Formation of a nonlinear high-amplitude wave in the development of two-beam instability of an ion-ion plasma

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The time variation of the electric field and potential of a wave excited during the development of drift two-beam instability of a plasma consisting of positive and negative ions, and also the ion velocity distribution functions, are investigated experimentally. It is shown that the evolution of a small initial perturbation of sinusoidal shape leads to the formation of a nonlinear wave in the form of spikes of high-amplitude electric field pulses, including the capture of a certain amount of particles of both signs and the formation of captured particle clusters or "macroparticles."

In connection with the difficulties that arise in the consideration of nonlinear waves of high amplitude in plasma theory,[1-3] there is interest in the experimental investigation of the formation of such waves in a relatively simple system in which there is an unstable ion-ion plasma.[4,5]

EXPERIMENTAL METHOD

The investigations were carried out on apparatus previously described.[4] The experimental scheme is shown in Fig. 1. The ion-ion plasma consisted of beams of positive and negative ions of hydrogen, propagated along the z axis with velocities $v_1 = v_0 \pm \Delta v$ and $v_2 = v_0 \mp \Delta v$, where $2\Delta v \ll v_0$ is the adjustable value of the relative beam velocity. At the point $z = 0$, a sinusoidal perturbation of the velocity of the beams, with amplitude $\tilde{v}_0$, was produced by means of a modulator to which was supplied an alternating potential of specified frequency from a high-frequency generator. The energy of each beam was $W_0 \approx 13$ keV ($v_0 \approx 1.6 \times 10^8$ cm/sec), and the current of each beam reached 3 mA.

The spatial evolution of the initial perturbation was observed by means of a movable probe, which was the end of a high-frequency cable covered with a quartz capillary and matched to a resistor. In this case, the signal picked off the resistor is proportional to the time derivative of the plasma potential $\partial\varphi/\partial t$ and, by virtue of the relation

$$\frac{\partial\varphi}{\partial t}=\frac{\partial\varphi}{\partial z}\frac{\partial z}{\partial t}=-v_0E(z,t)$$

it is proportional to the electric field of the wave. An integrating circuit made it possible to obtain a voltage proportional to the wave potential $\varphi(z, t)$. The time dependences of both quantities were observed simultaneously on screens of oscilloscopes synchronized with the modulator.

The intensity of the electric field of the wave was also measured directly by the deflection of a thin electron beam with energy 1 keV, which penetrated the plasma at the fixed point $z = 100$ cm. Both these measurements have been briefly reported.[6] With the help of a Hughes-Rojansky analyzer placed at the end of the beams ($z = 110$ cm), the time average of the energy distribution function of the particles of each of the beams was separately measured. To eliminate the effect of pulsations of the velocity of the ion beams at the line frequency 50 Hz, the measurements were carried out over a time interval of 2 millisec, for which the

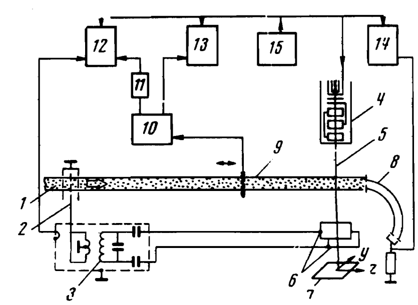

FIG. 1. Experimental setup: 1—ion-ion plasma; 2—velocity modulator; 3—high-frequency generator circuit; 4—electron beam; 5—probing beam; 6—plates for time sweep of the beam; 7—screen; 8—ion velocity analyzer; 9—movable probe; 10—broadband amplifier (up to 150 MHz); 11—integrating circuit; 12, 13, 14—oscillographs; 15—generator of intensifier pulses.

beams of the oscillographs and the electron beam were turned on by rectangular pulses of the indicated duration at the repetition rate of the pulsation frequency.

EXPERIMENTAL RESULTS AND THEIR DISCUSSION

The waves excited as a result of the development of two-beam instability in an ion-ion plasma can be observed by changing the relative velocity of the beams. This leads to a change in the increment of the instability,[4] which differs from zero only in the range $\Delta v < \Delta v_{cr}$ with a maximum at $\Delta v = \Delta v_{opt} = 0.87\,(f_{pl}/f)\,v_0$ (here f and f_{pl} are the modulation and plasma frequencies). The change in the time profile of the electric field of the wave (measured with the help of an electron beam) with change in the relative velocity of the beams is shown in Fig. 2. It is seen that in the entire range of relative velocities, with the exception of a small region near the optimal velocity, the electric field of the wave has an almost sinusoidal form, and its amplitude is small. At optimal relative velocity, a wave of large amplitude of essentially nonsinusoidal shape is excited in the plasma with unipolar spikes of the electric field, which is retarding for particles of the fast beam and, correspondingly, accelerating for the slow beam.

The formation of a nonlinear wave of the shape shown above at a given propagation length has been observed at sufficiently high amplitude of the initial modulation (Fig. 3). As is seen from the figure, the asymmetry of the field is due to the formation of closely spaced clusters of particles of opposite sign; initially, the cluster of the slow beam moves forward and then the cluster of

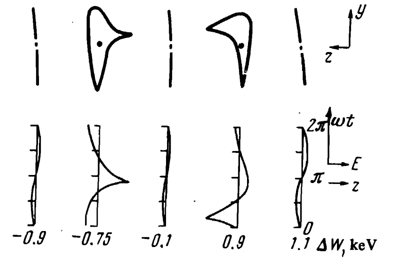

FIG. 2. Lissajou figures and the corresponding time profiles of the electric field of the wave. z = 100 cm, f_{mod} = 28 MHz, $2\Delta W = W_+ - W_-$ ($2\Delta v/v_0 = \Delta W/W_0$).

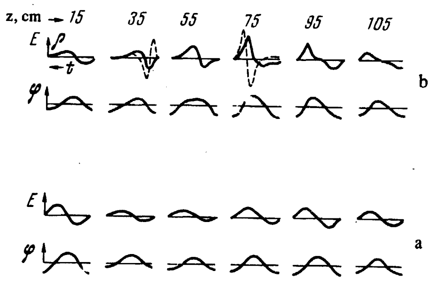

FIG. 3. Spatial evolution of a sinusoidal excitation: a) for $\widetilde{W}_0$ = 80 eV, b) for $\widetilde{W}_0$ = 640 eV (the dashed curve indicates the profile of the wave of excess density of charges $\rho = \rho_+ - \rho_-$, obtained by means of the Poisson equation $\partial E/\partial z = 4\pi\rho$); f_{mod} = 25 MHz, ΔW = −0.8 keV (the gain in case a is larger by 15 dB).

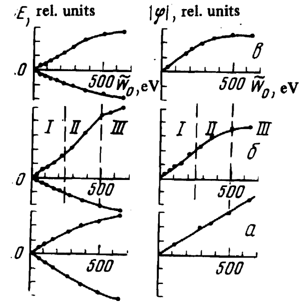

FIG. 4

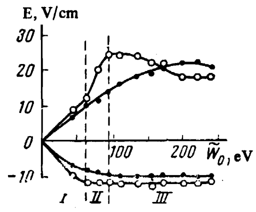

FIG. 5

FIG. 4. Dependence of the amplitude of the electric field and the potential of the wave on the amplitude of the initial modulation: a) for z = 25 cm, b) for z = 80 cm, c) for z = 110 cm; f_{mod} = 25 MHz, W = −0.8 keV.

FIG. 5. Dependence of the amplitude of the electric field (measured by beams of an electron beam) on the amplitude of the initial modulation: ○—ΔW = −0.8 keV, ●—ΔW = −0.6 keV, f_{mod} = 50 MHz, z = 100 cm.

the fast beam. It is characteristic that in the process of their interaction, the density of the clusters increases significantly and reaches a maximum value at some point which is at the phase focus for the particles. An interesting feature of the behavior of the wave up to the phase focus is that amplification along the electric field is much greater than in the case of propagation of a linear wave ($\widetilde{W}_0$ = 80 eV), i.e., there is an effect of nonlinear amplification of the field. Beyond the phase focus, both the amplitude of the wave and its asymmetry decrease.

The effect of nonlinear amplification is observed especially clearly in the dependence of the amplitude of the field of the wave on the value of the initial modulation (Figs. 4 and 5). As follows from the figures, the entire range of change of the field of the wave in the region of the phase focus (Fig. 4b and the open circles of Fig. 5) can be split into three characteristic regions, corresponding to three stages of evolution of the wave. The region I corresponds to the linear stage. In region II, because of the nonlinear amplification, the amplitude of the field increases more rapidly than in region I. As to the amplitude of the potential, its growth in region II is slowed (Fig. 4).

For an explanation of the feature of the interaction of the particles with the wave, we consider the change in the velocities of the particles in the course of their evolution. In a set of coordinates in which the wave is at rest (this system moves with the velocity $v_0 = (2W_0/M)^{1/2}$), the velocities of the particles of the beams are specified at the initial instant, and are equal to $u_0 \pm \Delta v \pm \widetilde{v}_0 \cos z'$, while the waves will change with increase in the potential according to the relation

$$^1/_2 M u_0^2 = {}^1/_2 M u^2 \pm e\varphi(z'). \tag{1}$$

This change in the velocities has been observed on the distribution functions shown in Fig. 6, where the amplitude of the initial modulation is measured along the $\widetilde{W}_0$ axis, and the dependence on it of the maximum and minimum velocities of the particles is shown by the dashed lines. As is seen, this dependence has the same three regions as the analogous dependence of the electric field. The most important feature of the propagation of the particles in region II is the appearance of particles with opposite velocities, i.e., particles captured by the electric field. The capture of these particles leads to an amplification of the grouping into clusters and is connected with this amplification of the electric field, retarding the clusters of the fast beam and accelerating the clusters of the slow beam.

The relation (1) allows us to determine the absolute value of the amplitude of the potential of the nonlinear wave by substitution in it of the measured values of the maximum or the minimum velocity of the particles. At that moment when $u_{min} = 0$ (this is the point of inflection on all the amplitude dependences) we have

$$e\varphi = e\varphi_{capt} = {}^1/_2 M u_0^2. \tag{2}$$

As is seen from Figs. 4 and 6, the potential continues to increase and after the beginning of capture (this is seen in Fig. 6 from the change in the maximum velocity) and reaches a maximum value which exceeds φ_{capt} by 20—30%.

The maximum field intensity as a function of the wavelength of the initial perturbation is shown in Fig. 7. The dashed lines indicate the dependence of the field corresponding to complete separation of the charges distributed over a half wavelength, and is equal in the one-dimensional case to

$$E = 2\pi e n \lambda. \tag{3}$$

The linear character of the experimental dependence indicates that radial swelling of the clusters, which should lead to a decrease in the field in comparison with the field of one-dimensional clusters, does not manifest itself as yet. It is seen that the degree of grouping of the particles into clusters as a result of their capture is less than one half the maximum possible.

On the basis of the aggregate of experimental data,

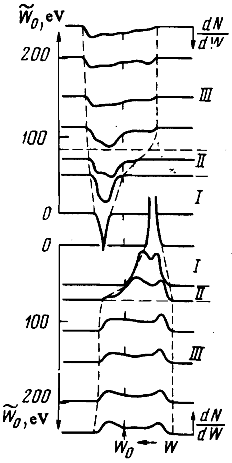

FIG. 6

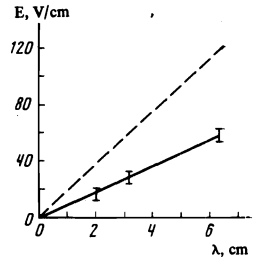

FIG. 7

FIG. 6. Change in the velocity distribution function of positive and negative ions upon increase in the amplitude of the initial modulation; $f_{mod} = 50$ MHz, $\Delta W = -0.6$ keV, $z = 110$ cm.

FIG. 7. Calculated (dashed curve) and experimental dependences of the maximum electric field on the wavelength of the excitation.

we can conclude that a nonlinear interaction of wave and particle takes place during the process of development of a two-beam instability. The capture of the ions by the wave is such an interaction. As a result of the capture of a definite part of the particles, the formation of clusters of captured particles takes place—"macroparticles"[7] of two types, with which are connected the effect of nonlinear amplification of the field, the specific profile of the wave, and the maximum values of its amplitudes. The experimental data of this research are in agreement with the results of mathematical modeling of the nonlinear interaction of beams of positive and negative ions, given in[8].

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[1] B. B. Kadomtsev and V. I. Karpman, Usp. Fiz. Nauk 103, 193 (1971) [Sov. Phys.-Uspekhi 14, 40 (1971)].
[2] G. M. Zaslavskiĭ, Usp. Fiz. Nauk 111, 395 (1973) [Sov. Phys.-Uspekhi 16, 761 (1974)].
[3] A. A. Galeev and R. Z. Sagdeev, in: Problemy teorii plazmy (Problems of Plasma Theory) (Acad. M. A. Leontovich, ed.) vol. 7, Atomizdat, 1973.
[4] M. D. Gabovich and A. P. Naida, Zh. Eksp. Teor. Fiz. 60, 965 (1971) [Sov. Phys.-JETP 33, 522 (1971)].
[5] M. D. Gabovich and A. P. Naĭda, ZhETF Pis. Red. 14, 1 (1971); Zh. Eksp. Teor. Fiz. 62, 183 (1972); [JETP Lett. 14, 3 (1971); Sov. Phys.-JETP 35, 98 (1972)].
[6] M. D. Gabovich, A. M. Gladkiĭ, V. P. Kovalenko, Yu. N. Kozyrev, and A. P. Naĭda, ZhETF Pis. Red. 18, 343 (1973) [JETP Lett. 18, 202 (1973)].
[7] B. B. Kadomtsev and O. P. Pogutse, Phys. Fluids 14, 2474 (1971).
[8] V. S. Imshennik, O. V. Lokutsievskiĭ, L. G. Khazin, M. D. Gabovich, and A. P. Naĭda, Preprint No. 41, Inst. of the Problems of Mechanics, Acad. Sci. USSR, Moscow, 1974.

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