Qualitative picture of the two-dimensional mixed state in type-I superconductors

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A structure is proposed for the thin superconducting layer arising on the surface of a hollow cylinder with current, the proposed structure having the form of alternating rings of superconducting phase separated by narrow rings of normal phase. Formulas are derived for the additional nondissipative current associated with this structure.

An unusual two-dimensional mixed (TM) state of a superconductor, whose possible occurrence was pointed out by L. Landau,[2] was observed in the experiments of I. Landau and Sharvin[1] concerning the intermediate state in hollow cylinders with current. In a hollow cylinder with a current exceeding the value at which the thermodynamic structure of the intermediate state vanishes, the magnetic field is nevertheless equal to zero near the inside surface of the cylinder. Therefore, in this region the conditions for the formation of a thin superconducting layer on the inside boundary of the metal still exist. In fact, in experiment[1] it was found that the current-voltage characteristic in the indicated current range is given by $I = R^{-1}V + I_S$, where $R^{-1}V$ is the ohmic part, and over a wide range of voltages (currents) the value of I_S is close to the critical current, calculated with the radius of the inner region. In other words this somewhat unexpected result indicates that the nondissipative current I_S is large and close to the surface density of the superconducting current, which is flowing on the boundary between the normal and the superconducting phases. In this connection the magnetic field increases jumpwise in the layer to the value H_C.

The presence of an electric field in the cylinder complicates an understanding of the situation, i.e., the poorly investigated role of dissipative processes. Andreev and Tekel'[3] proposed a simple, thermodynamic theory of the TM layer, in which they obtained the value

$$d=\frac{\xi}{2\sqrt{3}}\ln\left\{\frac{64\sqrt{2}D}{\xi}\frac{I-I_c}{I_c}\right\}$$

for the thickness of the layer (ξ is the microscopic correlation radius and D is the thickness of the cylinder). The direction of the currents was assumed by Andreev and Tekel'[3] to be along the axis of the cylinder, and the quantity E was completely neglected in comparison with H. Since the total current I ($I \sim I_C$) is distributed over the entire cross section of the sample, the electric field is small, $E \sim H_C(\xi/D)$. Of course, in thermodynamic relationships one can always neglect the contribution from the energy of the electric field. In actual fact, however, the role of the electric field is quite different and, as is shown in the proffered article, strictly speaking it is not legitimate to neglect it.

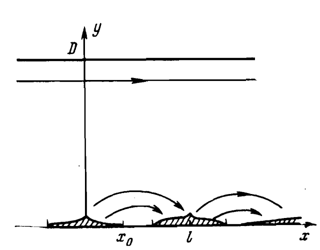
In short, the essence of the matter reduces to the fact that, on the surface of a macroscopic superconducting segment of the metal, the tangential component of the electric field is equal to zero whereas the normal component does not vanish. Because of this current flows in the superconducting segments and, by virtue of the Meissner effect, the current collects on the surface. The magnitude of this surface current density i_S cannot exceed the critical current density i_C corresponding to the jump of the magnetic field H_C. In the geometry of a two-dimensional thin layer[1,3] the inward leakage of the current from the normal part of the sample cannot be balanced by a growth of the surface, as in the case of intermediate structure, and therefore it leads to the breaking of the layer and the formation of rings of normal regions, in which the electric field is concentrated on the inner surface.

As will be clear from the following, there will be many such breaks under the conditions of the experiment[1]. We assume that they form a regular structure with a period l. For simplicity the case of a thin-walled cylinder, $D \ll R$, is considered below, as a consequence of which we can confine our attention to the two-dimensional problem. In addition, it is found that one can assume l to be small: $l \ll D$. This structure is depicted in the figure. The x axis is directed along the axis of the cylinder, the y axis corresponds to the direction along its thickness, and the azimuthal magnetic field is perpendicular to the plane of the figure. The distortion of the electric field and of the lines of current is indicated schematically. In the structure under consideration the distance 2λ ($x_0 = l/2 - \lambda$) between the superconducting regions is, for the time being, arbitrary. The possibility of neglecting the thickness of these regions in comparison with the period l of the structure is essential. The growth of the superconducting parameters in them occurs in a selfconsistent manner with the increase of the surface current; the slowness of the variation of the latter is due to the smallness of the volume current density.

Extremely pure samples were used in the experiments[1]. Nevertheless the problem is solved below for the simplest assumption of a local expression for the volume density of the normal current:

$$j=\sigma E. \tag{1}$$

The condition div j = 0 thus immediately leads to the usual problem of potential theory with the boundary conditions $E_t = 0$ on the segments of the real axis which correspond to the superconductor. For a two-dimensional problem, the distribution of the electric and magnetic fields can be found by the methods of complex variable theory.



Let the complex potential W(z) be chosen in the form

$$W(z)=\varphi-iA,\quad dW/dz=-E_x+iE_y. \tag{2}$$

The equation curl $\mathbf{H} = 4\pi\mathbf{j}/c$ together with Eq. (1) in the geometry of the figure gives

$$H=4\pi\sigma A/c. \tag{3}$$

The choice of the integration constant is determined by the condition that the field vanishes on the normal segments at $y = 0$. Assuming $l \ll D$, it is easy to verify that the requisite mapping realized the potential

$$W(z)=Ci\ln\left\{\frac{\cos(\pi z/l)+[\cos^2(\pi z/l)-\cos^2(\pi x_0/l)]^{1/2}}{\cos(\pi x_0/l)}\right\}. \tag{4}$$

By examining its asymptotic behavior as $y \to \infty$, we find

$$C=-E_\infty l/\pi. \tag{4'}$$

It is convenient to express x_0 in terms of λ:

$$W(z)=-i\frac{E_\infty l}{\pi}\ln\left\{\frac{\cos(\pi z/l)+[\cos^2(\pi z/l)-\sin^2(\pi\lambda/l)]^{1/2}}{\sin(\pi\lambda/l)}\right\}. \tag{5}$$

Expression (5) gives a concentration of the electric field at the ends of the superconducting segments. Formula (5) is not directly applicable in this region; however, for the results obtained below the latter fact is of little importance. The subsequent microscopic estimates indicate that $\lambda \ll l$. Therefore, we call attention to the fact that expression (5) contains a large logarithm: $\ln(l/\lambda) \gg 1$.

In expressions (4) and (5) E_∞ has the meaning of a uniform electric field far from the TM layer. Therefore, $\Delta\varphi = E_\infty l$ is the jump in the voltage potential between the isolated superconducting "islets." Now let us examine the behavior of W(z) for $y \gg l$ in more detail. According to Eqs. (3) and (5) we have

$$H\approx\frac{4\pi\sigma E_\infty}{c}\left\{y+\frac{l}{\pi}\ln\frac{l}{\pi\lambda}\right\}. \tag{6}$$

Since for $y = D$ we have $H = H^* = 2I/cR$, where I is the total current through the entire cross section of the cylinder, in the expression

$$\frac{I}{2\pi R}=\sigma E_\infty D+\frac{\sigma E_\infty l}{\pi}\ln\frac{l}{\pi\lambda} \tag{7}$$

the second term represents the additional nondissipative current due to the TM layer. In order to determine its value it is necessary to know the relation of the structure parameters l and λ with the field E_∞. For $l \gg \lambda$ expression (5) simplies considerably, in particular, the magnetic field H is given by

$$H=\frac{2\sigma E_\infty l}{c}\ln\left\{\frac{4l^2}{\pi^2\lambda^2}\left(\operatorname{sh}^2\frac{\pi y}{l}+\cos^2\frac{\pi x}{l}\right)\right\}. \tag{8}$$

For $y = 0$ its maximum value is reached at $x = 0$. Assuming this value to be equal to the critical value, we find

$$\frac{\sigma E_\infty l}{\pi}\ln\frac{2l}{\pi\lambda}=i_c. \tag{9}$$

It is convenient to rewrite this relationship so that the total currents flowing through the cylinder ($I_S \approx I_C$) appear in it:

$$l=\pi D\frac{I_c}{I-I_c}\Big/\ln\left(\frac{2D}{\lambda}\frac{I_c}{I-I_c}\right). \tag{10}$$

Finally, let us bring the current-voltage characteristic (7) to the following form:

$$I=R^{-1}V+I_c\left[1-\ln 2\Big/\ln\left(\frac{2D}{\lambda}\frac{I_c}{I-I_c}\right)\right]. \tag{11}$$

The following estimate will be obtained below for λ:

$$\lambda\sim\frac{2\xi}{3\pi}\ln^2\left(\frac{2D}{\lambda}\frac{I_c}{I-I_c}\right). \tag{12}$$

According to Eq. (9) the jump in the voltage potential between the isolated superconducting segments is a slowly varying function of the magnitude of the current:

$$\Delta\varphi=\frac{\pi i_c}{\sigma}\ln^{-1}\left(\frac{2D}{\lambda}\frac{I_c}{I-I_c}\right). \tag{13}$$

In the experiments[1] the value of $\Delta\varphi$ amounted to between 1×10^{-8} and 2×10^{-8} V, and l varied from 6×10^{-2} to 1×10^{-2} cm. We did not investigate the possibility of motion of the proposed structure.

Let us discuss certain details of our picture. In the first approximation we neglected the thickness d of the superconducting regions. By using the following parametric relations derived by Andreev and Tekel'[3] between d, the value of the superconducting parameter $\Delta(0)$, and the magnetic field H_0 on the surface of the layer:

$$d=\xi(1+k^2)^{1/2}K(k),\quad \Delta(0)=\frac{\sqrt{2}k}{(1+k^2)^{1/2}},\quad H_0=H_c\frac{2k}{1+k^2}, \tag{14}$$

one can immediately determine the dependence d(x) of the thickness of the superconducting regions with regard to the given distribution of the field $H_0(x)$ (which is given by Eq. (8)). According to (14) it is small everywhere. The region near $x = 0$ constitutes an exception, if relation (9) is satisfied. In fact, if $(H_C - H_0)/H_C \ll 1$, then

$$d\approx2^{-1/2}\xi\ln[H_c/(H_c-H_0)]$$

and increases with approach to $x = 0$. However, this growth is extremely slow and d still remains small in comparison with l even if (9) is satisfied with great accuracy. This fact justifies our fundamental assumption, according to which the period of the structure is determined by the fact that the superconducting segments are able to carry surface currents right up to the critical value.

The natural question arises: Is such a structure favorable from a thermodynamic point of view? For a given volume current density, relations (11)—(13) do not depend on D, and therefore in this case may be regarded as certain elementary solutions which are only associated with the surface itself. In order to show that the formation of structure is advantageous, let us compare the magnetic field distribution (8) with the field distribution in the Andreev-Takel' model,[3] in which the field H_0 on the surface of the layer is assumed to be close to the critical field, which corresponds to (8) and (9) in the first approximation, since in (8) the field is constant (to within logarithmic accuracy) along a superconducting segment over distances of order $l \gg \lambda$. However, the magnetic field in[3] is created by uniform currents, its distribution along the y axis is completely independent of x, and has the form

$$H(y)=H_0+(H^*-H_0)\frac{y-d}{D-d}, \tag{15}$$

where $H^* = 2I/cR$ and d is the thickness of the superconducting layer. Thus, in Eq. (15) it is considered that the field distribution is "shifted" towards the normal phase side. The minimization of the potential

$$\tilde{\mathcal{F}}_n=-\int H^2\,dV/8\pi$$

at a given current ($H^* = \text{const}$), in conjunction with (14), has determined the extremum of the potential because the energy gain due to the increase of the field around the layer is cancelled by the absence of a field inside the superconducting layer d.

Mathematically the extremum of the potential $\tilde{\mathscr{F}}$ in[3] is due to the fact that, although the contribution to the energy from the "displacement" of the field over a distance d is itself small, its change upon variation is very large. This fact is a consequence of the assumptions made in[3] that H_0 and d are independent of x. One can easily verify that in fact in Eq. (10) from[3] the quantity

$$\frac{d}{dk}d(k)\sim\frac{1}{1-k}\gg 1$$

is bounded by the values

$$\frac{1}{1-k}\sim\ln\frac{l}{\lambda}.$$

Rewriting (6) in the form

$$H_a(y)=H^*\frac{y+y_0}{D+y_0},\quad y_0=\frac{l}{\pi}\ln\frac{l}{\pi\lambda}\gg l, \tag{16}$$

one can single out in (8) a part h(xy), which is concentrated near the surface for $y\sim l$:

$$H(xy)=h(xy)+H_a(y),$$

where h(xy) is of the order of $h\sim H^*l/(D+y_0)$ and therefore gives a small contribution to

$$-\int H^2\,dV/8\pi.$$

Expression (16) corresponds to an effective increase in the area of the cylinder $D\to D+y_0$, in the background of which the displacement of the field over a distance of order $d\ll l$ gives small corrections.

Expression (16) represents the major contribution to the thermodynamic potential, and therefore it is clear that the formation of structure is advantageous for arbitrary y_0. However, the parameters l and λ are not determined by the conditions for a minimum of the potential, but must emerge as a result of the solution of the microscopic equations for the posed semi-infinite problem with a given current density at infinity. According to the results of the experiments[1], such solutions evidently exist.

Mathematical complexities do not allow us to find these solutions even for the simplest model equations, extending the Ginzburg-Landau equations to the case when an electric field is present. Such equations can be written down successfully for alloys containing paramagnetic impurities.[4] In the dimensionless variables of the Ginzburg-Landau theory they take the form[5]

$$\begin{gathered}12\dot{\Delta}+(\Delta^2-1+Q^2)\Delta-\varkappa^{-2}\nabla^2\Delta=0,\\ 12\Delta^2\mu+\varkappa^{-1}\operatorname{div}(\Delta^2\mathbf{Q})=12\Delta^2\mu-\varkappa^{-2}\nabla^2\mu=0,\\ \mathbf{j}=\operatorname{rot}\operatorname{rot}\mathbf{Q}=-\dot{\mathbf{Q}}-\varkappa^{-1}\nabla\mu-\Delta^2\mathbf{Q}.\end{gathered}\tag{17}$$

In the usual notation the gauge invariant combination μ is related to the scalar potential φ and the phase θ of the order parameter by the equation:

$$\mu=\theta+2e\varphi. \tag{18}$$

The boundary condition for μ on the surface of the metal is given by $(\partial\mu/\partial n)_S=0$. We shall assume $\kappa\ll 1$, in accordance with the fact that the superconductor is of type I.

Let us assume that the problem is time-independent ($\dot{\Delta}=\dot{Q}=0$). In this case the system of equations (17), describing the properties of the superconducting layer, differs from the equations of the Ginzburg-Landau theory by the presence of an additional normal contribution $-\kappa^{-1}\nabla\mu$ to the current and by the specific equation for μ. Assuming the electric field to be weak, we see that the magnetic field distribution and Δ in the layer coincide with the solutions (14).[3]

From the equations for μ and the boundary condition $(\partial\mu/\partial n)_S=0$ it follows that $E_t=0$ on the two-dimensional surface of a superconducting segment, but E_n and μ vary over distances $\xi\sim 1/\kappa$. If $d\gg\xi$, then μ is attenuated in the depths of the superconductor, leading to the Josephson condition (18). The thickness of the layer, at which the inleaking normal current changes into a Meissner surface current, coincides with the screening depth of a strong field. The latter was estimated by Ginzburg and Landau[6] in connection with the solution of the problem of surface tension between the phases and was found to be equal to $1/\kappa^{1/2}$ (i.e., $(\delta\xi)^{1/2}$). Thus, the width of the normal region, where the current near $y\sim d$ is still not able to collect into a surface current density, is of the order of $r\sim(\delta\xi)^{1/2}\sim\xi\kappa^{1/2}$. In this region the value of the order parameter is equal to $\kappa^{1/2}$.[6] The dimension r should not be confused with the λ appearing in formulas (5)—(13). The latter is determined by the position where the condition $E_t\ll E_n$, utilized in the derivation of formula (5), ceases to be satisfied.

Let $\kappa\ll\Delta^2\ll 1$ in Eq. (17). Then the first of the equations in (17):

$$\frac{1}{\varkappa^2}\frac{d^2\Delta}{dy^2}+\Delta=0,$$

gives the solution $\Delta=\Delta(0)\cos(\kappa y)$. By integrating the second equation with respect to y:

$$12\Delta^2\mu-\frac{1}{\varkappa^2}\frac{\partial^2\mu}{\partial y^2}=0,$$

we obtain

$$\mu_0=E_n/3\pi\varkappa\Delta^2(0). \tag{19}$$

Here μ_0 is the potential on the surface of the superconductor (i.e., for $y=\pi/2\kappa$) and E_n is the electric field.

According to Eq. (5), for $|x-x_0|\ll l$ we have $E_n\propto 1/(x-x_0)$.

By differentiating (19) along the surface, we obtain

$$E_t\sim E_n/3\pi\varkappa\Delta^2(0)\,|x-x_0|,$$

or

$$\lambda\sim 1/3\pi\varkappa\Delta^2(0).$$

Since in the region $|x-x_0|\sim\lambda$ we have

$$\Delta(0)\sim H_0/\sqrt{2}H_c\sim 1/\sqrt{2}\ln(l/\lambda),$$

according to expressions (14), in formulas (5)—(13) λ will be of the order of

$$\lambda\sim\frac{2\xi}{3\pi}\ln^2\frac{l}{\xi}. \tag{20}$$

We note, finally, that in fact because of the Josephson condition (18), oscillations with frequency

$$\omega=\frac{2e\pi i_c}{\hbar\sigma}\ln^{-1}\left(\frac{2D}{\lambda}\frac{I_c}{I-I_c}\right), \tag{21}$$

appear in the system, corresponding to the weak interaction between the separated superconducting segments through the normal region. Therefore, on the periphery of the structure shown in the figure, the picture is not time-independent. These oscillations emerge into the internal cavity of the cylinder with segments, where the superconducting order parameter is small ($\Delta \lesssim \kappa^{1/2}$). The dimensions of these regions were estimated above as $r \sim \sqrt{\delta\xi} \sim 10^{-5}$ to 10^{-4} cm. It is difficult to estimate the magnitudes of the corresponding alternating currents. In the experiments[1] the frequencies (21) should vary from 3×10^6 to 7×10^6 Hz.

Experimental verification of the details of the conjectured structure might consist of a measurement of the difference between the nondissipative part of the current I_s and the critical current I_c and an investigation of the weak dependence of I_s on the total current I, in accordance with Eq. (11). These corrections are found within the limits of error of the experiment[1]. Analogous terms also exist in the value for the surface impedance of the layer (compare with[3]):

$$\frac{Z-Z_s}{Z_s}=\frac{1}{\sqrt{2}l}\int\limits_{-l/2}^{l/2} dx\left[\ln\left(1\Big/\cos\frac{\pi x}{l}\right)\ln^{-1}\frac{2l}{\pi\lambda}\right]^{1/2}.$$

In addition, the normal segments with dimensions r, emerging on the inner surface of the cylinder, also make a contribution to the impedance.

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