Solitary charge density waves in a magnetoactive plasma

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We show that a charge density wave propagating at an angle to the magnetic field can have the shape of an isolated bunch of electrons (charge density soliton). We study, both theoretically and experimentally, the dispersion characteristics of this soliton, the adiabatic damping of the soliton due to its interaction with resonance particles, and its amplification by an electron beam.

INTRODUCTION

A periodic charge density wave (periodic bunching and rarefactions of the electron density) in a plasma in a magnetic field has a phase velocity v_{ph} in the direction of the magnetic field which is less than the critical velocity

$$v_{ph,cr}=\frac{\omega_{p0}}{k_\perp(1+\omega_{p0}^2/\omega_H^2)^{1/2}}$$

(ω_H is the cyclotron frequency, ω_{p0} the plasma frequency corresponding to the equilibrium density, and $k_\perp$ the transverse component of the wave vector). When $v_{ph} > v_{ph,cr}$ the charge density wave has the form of an isolated bunch of electrons (solitary wave or soliton). At each given time such a bunch is formed because of the thermal electrons in the plasma which are in the region where the electrical field is a maximum, and constitutes a hump in the potential energy, from which the resonance electrons with velocities close to the phase velocity of the wave can be reflected.

Under well-defined conditions the interaction with such resonance electrons leads to an adiabatic change in the amplitude of the solitary wave (damping or amplification). Amplification occurs when the resonance particles form a beam moving with a velocity $\bar{v} > v_{ph}$. As the phase velocity of the soliton increases with increasing amplitude, such an amplification is accompanied by an acceleration of the soliton. This fact makes it possible to use the interaction of a solitary wave with an electron beam to accelerate ions trapped in the potential well produced by the wave.[1)]

In the present paper we study, theoretically and experimentally, the propagation of solitary waves in a magneto-active plasma and their amplification when they interact with electron beams.

STATIONARY CHARGE DENSITY SOLITON

It was shown in[2,3] that in the limiting case of a strong magnetic field ($\omega_H \gg \omega_{pe}$) a charge density soliton does exist. We consider the case of finite magnetic field strengths $\omega_H \lesssim \omega_{p0}$. In that case we can write the non-linear set of equations which describe the two-dimensional stationary charge density wave $\varphi(x,z-v_{ph}t)$ in the following form:

$$(v_z-v_{ph})\frac{\partial v_z}{\partial\zeta}+v_x\frac{\partial v_z}{\partial x}=\frac{e}{m}\frac{\partial\varphi}{\partial\zeta}, \tag{1}$$

$$(v_z-v_{ph})\frac{\partial v_x}{\partial\zeta}+v_x\frac{\partial v_x}{\partial x}=\frac{e}{m}\frac{\partial\varphi}{\partial x}-\omega_H v_y, \tag{2}$$

$$(v_z-v_{ph})\frac{\partial v_y}{\partial\zeta}+v_x\frac{\partial v_y}{\partial x}=\omega_H v_x, \tag{3}$$

$$\frac{\partial}{\partial\zeta}n(v_z-v_{ph})+\frac{\partial}{\partial x}nv_x=0, \tag{4}$$

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial\zeta^2}=4\pi e(n-n_0),\quad \zeta=z-v_{ph}t. \tag{5}$$

We restrict our considerations to a small amplitude soliton $e\varphi/mv_{ph}^2 \ll 1$, for which $|\varphi^{-1}\partial\varphi/\partial z| \ll k_\perp$ and hence $|\varphi^{-1}\partial\varphi/\partial t| \ll \omega_H$. The equations of motion (1) to (3) for the electrons can then easily be integrated in the drift approximation:

$$v_y=\frac{c}{H_0}\frac{\partial\varphi}{\partial x},\quad v_x=\frac{c}{H_0}\frac{v_z-v_{ph}}{\omega_H}\frac{\partial^2\varphi}{\partial x\partial\zeta},$$

$$v_z=-\frac{e\varphi}{mv_{ph}}+\frac{e^2\varphi^2}{2m^2v_{ph}^3}+\frac{e^2}{2m^2\omega_H^2v_{ph}}\left(\frac{\partial\varphi}{\partial x}\right)^2. \tag{6}$$

Integrating the continuity equation (4) over ζ we have for the electron density n the formula

$$n=-\frac{n_0v_{ph}}{v_z-v_{ph}}\left(1-\frac{c}{H_0}\frac{1}{\omega_H}\frac{\partial^2\varphi}{\partial x^2}\right), \tag{7}$$

which we can use to write finally the equation for the potential of the wave $\varphi(x,\zeta)$ in the following form:

$$\frac{\partial^2\varphi}{\partial x^2}+k_\perp^2\varphi=\frac{1}{1+\omega_{p0}^2/\omega_H^2}\left\{-\frac{\partial^2\varphi}{\partial\zeta^2}+\frac{\omega_{p0}^2}{v_{ph}^2}\delta\varphi+\frac{3}{2}\frac{\omega_{p0}^2}{v_{ph}^2}\frac{e\varphi^2}{mv_{ph}^2}+\frac{\omega_{p0}^2}{2\omega_H^2}\frac{e}{mv_{ph}^2}\left(\frac{\partial\varphi}{\partial x}\right)^2+\frac{\omega_{p0}^2}{\omega_H^2}\frac{e\varphi}{mv_{ph}^2}\frac{\partial^2\varphi}{\partial x^2}\right\}. \tag{8}$$

To get that equation we used the fact that for a small amplitude soliton

$$v_{ph}^2=\frac{\omega_{p0}^2}{k_\perp^2}\frac{1+\delta}{1+\omega_{p0}^2/\omega_H^2},\quad 0<\delta\ll1.$$

We transferred all small quantities of order δ to the right-hand side of Eq. (8). We look for its solution in the form of an expansion in the parameter δ: $\varphi=\varphi^{(0)}+\varphi^{(1)}+\ldots$. We have in zeroth approximation

$$\varphi^{(0)}=\psi(\zeta)\cos k_\perp x. \tag{9}$$

To simplify matters we chose an even solution $\varphi(-x) = \varphi(x)$. The $k_\perp$-spectrum is determined from the boundary conditions in x. For example, if the wave propagates in a rectangular waveguide with conducting walls, we have $k_\perp a = 1/2\pi(2s+1)$, s = 0, 1, ...

We can, as usual, get the equation for $\psi(\zeta)$ from the condition that the right-hand side of (8) is orthogonal to $\varphi^{(0)}(x)$:

$$\frac{d^2\psi}{d\zeta^2}-k_\perp^2\psi\delta\left(1+\frac{\omega_{p0}^2}{\omega_H^2}\right)-2\frac{k_\perp}{a}(-1)^s\frac{e\psi^2}{mv_{ph}}\left(1+\frac{\omega_{p0}^2}{2\omega_H^2}\right)=0. \tag{10}$$

This equation has a solution in the form of a soliton:

$$\psi(\zeta)=(-1)^{s+1}\frac{mv_{ph}^2}{e}\alpha\,\mathrm{ch}^{-2}\frac{\zeta}{\Delta},\quad 0<\alpha\ll1. \tag{11}$$

On the waveguide axis x = 0 the electron density is a maximum ($\varphi(x = 0) < 0$) for harmonics with even s and a minimum ($\varphi(x = 0) > 0$) for odd s. The width of the soliton is

$$\Delta=2k_\perp/\sqrt{\delta}, \tag{12}$$

and we have for the connection between the soliton phase velocity and its amplitude the equation[2)]

$$v_{\rm ph}=\frac{\omega_{p0}^2}{k_\perp^2(1+\omega_{p0}^2/\omega_H^2)} \times\left(1+\frac{4}{3k_\perp a}\alpha\frac{1+\omega_{p0}^2/2\omega_H^2}{1+\omega_{p0}^2/\omega_H^2}\right). \tag{13}$$

All we have said so far referred to a soliton with a sufficiently small amplitude for which the Mach number

$$M=\frac{k_\perp v_{\rm ph}}{\omega_{p0}}\left(1+\frac{\omega_{p0}^2}{\omega_H^2}\right)^{1/2}=\frac{v_{\rm ph}}{v_{\rm ph}}\approx 1.$$

One can consider the two-dimensional charge density soliton for arbitrary Mach numbers by means of numerical methods. We show in Fig. 1 the dispersion relation for a solitary wave for arbitrary M in the limiting case as $\omega_{p0}/\omega_H \to 0$ which was obtained in this way. The maximum amplitudes $|\varphi|^{\max} = mv_{\rm ph}^2/2e$, corresponding to an intersection of the electron trajectories at the crest of the wave, are reached when $M \approx 1.3$. See[5] for detailed results of calculations for a two-dimensional soliton for arbitrary Mach numbers.

ADIABATIC DAMPING OF A SOLITON

For the sake of simplicity we restrict ourselves in the discussion of the damping to the first harmonic s = 0 in the case of a strong magnetic field $\omega_H \gg \omega_{p0}$. The damping is caused by the interaction with the resonance particles with velocities close to the phase velocity of the soliton. If its amplitude is small $e|\psi|/mv_{\rm ph}^2 \ll 1$ the number of resonance particles is also small and the damping is adiabatic, i.e., it takes place while the soliton retains its shape (11). The quantities Δ and α are then slowly varying functions of the time which are connected with one another and with the phase velocity $v_{\rm ph}$ through Eqs. (12) and (13).

A charge density soliton is a "hump" in the electrical field for the electrons and we can hence separate two groups of resonance particles. Particles with a sufficiently high energy in the frame of the wave $\mathscr{E} > -e\varphi_0$ pass through the hump and are untrapped (trajectories I^a, I^b in Fig. 2). Such particles move successively in the accelerating and in the decelerating phase of the field and over times long compared to the time of flight through the soliton, $t \gg \Delta/v_{\rm ph}$, do not contribute to the damping. The dissipation of the soliton energy is basically connected with particles reflected from the potential hump, $\mathscr{E} < -e\varphi_0$.[6] On trajectories of the type II^a such particles move at all times in the

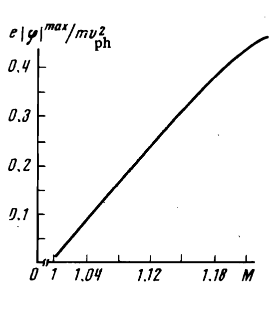
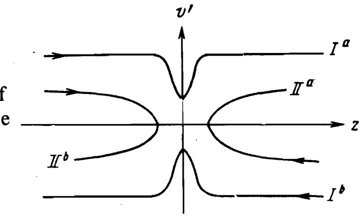


FIG. 1. Dispersion relation (amplitude as function of Mach number) for a two-dimensional solitary charge density wave.

FIG. 2. Phase trajectories of resonance electrons in the frame of the wave.



accelerating phase of the field, but on trajectories of the type II^b in the decelerating phase. If $\partial f_0/\partial v_{\rm ph} < 0$ ($f_0(v)$ is the equilibrium plasma distribution function), there are more particles in the accelerating phase and the soliton is damped. It is important that there is no nonlinear stabilization of the damping caused by phase mixing[7, 8] for the reflected particles and the amplitude of the soliton is damped to become zero due to the interaction with such particles. When $\partial f_0/\partial v_{\rm ph} > 0$, i.e., when there is a beam in the plasma, the interaction with the reflected particles leads to amplification of the soliton. According to (13) the soliton amplitude increases to a magnitude $e|\psi|_{\max} \sim mv_{\rm ph}(\bar{v} - v_{\rm ph})$ when the soliton velocity becomes comparable with the beam velocity.

We consider first of all the damping of a solitary charge density wave in a Maxwellian plasma. The damping leads to the wave potential φ becoming a slowly changing function of the time $\varphi = \varphi(t,x,\zeta)$. Taking this into account we get the equations

$$v_z=-\frac{e\varphi}{mv_{\rm ph}}+\frac{e^2\varphi^2}{2m^2v_{\rm ph}^3}+\frac{e}{mv_{\rm ph}^2}\int_\zeta^\infty\frac{\partial\varphi}{\partial t}d\zeta,$$

$$n=\frac{n_0v_{\rm ph}}{v_{\rm ph}-v_z}+n_0\frac{e}{mv_{\rm ph}^3}\int_\zeta^\infty\frac{\partial\varphi}{\partial t}d\zeta, \tag{14}$$

for the velocity and the density of the thermal electrons in the limiting case as $\omega_{p0}/\omega_H \to 0$; using these we get, instead of (10) for $\psi(t,\zeta)$ the following equation where

$$\frac{\partial^2\psi}{\partial\zeta^2}-k_\perp^2\delta\psi-2\frac{k_\perp}{a}\frac{e\psi^2}{mv_{\rm ph}^2}=2\frac{\omega_{p0}^2}{v_{\rm ph}^3}\int_\zeta^\infty\frac{\partial\psi}{\partial t}d\zeta+\frac{4\pi e}{a}\int_{-a}^{a}n_{res}\cos k_\perp x\,dx, \tag{15}$$

$n_{\rm res}(t,x,\zeta)$ is the density of the resonance electrons with $v \approx v_{\rm ph}$.

The condition that the damping of the soliton is adiabatic is $\epsilon = \gamma\Delta/v_{\rm ph}\delta \ll 1$ (γ is the damping rate). When this condition is satisfied the terms transferred to the right-hand side of Eq. (15) are small and the solution of this equation can be looked for in the form of an expansion in the parameter ϵ. In the zeroth approximation we get Eq. (10) for ψ (for $\omega_{p0}/\omega_H \to 0$, s = 0) with the solution $\psi^{(0)}(t,\zeta)$ determined by Eq. (11), where α and Δ are slowly varying functions of the time. In the next approximation

$$\frac{\partial^2\psi^{(1)}}{\partial\zeta^2}-k_\perp^2\delta\psi^{(1)}-\frac{4k_\perp}{a}\frac{e\psi^{(0)}\psi^{(1)}}{mv_{\rm ph}^2}=\frac{4\pi e}{a}\int_{-a}^{a}n_{res}\cos k_\perp x\,dx+2\frac{\omega_{p0}^2}{v_{\rm ph}^3}\int_\zeta^\infty\frac{\partial\psi^{(0)}}{\partial t}d\zeta. \tag{16}$$

From the condition that the zeroth and first approximations are compatible it follows that the right-hand side of (16) must be orthogonal to $\partial\psi^{(0)}/\partial\zeta$:

$$\int_{-\infty}^{\infty}d\zeta\frac{\partial\psi^{(0)}}{\partial\zeta}\int_\zeta^\infty d\zeta\frac{\partial\psi^{(0)}}{\partial t}=-\frac{2\pi ev_{\rm ph}^3}{\omega_{p0}^2a}\int_{-\infty}^{\infty}d\zeta\frac{\partial\psi^{(0)}}{\partial\zeta}\int_{-a}^{a}dx\cos k_\perp x\,n_{res}. \tag{17}$$

This is the required equation determining the change in the soliton parameters as a result of the interaction with resonance particles. Integrating by parts, we can write this equation in the form of the energy conservation law in the soliton-resonance particles system:

$$\frac{1}{4\pi}\frac{d}{dt}\int_{-a}^{a}dx\int_{-\infty}^{\infty}d\zeta k_\perp^2\varphi^2=-ev_\Phi\int_{-a}^{a}dx\int_{-\infty}^{\infty}d\zeta\frac{\partial\varphi}{\partial\zeta}n_{res}=-\int_{-a}^{a}dx\int_{-\infty}^{\infty}d\zeta j_z^{res}E_z. \tag{18}$$

Perturbation theory methods used to solve Eq. (15) are, strictly speaking, inapplicable in the region behind the soliton ($\zeta\to-\infty$), where

$$\varphi\to0,\quad v_z\neq0,\quad n\sim\int_{-\infty}^{\infty}\frac{\partial\varphi}{\partial t}d\zeta\neq0.$$

This last fact is connected with the adiabatic perturbation of the velocity and the density of the thermal electrons passing through the soliton, the parameters of which change with time. Such a perturbation leads to the appearance of a "tail" for a soliton with an amplitude of the potential $\varphi\sim\gamma$. One can show that the contribution from this region to the soliton energy and, hence, to Eq. (18) for times $t\sim1/\gamma$ is also proportional to γ, i.e., negligibly small.

We now turn to a study of Eq. (17). We substitute into it the density of the resonance particles in the form

$$n_{res}=\int f_{res}dv.$$

We transform the right-hand side of (17) using the Liouville theorem about the conservation of phase volume: $d\zeta dv=d\zeta_0 dv_0$ (ζ_0, v_0 are the original coordinates of particles in phase space which pass at time t through the point ζ, v) and the condition that the distribution function along the trajectories of the resonance particles must be constant $f_{res}(t,\zeta,v)=f_0(v_0'+v_{ph})$; f_0 is the equilibrium distribution function; we can, as usual, neglect its initial perturbation (see, e.g.,[8]). As $v_0'\ll v_T^2/v_{ph}$ for resonance particles, we can expand the distribution function $f_0(v_0'+v_{ph})$ in a power series in v_0', and only the odd terms in the expansion will be important by virtue of the symmetry property $\zeta(-\zeta_0,-v_0,t)=-\zeta(\zeta_0,v_0,t)$. Finally, substituting $\psi^{(0)}$ and $\partial\psi^{(0)}/\partial\zeta$ from (11) and integrating over ζ in the left-hand side of (17) we are finally led to the following equation for the soliton amplitude:

$$\frac{d\alpha}{dt}=-4\left(\frac{2}{3\pi^3}\right)^{1/2}\frac{\omega_{p0}}{n_0}\frac{\partial f_0}{\partial v_{ph}}\alpha^{1/2}\int_0^{\pi/2}d\eta\cos\eta\int_0^{\infty}d\xi_0\int dv_0'v_0'\frac{\operatorname{sh}\xi(t,\eta,\xi_0,v_0')}{\operatorname{ch}^3\xi(t,\eta,\xi_0,v_0')}, \tag{19}$$

where we have used the notation $\eta=k_\perp x$, $\xi=\zeta/\Delta$.

When the condition for adiabaticity is satisfied the change in the soliton amplitude during the time of flight of the resonance particles is negligibly small and we can find the trajectories of these particles $\xi(t,\xi_0,v_0')$ from the energy integral:

$$\frac{m}{2}\left(\frac{d\zeta}{dt}\right)^2=\mathscr{E}-\frac{w}{\operatorname{ch}^2\zeta/\Delta},\quad w=\alpha m v_{ph}^2\cos\eta. \tag{20}$$

Integrating over t we get equations for the trajectories of the resonance particles in the field of the soliton:

$$u=u_0\pm(2\mathscr{E}/m)^{1/2}t/\Delta; \tag{21}$$

the $\pm$ sign refers to particles with $v_0'>0$ and $v_0'<0$. For untrapped particles with energies $\mathscr{E}>w$ the quantity u is connected with ζ through the equation

$$\operatorname{sh}(\zeta/\Delta)=\mu\operatorname{sh}u,\quad \mu^2=(\mathscr{E}-w)/\mathscr{E}. \tag{22}$$

The region where these particles can move is infinite in ζ. For reflected particles with energies $\mathscr{E}<w$, the region of ζ where the particles can move is limited by the reflection condition $\sinh^2(\zeta/\Delta)>(w-\mathscr{E})/\mathscr{E}$ and the connection between the quantities u and ζ has the form

$$\operatorname{sh}(\zeta/\Delta)=(-\mu^2)^{1/2}\operatorname{ch}u. \tag{23}$$

Once we have the equations for the trajectories of the resonance particles, we can easily find from (19) the contribution from these particles to the soliton damping. The contribution of the reflected particles is determined by the equation

$$\frac{d^{(R)}\alpha}{dt}=4\left(\frac{2}{3\pi^3}\right)^{1/2}\frac{\omega_{p0}}{mn_0}\frac{\partial f_0}{\partial v_{ph}}\alpha^{1/2}\int_0^{\pi/2}d\eta\cos\eta\int_0 d\mathscr{E}\mu^2\sum_{j=\pm1}j\int_0 du_0\operatorname{ch}\left[u_0+j\left(\frac{2\mathscr{E}}{m}\right)^{1/2}\frac{t}{\Delta}\right]\left\{1-\mu^2\operatorname{ch}^2\left[u_0+j\left(\frac{2\mathscr{E}}{m}\right)^{1/2}\frac{t}{\Delta}\right]\right\}^{-3/2}\frac{\operatorname{sh}u_0}{(1-\mu^2\operatorname{ch}^2u_0)^{1/2}}. \tag{24}$$

For times t large compared to the flight time of the resonance particles through the soliton, $t\gg\Delta(2w/m)^{-1/2}$, only particles with $v_0'<0$ ($j=-1$) are important. One can easily evaluate in that limiting case the integrals on the right-hand side of Eq. (24). The result has the following form:

$$\frac{d^{(R)}\alpha}{dt}=\left(\frac{2}{3\pi}\right)^{1/2}\frac{\omega_{p0}}{n_0}v_{ph}^2\frac{\partial f_0}{\partial v_{ph}}\alpha^{3/2}. \tag{25}$$

We determine similarly the contribution from the untrapped particles to the damping

$$\frac{d^{(UR)}\alpha}{dt}=-4\left(\frac{2}{3\pi^3}\right)^{1/2}\frac{\omega_{p0}}{mn_0}\frac{\partial f_0}{\partial v_{ph}}\alpha^{1/2}\int_0^{\pi/2}d\eta\cos\eta\int_w^{\infty}d\mathscr{E}\mu^2\sum_{j=\pm1}j\int_0^{\infty}du_0\operatorname{sh}\left[u_0+j\left(\frac{2\mathscr{E}}{m}\right)^{1/2}\frac{t}{\Delta}\right]\left\{1+\mu^2\operatorname{sh}^2\left[u_0+j\left(\frac{2\mathscr{E}}{m}\right)^{1/2}\frac{t}{\Delta}\right]\right\}^{-3/2}\frac{\operatorname{ch}u_0}{(1+\mu^2\operatorname{sh}^2u_0)^{1/2}}. \tag{26}$$

It follows from this equation that when $t\gg\Delta(w/m)^{-1/2}$ the contribution from the untrapped particles to the damping is exponentially small (of the order of $\exp\{-2(2w/m)^{1/2}t/\Delta\}$) and therefore the solution of Eq. (25)

$$\alpha(t)=\alpha(0)\left[1+\left(\frac{\alpha(0)}{6\pi}\right)^{1/2}\gamma_L t\right]^{-2},\qquad \gamma_L=-\frac{\omega_{p0}}{n_0}v_{ph}^2\frac{\partial f_0}{\partial v_{ph}} \tag{27}$$

determines for $t\gg\Delta(w/m)^{-1/2}$ the adiabatic damping of the charge density soliton which is due to the interaction with the resonance particles.

AMPLIFICATION OF THE SOLITON BY ELECTRON BEAM

When the adiabaticity condition $v_{ph}\alpha/\Delta\gg\gamma$ is satisfied the growth rate γ of the amplitude of soliton interacting with an electron beam can also be found from Eq. (19). Assuming that the beam velocity $\bar{v}$ is close to the phase velocity of the wave, we substitute into that equation the equilibrium distribution function of the resonance particles in the form

$$f_0=\frac{n_1}{\Delta v}\frac{1}{\pi^{1/2}}\left[1-\frac{(v-\bar{v})^2}{(\Delta v)^2}\right],$$

where n_1 is the beam density and Δv the spread in velocities in the beam. One must in (19) also take into account the dependence of the phase velocity of the solitary wave on its amplitude, given by Eq. (13), as the increase in the wave amplitude is limited by the fact that for sufficiently large amplitudes the wave velocity exceeds $\bar{v}$ and the sign of the derivative $\partial f_0/\partial v_{ph}$ is changed (amplification is changed to damping). As a result we get the following equation for the wave amplitude:

$$d\alpha/d\tau=\alpha^{3/2}(\nu-\alpha), \tag{28}$$

where we have used the dimensionless variables

$$\tau=\frac{8\sqrt{2}}{\sqrt{27}\,\pi^2}\,\omega_{p0}t\,\frac{n_1}{n_0}\,\frac{\bar{v}^3}{(\Delta v)^3}\equiv\beta t,$$

$$\alpha=\frac{e|\psi|_{max}}{mv_{ph}^2},\qquad \nu=\frac{3\pi}{4}\,\frac{\bar{v}-v_{ph}(t=0)}{\bar{v}}.$$

The solution of this equation

$$\frac{1}{\sqrt{\alpha_0}}-\frac{1}{\sqrt{\alpha}}+\frac{1}{2\sqrt{\nu}}\ln\left[\frac{\sqrt{\nu}-\sqrt{\alpha_0}}{\sqrt{\nu}-\sqrt{\alpha}}\,\frac{\sqrt{\nu}+\sqrt{\alpha}}{\sqrt{\nu}+\sqrt{\alpha_0}}\right]=2\nu\tau \tag{29}$$

describes the increase in the soliton amplitude from an initial value α_0 to its maximum $\alpha_{max}=\nu$.

Substituting the growth-rate γ from (28), $\gamma\sim\beta\alpha^{3/2}$, into the adiabaticity condition $v_{ph}\alpha/\Delta\gg\gamma$, we see that the condition is satisfied for beams with sufficiently large thermal spreads:

$$n_1\bar{v}^3/n_0(\Delta v)^3\ll 1.$$

The maximum value of the amplitude of a solitary wave, determined by Eq. (29),

$$\frac{e|\psi|_{max}}{mv_{ph}^2}=\frac{3\pi}{4}\,\frac{\bar{v}-v_{ph}(t=0)}{\bar{v}}, \tag{30}$$

is larger by a factor $k^2\bar{v}\Delta v/\gamma^2$ than the maximum amplitude of a periodic wave excited by an electron beam (see[9]). Such an increase is connected with the fact that there is in the case of a soliton no non-linear mechanism for the stabilization of the instability caused by the phase "mixing" of the resonance particles. The maximum energy density in the soliton, corresponding to (30), also turns out to be very considerable:

$$E_{max}^2/4\pi={}^9/_{16}\pi^2 n_0 m(\bar{v}-v_{ph}(t=0))^2 \tag{31}$$

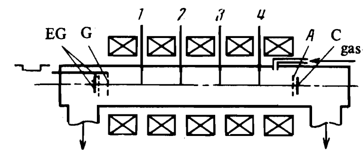
and by virtue of the spatial localization of the field in such wave this can also exceed the energy density in the beam by even a large amount.

EXPERIMENTAL RESULTS

1. Description of the experiment

The apparatus which we used for our studies (Fig. 3) is a cylindrical plasma waveguide placed in a constant, uniform longitudinal magnetic field. The primary electrons, emitted by the heated cathode C with an energy of 50 eV, ionize the neutral gas. We used a tungsten grid A electrically joined to the vacuum chamber. We normally used argon at a pressure of 1 to 5 × 10^{-4} mm Hg as the working gas. For those pressures we can neglect the collisions between the electrons and the neutral particles. The plasma density was measured by Langmuir probes and varied within the limits $n_0 = 10^7$ to 10^8 cm^{-3}. The electron temperature was $T_e \lesssim 3$ eV. The transverse dimensions of the plasma column were 3 cm, the diameter of the copper tube surrounding the plasma column was 7.5 cm. The length of the uniform part of the magnetic field was 50 cm. The diverging magnetic field at the ends of the apparatus lowered

FIG. 3. Experimental setup: C—cathode, A—anode, G—exciting grid, 1 to 4—probes, EG—electron gun.



significantly the reflection of waves by the ends.

Solitary waves were excited by means of a grid of diameter 5 cm. The grid was prepared from tungsten wire of diameter 0.12 mm. The dimensions of the mesh were 2 × 2 mm which was smaller than or of the order of the Debye length under the conditions of the experiment. The grid was coupled to a generator of square pulses of negative polarity (rise time 10 ns) by means of high-frequency vacuum leads. The directions of propagation of the waves and of the drift in the plasma were in opposite directions while the drift velocity $v_{dr}\ll v_{ph}$ so that the presence of drift did not lead to any significant changes in the dispersion characteristics of the plasma waves. A study of these characteristics for small amplitudes of the excited signal showed that the first radial harmonic of the Langmuir oscillations propagated in the system. When the critical value $v_{ph,cr}$ of the phase velocity is exceeded there appears a solitary charge density wave in the system. We studied the dynamics of such a wave using high-frequency capacitive probes the signals of which were registered through an integrating element and a wide-band amplifier by an oscillograph. For this we used matched capacitive probes placed at distances of 10 cm from one another along the apparatus and oriented toward the transverse component of the high-frequency electrical field.

For the study of the amplification of the solitary waves by electron beams we placed in the apparatus a two-electrode gun EG at a distance of a few cm from the exciting grid G. We used a tungsten grid as anode in this gun, electrically joined to the vacuum chamber. As cathode we used a lanthanum boride disk of diameter 0.5 cm. The electron gun produced an electron beam (current up to 100 μA, energy up to 100 eV) in which the particle density n_1 was usually less than in the plasma so that the presence of such a beam did not lead to a noticeable change in the dispersion properties of the plasma waveguide.

2. Charge density soliton in a finite magnetic field

When we supply to the exciting grid a negative potential step of magnitude more than 5 V a solitary pulse is excited in the system of negative polarity, the amplitude and width of which depend on the amplitude of the pulse on the grid and on the magnetic field strength. We changed the magnetic field strength within the range H = 10 to 1000 Oe so that during the experiment there could be no transition from the finite field regime ($\omega_H/\omega_{p0}\sim 1$) to the infinitely strong field regime ($\omega_H/\omega_{p0}\gg 1$).

Our experiments confirmed satisfactorily the theoretical prediction that in an unmagnetized plasma $\omega_H\sim\omega_{p0}$ there can propagate two-dimensional solitary charge density waves $f(r,z-v_{ph}t)$ provided the phase velocity of the wave satisfies the condition

$$v_{ph}>v_{ph,cr}=\omega_{p0}/k_\perp(1+\omega_{p0}^2/\omega_H^2)^{1/2}.$$

We give in Fig. 4 the experimental dependence of the propagation velocity of the solitary pulse v_{ph} on the

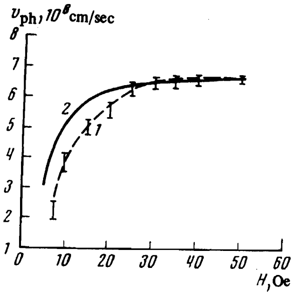


FIG. 4. The phase velocity of the solitary wave as function of the magnetic field strength for constant amplitude of the exciting signal: 1—experiment, 2—theory.

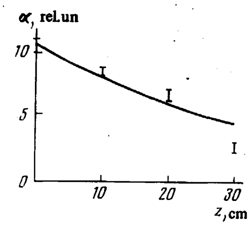


FIG. 6. Damping of the amplitude of a solitary wave in the plasma; z—distance from the exciting grid.

magnetic field strength, plotted for a constant amplitude of the exciting signal (curve 1) and also the theoretical curve $v_{ph}(H)$ normalized to the experimental value of v_{ph} as $H \to \infty$ (curve 2). The theoretical function was calculated using the dispersion characteristics of a charge density soliton which is analogous to (13). Equation (13) is changed as follows for a cylindrical soliton such as is excited in the experiment:

$$v_{ph}^2=\frac{\omega_{p0}^2}{k_\perp^2(1+\omega_{p0}^2/\omega_H^2)}\left[1-\frac{3}{2}I\frac{e\psi}{mv_{ph}^2}\frac{1+\omega_{p0}^2/2\omega_H^2}{1+\omega_{p0}^2/\omega_H^2}\right],$$

$$I=\int_0^a J_0^3(k_\perp r)r\,dr\Big/\int_0^a J_0^2(k_\perp r)r\,dr=1.2,\quad J_0(k_\perp a)=0. \tag{32}$$

The small difference between the theoretical and the experimental curve in the region of small phase velocities is connected with the damping of the wave which was not taken into account when deriving the theoretical Eqs. (13) and (32).

3. Damping and amplification of a charge density soliton in the infinite magnetic field case

We give here the results of the experimental studies of a solitary wave for large magnetic field strengths $H > 200$ Oe, corresponding to $\omega_H/\omega_{p0} > 10$. In that case the dispersion characteristics of a solitary wave (the dependence of the soliton phase velocity and width on its amplitude) have been studied before.[2] Our aim was to study the resonance damping of the wave in an equilibrium plasma and the possibility that it would be amplified when there is an electron beam present. We showed experimentally that for sufficiently small amplitudes the damping and amplification of the soliton is adiabatic so that while the amplitude changes the dispersion relation (32) is all the time approximately satisfied.

We show in Fig. 5 a series of oscillograms for the case of the damping of a soliton (oscillograms a) and when it is amplified by an electron beam (oscillograms b to e). In the left-hand series of oscillograms we show the soliton registered by the probe situated 5 cm from the exciting grid. On the right-hand series we give the oscillograms of the soliton after it has passed a distance of 20 cm along the system. The electron beam velocity $\bar{v}$ is a parameter for the oscillograms b to e. The preparation of such kinds of oscillograms enables us to determine the spatial distribution of the soliton amplitude. We show in Fig. 6 the results of measurements, for the case where the soliton is damped, and we give for a comparison also the theoretical behavior (solid curve) obtained from Eq. (27) (it is here necessary to replace in Eq. (27) t by z/v_{ph}).

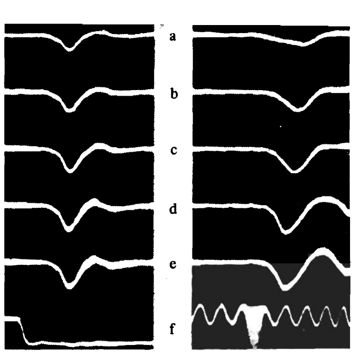


FIG. 5. Oscillograms of a charge density soliton plotted at different distances from the exciting grid: a—when there is no beam, b to e—for different values of the beam velocity, e—form of the pulse produced at the grid and time marks.

The presence of an electron beam in the plasma leads to the amplification of the soliton. The soliton is amplified until its phase velocity ceases to be larger than the beam velocity $\bar{v}$. In accordance with this we found in the experiment that the maximum value of the Mach number was proportional to the beam velocity. We also obtained curves determining the spatial distribution of the amplitude of a soliton, amplified by a beam, for different values of the beam density while its energy was fixed. For small beam densities $n_1/n_0 \ll 1$ these curves agree qualitatively with the theoretical ones obtained from Eq. (29). For larger beam densities $n_1 \sim n_0$ the beam changes the parameters of the plasma through which the soliton propagates, and the spatial growth rate of the amplitude increases, more slowly than in (29), when the beam density increases.

The authors are grateful to R. Z. Sagdeev for useful hints and for discussions of the results of this paper.

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1)Faĭnberg [1] was the first to consider the possibility of using solitons to accelerate charged particles in a plasma.

2)One should not confuse the charge density solitary wave (soliton) considered in the present paper with the soliton which occurs in the problem of the modulation instability of a plasma and which is the envelope filled with Langmuir oscillations,

$$E(\mathbf{r},t)\sim\exp[i(\mathbf{kr}-\omega t)]f\left(\mathbf{r}-\frac{\partial\omega}{\partial\mathbf{k}}t\right),$$

and which is called in the established terminology a Langmuir soliton (see, e.g. [4]).

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223