## Effect of domain structure on the nonlinear properties of ferrites

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The spin-wave instability threshold in, and the susceptibility of, unsaturated samples of yttrium iron garnet located in arbitrarily oriented—with respect to each other—alternating and constant magnetic fields are investigated in the three-centimeter wave band. It is established that the instability threshold is determined by the orientation of the microwave field relative to the magnetization of the group of domains closest in direction to the microwave field. The dynamics of the variation of the domain structure as the crystal is magnetized along the principal axes is studied. For example, orientation of the external magnetic field along one of the easy axes results in the sudden appearance, near the saturation field, of domains magnetized along the other easy axes.

The present paper is devoted to experimental threshold and nonlinear-susceptibility studies in crystals possessing cubic symmetry and domain structures under conditions of first-order spin-wave instability. The difference between our experiments and the published experiments<sup>[1]</sup> lies, first, in the measurement in the unsaturated crystals of the imaginary part  $\chi''$  of the nonlinear susceptibility and, second, in the use of variable h and constant H<sub>0</sub> magnetic fields arbitrarily oriented with respect to each other: only the h $\parallel$ H<sub>0</sub> and h $\perp$ H<sub>0</sub> spin-wave instability pumping methods have been used before.

## EXPERIMENT

1. The experimental setup used by us is described in detail  $in^{[2]}$ ; here we only note that the pump frequency was 9370 MHz, the pump generator operated in the pulsed regime, the pulse duration was 200  $\mu$ sec, and the pulse repetition rate was 1-50 Hz. The resonator with the ferrite sphere was located at the center of an electromagnet that could rotate relative to the resonator, thereby changing the angle between the constant and variable magnetic fields. The ferrite sample under investigation was oriented such that  $H_0$  and h lay in the  $(1\overline{1}0)$  crystallographic plane (see Fig. 1); furthermore, the sample located inside the uhf resonator was rigidly fixed to the electromagnet, so that the rotation of the latter changed only the orientation of the pump field relative to the crystallographic axes and  $H_0$ , which orientation was determined by the angles  $\gamma$  and  $\beta$  ( $\alpha + \beta = \gamma$ ), and not the direction of  $H_0$  relative to the crystallographic axes of the ferrite (i.e., the angle  $\alpha$  = const). The angles could be adjusted and set to within  $\pm 1^{\circ}$ . The investigations were carried out on spheres of diameter  $\sim 2.5 \text{ mm}$ that were fabricated from a single crystal of yttrium iron garnet (YIG).

2. The results of the investigation of the dependence of the spin-wave instability threshold power  $P_{thr}$  on the strength of the external constant magnetic field for a YIG sphere are shown in Fig. 2 for H<sub>0</sub> directed along the [111] axis of easy magnetization ( $\alpha \approx 55^{\circ}$ ) and in Fig. 3 for H<sub>0</sub> directed along the axis [001] of difficult magnetization ( $\alpha = 0$ ) and for different pump angles.

The behavior of the experimental curves in fields  $H_0$  stronger than the saturation field  $H_S$  corresponds to the theoretical ideas<sup>[3]</sup>, according to which the threshold of the first-order parametric instability is minimum for a parallel (i.e.,  $\beta = 0$ ) pump and maximum for a perpen-



FIG. 1. Definition of the angles between the variable h and constant  $H_0$  magnetic fields and the crystallographic axes of the crystal.

dicular ( $\beta = 90^{\circ}$ ) pump. The growth of the instability threshold with decreasing H<sub>0</sub> is connected with the dependence of the spin-wave line width on the wave vector. The acceleration of the growth of P<sub>thr</sub> near the field H<sub>s</sub> (see, for example, the curves 2–4 in Fig. 2 near H<sub>0</sub>  $\approx$  700 Oe) occurs as a result of the effect of the threemagnon splitting process<sup>[4]</sup>.

In the region  $H_0 \leq H_s$ , where the domain structure can exist, the behavior of the threshold curves differs significantly from the above-described behavior, it being only sufficient to say that when the crystal is magnetized along a difficult axis (see Fig. 3) the parallel instability pump has the maximum threshold, while the minimum threshold, which coincides with the threshold of the parallel pump in Fig. 2, is observed at pump angles  $\beta = 55^{\circ}$ .

3. The behavior of the curves in Figs. 2 and 3 can be explained if we assume that the spin-wave instability in cubic ferromagnets located in fields weaker than the saturation field possesses the following property: the threshold for the first-order spin-wave instability depends only on the orientation of the external uhf magnetic field h relative to the magnetization  $M_d$  of the group of domains closest in direction to **h**, this orientation dependence being analogous to the dependence of the spin-



FIG. 2. Dependence of the threshold P<sub>thr</sub> for first-order spin-wave instability on the strength H<sub>0</sub> of the constant magnetic field for different orientations of the pump field. The crystal is magnetized along an easy axis. The sample is a YIG sphere of diameter 2.58 mm. The zero mark on the ordinate corresponds to an uhf field amplitude of 1.2 Oe. The curve 1) corresponds to a pump orientation  $\beta = 0$ ; 2)  $\beta = 55$ ; 3)  $\beta = -55$ ; 4)  $\beta = 71$ ; 5)  $\beta = 90^{\circ}$ .



FIG. 3. Same as in Fig. 2 for crystal magnetization along a difficult axis. The curve 1 corresponds to  $\beta = 0$ ; 2)  $\beta = 30$ ; 3)  $\beta = 55$ ; 4)  $\beta = 90^{\circ}$ .

wave excitation threshold on the angle between the constant field and the pump field in a saturated sample: the threshold is minimum at zero angle between the constant and variable magnetic fields. In such a case the behavior of the curves in Figs. 2 and 3 for  $H_0 = 0$  are quite natural.

Let us first consider the case of crystal magnetization along the [111] axis, i.e., the  $\alpha = 55^{\circ}$  case. As is well known, as H<sub>0</sub>  $\rightarrow$  0, the magnetization of each of the domains strives to align itself along one of the easy axes, of which there are four-the [111], [111], [111], and [111] directions (see Fig. 1)-in cubic crystals. For  $\alpha = 55^{\circ}$  and  $\beta = 0$ ,  $\gamma = 55^{\circ}$ , and the domain magnetization M<sub>d</sub> directed along the [111] axis turns out to be parallel to h, i.e., the instability threshold has its minimum value. For  $\alpha = 55^{\circ}$  and  $\beta = 71^{\circ}$ ,  $\gamma \approx 126^{\circ}$ , and, to within the error in the angle adjustment, the field h is parallel to the group of domains whose magnetization  $\mathbf{M}_{\mathbf{d}}$  is directed along the  $[11\overline{1}]$  axis: The threshold in this case is clearly the same as in the preceding case, i.e., it is also the minimum threshold. For  $\alpha = 55^{\circ}$  and  $\beta = -55^{\circ}$ ,  $\gamma = 0$ , and h is directed along the [001] axis, in which case the angle between h and the nearest easy axes, [111] and  $[1\overline{1}1]$ , along which the magnetization is directed, is  $55^{\circ}$ . This angle is the largest angle that can be formed by the vector and any easy axis of the cubic crystal; therefore, the threshold in this case will be the maximum threshold.

The difference between the  $H_0 = 0$ ,  $\beta = \pm 55^{\circ}$  thresholds (the curves 2 and 3 in Fig. 2) is due to the fact that for  $\beta = 55^{\circ}$  the vector h lies in the plane (110) between the [110] and [111] axes, making with the latter axis and, consequently, with the direction of  $M_d$  an angle of 16°, and not 55°, as obtained in the  $\beta = -55^{\circ}$  case.

We can explain in a completely similar fashion all the dependences obtained for ferrite magnetization along a difficult axis (i.e., for  $\alpha = 0$ ) and shown in Fig. 3, on account of which we restrict ourselves to the explanation of only the presence of the threshold peak for the parallel pump (i.e., for  $\alpha = 0$ ,  $\beta = 0$ , and  $\gamma = 0$ ). In this case h is directed along the [001] axis, and we have a situation similar to the ( $\alpha = 55^{\circ}$ ,  $\beta = -55^{\circ}$ , and  $\gamma = 0$ ) case considered above.

4. All the  $0 \leq H_0 \lesssim H_{\mathbf{S}}$  curves in Fig. 2 depend insignificantly on  $H_0$ . This is especially clear from the behavior of the curve 1, which corresponds to  $\alpha = 55^{\circ}$  and  $\beta = 0$  (i.e., the variable and constant magnetic fields are parallel to an easy axis). The picture is more complicated for magnetization along a difficult axis (Fig. 3), and will be explained below; but for fields  $H_0 \leq 350$  Oe, there is also observed here virtual independence of  $P_{thr}$  on  $H_0$ , the primary variable in this region of  $H_0$ being the orientation of h relative to the easy axes. This is confirmed by Fig. 4, which shows the dependence of  $P_{thr}$  on  $\gamma$ , the angle between h and the [001] axis for different directions of  $H_0$ . It can be seen from Fig. 4 that P<sub>thr</sub> virtually does not depend on the direction of the constant field  $H_0$ : the threshold value for  $H_0$  oriented along an easy axis of the crystal is the same as for  $H_0$ oriented along a difficult axis. The data of Fig. 4 were taken in a field  $H_0 = 200$  Oe; similar dependences have been obtained for fields  $H_0 = 0$ , 100, and 350 Oe.

For comparison of the instability threshold in saturated and unsaturated samples, we show in Fig. 4 (solid line) an experimentally obtained-for  $H_0 \gtrsim H_s$ -dependence of the spin-wave instability threshold for magnetization along the [111] axis on the orientation of the pump, i.e., on the angle  $\beta$ . Furthermore, the zero point on the  $\beta$ -angle scale (at which for  $H_0 > H_S$  the highfrequency field  $\mathbf{h}$  is parallel to  $\mathbf{H}_0$  and the saturation magnetization) corresponds to the angle  $\gamma = 55^{\circ}$  (at which for  $H_0 \leq H_s$  the field h is parallel to the [111] axis and the domain magnetization  $M_d$ ). It can be seen that the results for the saturated and unsaturated samples are in quite good agreement, which is a convincing proof of the fact that there occurs in an unsaturated sample oblique pumping of the first-order spin-wave instability, whose threshold depends on the angle between  $M_d$  and h, and that this dependence is similar to the dependence of the threshold on the angle  $\beta$  for  $H_0 \gtrsim H_s$ .



FIG. 4. Dependence of the spinwave instability threshold on the  $\gamma$ -orientation of the uhf field relative to the [001] axis of the crystal. The constant magnetic field was equal to 200 Oe and was directed along: (O) an easy axis, ( $\Delta$ ) a difficult axis. The solid line is the plot of the dependence of the threshold field on the angle  $\beta$  between the variable and the constant magnetic fields for a saturated sample.

In conclusion of the analysis of Fig. 4, let us note that results such as those shown in Fig. 4 can be obtained only for annealed samples free from internal strains and defects, since they can be nucleating centers for a domain structure that is uncharacteristic of a cubic crystal.

5. Since, in our opinion, we have produced in this paper conclusive proof of the dependence of the instability threshold on the direction of  $\mathbf{M}_d$ , we can extract from the shape of the  $P_{thr} = f(H_0)$  curves information about the dynamics of the domain structure of the investigated crystal. According to Fig. 2, upon the reduction below the saturation value of the field inside a crystal magnetized along an easy axis there immediately appear domains whose magnetization intensities are oriented along the [111] axis. (The field  $H_0$  is applied along the [111] axis.) This is shown by the curves 2, 4, and 5, which vary rapidly in the vicinity of  $H_S$ . For such crystal magnetization there are, in the entire field region  $H_0 \leq H_S$ , practically no domains magnetized along directions other than the easy axes.

An entirely different pattern of domain-structure behavior follows from Fig. 3, which corresponds to the magnetization of a spherical sample along a difficult axis. Now the threshold curves are smoother in the field H<sub>s</sub>, which is indicative of a slow variation of the direction of magnetization as we pass through the saturation field: as in the above-considered case, no abrupt changes in  $M_d$  occur. On this occasion, as the field is reduced from  $H_s$  to 400 Oe, the domain magnetization intensities Md turn gradually from the difficult axis to the easy axes. In  $H_0 \le 400$  Oe, each of the domains is magnetized along an easy axis. With the aid of the curve 1 in Fig. 3, we can easily construct the dependence of the direction of  $M_d$  on  $H_0$ , since the spin-wave instability threshold varies with the angle between h and  $\mathbf{M}_d$  according to a known law and the direction of h for the curve 1 is known (it is directed along the [001] axis). The direction of magnetization in the domains can also be determined for different fields H<sub>0</sub> by varying the orientation of the pump and determining the minimum instability threshold.

The aim of the present paper was not to investigate the domain structure in detail; therefore, let us only note here the fact that, as  $H_0$  is varied from 400 Oe to  $H_s$ ,  $M_d$  turns from an easy to a difficult axis almost linearly with the field.

The experiments performed in the case of magnetization along an intermediate axis showed that the variation of  $M_d$  as a function of  $H_0$  corresponds to theory<sup>[5]</sup>.

6. In Fig. 5 we show the external-field dependence of the imaginary part  $\chi''$  of the nonlinear susceptibility of ferrites for different pump angles in the case when the



FIG. 5. Dependence of the imaginary part of the nonlinear susceptibility on the constant magnetic field for a YIG sample of diameter 2.58 mm in the case of parallel (the curve 1) and perpendicular (the curve 2) pumping when the threshold is exceeded by 3 dB.

pump power exceeds the first-order spin-wave instability threshold. The behavior of the curves in Fig. 5 for  $H_0 > H_s$  is determined by three-magnon splitting, owing to which the susceptibility  $\chi''$  decreases sharply if the parametrically excited spin waves fall in the decay region of the spectrum<sup>[6]</sup>, where the splitting of each parametric spin wave into two new spin waves is possible. The effect of the three-magnon splitting on the spin waves that have the polar angle  $\theta_k = 90^\circ$  and that are excited during parallel pumping is less than on the waves excited during transverse pumping ( $\theta_k \approx 45^\circ$ ), on account of the fact that the decay region of the spectrum is narrower for waves with  $\theta_k = 90^\circ$  than for waves with  $\theta_k = 45^\circ$ .

For a pump frequency of 9370 MHz, the three-magnon splitting of the spin waves with  $\theta_{\mathbf{k}} = 90^{\circ}$  is possible in fields  $H_0 \leq H_s + 100$  Oe, while for waves with  $\theta_k = 45^\circ$ the splitting is possible in fields  $H_0 \lesssim H_{\rm S}$  + 300  ${\rm \ddot{O}e^{[4]}}.$ For arbitrary pump angles, it can be established that the width of the decay spectrum and, hence, the effect of the decay processes on the spin waves increase with increasing pump angle<sup>[4]</sup> (see, for example, Fig. 2: for  $H_0 \ge H_s$  the growth of the spin-wave instability threshold because of the three-magnon splitting increases with the pump angle). Experiment shows that near the saturation field the susceptibility  $\chi''$  for perpendicular pumping (the curve 2 in Fig. 5) is more than one order of magnitude less than the susceptibility for parallel pumping (the curve 1 in the same figure). Figure 5 corresponds to crystal magnetization along the easy axis [111]; the results for magnetization along the other axes are close to the results shown in Fig. 5.

From the consideration of Figs. 3 and 5 follows the following practical conclusion: Instruments using perpendicular uhf and constant magnetic fields near the saturation field will be very slightly exposed to the influence of the spin-wave instability, an especially propitious situation being realized in the case when  $H_0$  is oriented along a difficult axis of the crystal.

7. As was noted above, for  $H_0 \leq H_s$ , there obtains oblique pumping of the spin-wave instability. Because of the influence of the three-magnon splitting process, the susceptibility will also depend on the pump angle, the dependence being such that the larger the pump angle is, the smaller the value of  $\chi''$ . Furthermore, the susceptibility should also depend on the magnitude of the resultant magnetic moment of all the domains in which the spin-wave instability threshold has its minimum value. The behavior of the curves in Fig. 5 is understandable on the basis of these assumptions.

The susceptibility for parallel pumping decreases gradually with decreasing field strength, since the resultant magnetic moment of the domains, which is oriented along the [111] axis, decreases because of the appearance of domains magnetized along the other easy axes. The susceptibility  $\chi''$  for perpendicular pumping increases as  $H_0$  is decreased below the saturation value as a result of the appearance of domains magnetized along the [111] axis, for which domains the angle between  $M_d$  and **h** is ~ 19°, which is equivalent to the situation in which the pump is nearly a parallel pump and in which the susceptibility has its maximum value. Notice that both the susceptibility vs.  $H_0$  and susceptibility vs. sample orientation curves exhibit hysteresis, whereas the corresponding curves for the instability threshold exhibit practically no hysteresis. Thus, for example, for  $H_0$ = 0 the susceptibilities for  $\gamma$  = 55° and  $\gamma$  = 126° (h directed along the [111] or the  $[11\overline{1}]$  axis) may, depending on the "history" of the magnetization of the sample, differ by a factor of 1.2-1.5.

## CONCLUSIONS

1. It has been established, as a result of an investigation on cubic ferrites of the threshold for firstorder spin-wave instability arising under the action of constant and variable magnetic fields arbitrarily oriented relative to the crystal lattice, that the instability threshold in unsaturated samples depends, in the main, only on the angle between the variable magnetic field vector h and the magnetization of the group of domains closest in direction to h, this dependence being similar to the dependence in saturated samples of the instability threshold on the pump orientation relative to  $H_0$ .

2. The method of recording the spin-wave instability threshold has been used to study the dynamics of the domain structure of ferrites in spherical samples. It turned out that for crystal magnetization along a difficult axis the magnetization turns, as the field is reduced from  $H_s$  to 400 Oe, from the difficult to an easy axis almost linearly with the field. For magnetization along some easy axis, domains with magnetization along the other easy axes appear abruptly at a field strength below the field at which the domain structure arises.

3. The imaginary part  $\chi''$  of the nonlinear susceptibility for perpendicular pumping near the saturation field is more than an order of magnitude less than for parallel pumping in the same range of constant magnetic field strengths. As the field is decreased;  $\chi''$  for parallel pumping decreases, while  $\chi''$  for perpendicular pumping increases.

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 <sup>&</sup>lt;sup>1</sup>E. V. Lebedeva and A. I. Pil'shchikov, Fiz. Tverd. Tela 9, 3630 (1967) [Sov. Phys.-Solid State 9, 2862 (1968)].
<sup>2</sup>G. A. Melkov, Zh. Eksp. Teor. Fiz. 61, 373 (1971) [Sov. Phys.-JETP 34, 198 (1972)].
<sup>3</sup>Yu. M. Yakovlev, Fiz. Tverd. Tela 10, 2431 (1968) [Sov. Phys.-Solid State 10, 1911 (1969)].
<sup>4</sup>G. A. Melkov and V. L. Grankin, Fiz. Tverd. Tela 14, 3452 (1972) [Sov. Phys.-Solid State 14, 2916 (1973)].
<sup>5</sup>W. E. Courtney, Electron. Lett. 1, 21 (1965).
<sup>6</sup>V. S. L'vov, Preprint, Institute of Nuclear Physics, Siberian Division of the USSR Academy of Sciences, Nos. 69-72, Novosibirst, 1972.