac{3\pi^2}{4q\xi_0}\frac{\Delta(T)}{\Delta(0)}[1-2n_0(\Delta)]. \quad (36)$$

We can thus use expression (20) for the absorbed power by substituting the quantum-mechanical expressions for both the imaginary and the real parts of the response function. It is apparent that near T_c the real part decreases by a factor of the order of $q\xi_0$, while the imaginary part remains of the same order as in the case $q\xi_0 \ll 1$. Therefore, the region where the electromagnetic contribution prevails is $q\xi_0$ times wider than that given by the estimate (30).

Thus, the frequency and temperature dependence of transverse sound attenuation can provide information on the dispersion of the superconductor response function in a rather wide frequency region.

Unfortunately, despite the large number of experimental works on attenuation of transverse sound waves in superconductors, it is difficult to check quantitatively our results against experiment. The problem is that the neighborhood of the transition point, where the predicted dependences should be important, was not specially studied in most of the experimental works. Those works, where this neighborhood was investigated, contain no data on the mean free paths, which is necessary for a quantitative correlation with the theory. The obtained dependences agree with experiment qualitatively (see[5]).

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1)Such an attenuation is due to the fact that the electron mean velocity differs from that of the lattice motion. Therefore, electron scattering by impurities that move along with the lattice results in energy transfer from the lattice to electrons.

2)To be more precise, the criterion of validity of the kinetic equation requires that the parameters $\hbar/\bar{\epsilon}\tau$ and $\hbar q v_F/\bar{\epsilon}$ be small, where τ is the typical electron relaxation time and $\bar{\epsilon}$ is the typical quasiparticle excitation energy. In most cases one can assume that $\bar{\epsilon} \sim \max[T, \Delta(T)]$ and we then arrive at the estimates (2) and (3). If, however, the integrals for the quantities we are interested in depend on small excitation energies, then the conditions (2) and (3) must be modified. We shall discuss this question later on.

3)As was shown in [13], the expressions (5) and (6) should contain the gauge invariant combinations

$$\mathbf{p}_s = \frac{\hbar}{2}\left(\nabla\chi - \frac{2e}{\hbar c}\mathbf{A}\right), \quad \lambda_{ik}u_{ik} + e\varphi - \hbar\frac{\partial\chi}{\partial t},$$

where φ is the scalar potential of the electromagnetic field and χ is the phase of the order parameter. However, in the case of transverse sound and an isotropic electron spectrum, one can always choose a gauge where $\varphi = \theta$, $\chi = \theta$, and $\operatorname{div}\mathbf{A} = \theta$.

4)The narrow neighborhood of T, where condition (1) does not hold, is not considered here.

5)The condition $q\xi_0\Delta(0)/\Delta(T) \ll 1$ appears here in place of (2) since only small values of ξ_p are important in the integral.

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