Recoil effect in a strong resonant field

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In a strong resonant field the recoil effect is, generally speaking, of a classical nature, namely, gradient force causes the atom to move along a certain classical trajectory. A special case arises only at frequencies near exact resonance. Owing to saturation, the mean gradient force vanishes in this case and quantum fluctuations of the force must be taken into account. A correlation between the induced dipole moment and the atom velocity arises in this case. It is shown that an atomic beam incident on the vacuum-resonant field interface experiences birefringence. Some features of resonance fluorescence in an inhomogeneous field and of heating of atoms in a nonstationary resonance field are discussed.

1. INTRODUCTION

The recoil effect with spontaneous emission of atoms, as is well known, becomes noticeable when the resultant Doppler frequency shift is larger than the line width γ:

$$\hbar k^2/M > \gamma, \tag{1}$$

where M is the mass of the atom and k is the wave number. This condition is satisfied in spectroscopy only for very narrow resonances, $\gamma \sim 10^3 - 10^5$ Hz. The singularities in stimulated emission with allowance for recoil were considered in[1] as applied to the theory of the Lamp dip. The resonant field of the standing wave was assumed in this case to be weak in comparison with a certain characteristic quantity determined by the line width. The case when only one of the traveling waves is weak was studied by Stenholm[2].

It is of interest to consider the recoil effect in a strong field when the trajectory of the atom changes significantly during the course of the interaction with the field. The question is formulated in this manner: how does the motion of the atom in an inhomogeneous field influence its response, i.e., the average dipole moment p? The answer to this question becomes particularly clear in the case when p vanishes for an infinitely heavy atom.

In an external monochromatic field given by

$$E(\mathbf{r})e^{-i\omega t} + E^*(\mathbf{r})e^{i\omega t} \tag{2}$$

the polarizability α of the atom as a function of the frequency behaves approximately as shown in Fig. 1; $p = \alpha(\Delta, |E^2|)E$, and $\Delta = \omega - \omega_0$ is the detuning of the field frequency relative to the transition frequency ω_0 An approximate expression for α in a strong ihhomogeneous field and the exact value of α were obtained by the author earlier[3, 4].

In a strong field

$$dE \gg \hbar\gamma \tag{3}$$

the dipole moment at $\Delta = 0$ is small, owing to the saturation effect, $p \sim \hbar\gamma/E \ll d$, and the polarizability can be assumed to be equal to zero. In fact, this is true only for an atom with $M = \infty$. It will be shown below that for an atom with finite mass the dipole moment at exact resonance is of the following order of magnitude:

$$p \sim d^2E/Mv^2, \tag{4}$$

where d is the matrix element of the dipole moment and v is the velocity of the atom relative to the inhomogeneity of the field.

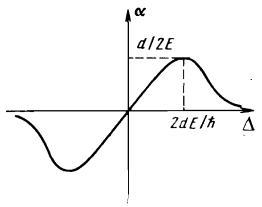

FIG. 1.

Since the average dipole moment vanishes in the resonant case, it is necessary to take its fluctuations into account. Therefore the problem of the motion of an atom at $\Delta = 0$ becomes essentially a quantum problem. In the nonresonant case $\Delta \gtrsim dE/\hbar$ the fluctuations of the dipole moment of the atom do not play so important a role, and the motion of the atom can be regarded as classical under the influence of an average gradient force.

We emphasize that the estimate (4) is suitable only for a sufficiently strong inhomogeneous field

$$d|\nabla E|/Mv > \gamma. \tag{5}$$

The left-hand side of this inequality determines the Doppler shift in a classical inhomogeneous field $E(\mathbf{r})$. The inequality (5) generalizes the criterion (1) for the case of a strong external electromagnetic field. By way of example, we estimate with the aid of (5) the value of the electromagnetic field for the case when the oscillator strength is of the order of unity, $M \sim 10$ a.u., $v \sim 10^4$ cm/sec, and $\gamma \sim 10^7$ Hz. If the inhomogeneity is determined by the resonant wavelength, then $E \sim 10^4$ V/cm, and the emission power is $\sim 5 \times 10^5$ W/cm^2.

In this paper we consider the behavior of atoms in a strong resonant field with allowance for the recoil effect. We analyze several examples in which the fluctuations of the gradient force are significant. In the second section we write down a quantum-kinetic equation for the Wigner density matrix. This equation is then used to study the passage of atoms through the boundary between vacuum and a strong resonant field. It is shown that birefringence of the atomic beam is produced here in the case of exact resonance. We also consider the nonresonant case and discuss the singularities of the resonance fluorescence in an inhomogeneous field.

A nonmonochromatic random field can be used to heat the atoms. The highest heating rate is obtained in the case when the average frequency of the field coincides with the frequency of the transition.

2. KINETIC EQUATION

We start from the quantum kinetic equation for the density matrix of the atoms $\rho(\mathbf{r}_1\mathbf{r}_2t)$ in an external resonant field in the form (2). In the approximation of two atomic states, the initial equation for ρ takes the form

$$\frac{\partial\rho(1,2)}{\partial t}+\hat{\gamma}\rho(1,2)=\frac{1}{i\hbar}[(H_0(1)+H(1))\rho(1,2)-\rho(1,2)(H_0(2)+H(2))]; \tag{6}$$

$$H_0=-\hbar^2\nabla^2/2M, \qquad H(\mathbf{r})={}^1/_2\sigma_3\hbar\Delta+d(\sigma_+E(\mathbf{r})+\sigma_-E^*(\mathbf{r})), \tag{7}$$

here γ denotes the operator of the relaxation of the atomic states as a result of the spontaneous emission. The matrix σ_3 is a diagonal Pauli matrix and $\sigma_+(\sigma_-)$ is the operator for the production of the atom in the upper (lower) working state.

Next, as usual, it will be more convenient to use the density matrix in the mixed representation (in coordinate space and in velocity space)

$$\rho(\mathbf{rvt})=(2\pi\hbar)^{-3}\int d^3\mathbf{r}'\rho\left(\mathbf{r}+\frac{\mathbf{r}'}{2},\mathbf{r}-\frac{\mathbf{r}'}{2},t\right)\exp\left\{\frac{iM\mathbf{v}\mathbf{r}'}{\hbar}\right\}. \tag{8}$$

Changing over to the equation for the Wigner density, we confine ourselves to the quasiclassical limit

$$\hbar k\ll Mv. \tag{9}$$

We then get from (6)

$$\frac{d\rho}{dt}+\hat{\gamma}\rho=\frac{1}{i\hbar}(H\rho-\rho H)+\frac{1}{2M}\left(\nabla H\frac{\partial\rho}{\partial\mathbf{v}}+\frac{\partial\rho}{\partial\mathbf{v}}\nabla H\right). \tag{10}$$

Here $d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$ is the total derivative with respect to time. The recoil effect in the kinetic equation (10) is described by the second term in the right-hand side. When this term becomes larger than $\hat{\gamma}\rho$, then the recoil effect becomes significant, and we arrive at the criterion (5).

We write down Eqs. (10) for individual components of the density matrix of the atoms. We denote by $f(\mathbf{rvt}) = \mathrm{Tr}(\rho(\mathbf{rvt}))$ the distribution function of the atoms, by $p(\mathbf{rvt}) = d\,\mathrm{Tr}(\sigma_-\rho(\mathbf{rvt}))$ the dipole moment induced by the field, and by $q(\mathbf{rvt}) = \mathrm{Tr}(\sigma_3\rho(\mathbf{rvt}))$ the difference between the populations of the upper and lower levels. We then obtain from (10) the following system of equations:

$$\frac{df}{dt}=\frac{(\nabla E^*)}{M}\frac{\partial p}{\partial\mathbf{v}}+\text{c.c.}, \tag{11}$$

$$\frac{dp}{dt}+\left(i\Delta+\frac{\gamma}{2}\right)p=\frac{id^2E}{\hbar}q+\frac{d^2}{2M}(\nabla E)\frac{\partial f}{\partial\mathbf{v}}, \tag{12}$$

$$\frac{dq}{dt}+\gamma q=-\gamma f+\frac{2i}{\hbar}(pE^*-\text{c.c.}). \tag{13}$$

The lower level is assumed to be the ground (or metastable) state; the relaxation operator $\hat{\gamma}$ is chosen accordingly.

3. BIREFRINGENCE OF ATOMIC BEAM

We now use Eqs. (11)–(13) to solve the following very simple boundary-value problem: assume that a monoenergetic beam of atoms with velocity $\mathbf{v}_0(v_{0x}, v_{0y}, 0)$ is incident on the interface x = 0 between vacuum and a half-space filled with an optical field $E(\mathbf{r})$. What are the trajectories of the atoms inside the region occupied by the light?

If the electromagnetic field propagates along the y axis, then we can use for E(x, y) approximately the expression

$$E(x,y)=E(x)e^{iky}, \tag{14}$$

where E(x) is a real function that varies from 0 at $x\to-\infty$ to E_∞ at $x\to+\infty$. The explicit form of the function E(x) is not important; all that matters is that the criterion (5) be satisfied in the transition region. To simplify the calculations we assume that it is satisfied with a large margin

$$\frac{d}{Mv}\frac{dE(x)}{dx}\gg\gamma. \tag{15}$$

In the transition layer, the atom is acted upon by two forces: a gradient force on the order of $ddE(x)/dx$ along the x axis, and a force on the order of $\hbar\gamma k$, which is connected with the spontaneous emission and which acts along the y axis (for details see[4]). When the condition (15) is satisfied, the gradient force along the x axis is much larger than the force along the y axis. The latter can therefore be disregarded. To this end it suffices to omit the factor e^{iky} in Expression (14).

In this approximation the problem becomes one-dimensional: the velocity along the y axis remains unchanged, and it is necessary to find only the change of v_x. We consider first the resonant case $\Delta \ll (d/Mv)dE/dx$. Then Eqs. (11)–(13) in the transition layer become much simpler and take the form

$$v_x\frac{\partial r_\pm}{\partial x}=\pm\frac{d}{M}\frac{dE(x)}{dx}\frac{\partial r_\pm}{\partial v_x}, \qquad r_\pm=\frac{1}{2}f\pm\mathrm{Re}\,\frac{p}{d}. \tag{16}$$

If the distribution function in vacuum as $x\to-\infty$ is $f_0 = A\delta(v_x - v_{0x})$, then outside the transition layer, at $E(x)\approx E_\infty$, we have

$$f=Av_{0x}[\delta(v_x^2-v_{-x}^2)+\delta(v_x^2-v_{+x}^2)],$$
$$\mathrm{Re}\,p=\frac{1}{2}dAv_{0x}[\delta(v_x^2-v_{-x}^2)-\delta(v_x^2-v_{+x}^2)]; \tag{17}$$
$$v^2{}_{\pm x}=v_{0x}{}^2\mp2dE_\infty/M.$$

Thus, a monoenergetic atomic beam in a strong resonant field splits into two beams of equal intensity with normal components $v_{\pm x}$. This is physically connected with the fact that although the quantum mean value of the dipole moment is indeed equal to zero at $\Delta = 0$, nevertheless such a dipole moment can arise as a result of fluctuations. The probabilities of occurring in phase or in counterphase with the field are equal in this case.

Assuming the splitting angle $\theta_2-\theta_1$ to be small (fig. 2), we obtain from (17)

$$\theta_2-\theta_1=2(dE_\infty/Mv_0^2)\,\mathrm{ctg}\,\theta. \tag{18}$$

Let us estimate the order of magnitude of this quantity for the following case: incident-atom energy on the order of 10^{-2} eV, $E\sim10^4$ V (power 5×10^5 W/cm^2), $\theta = 45°$. For the resonant transitions of the alkali-metal atoms we then have $\theta_2-\theta_1\sim10^{-2}$.

Under real conditions, however, the thickness of a light beam is quite small, and the picture of the splitting takes the form shown in Fig. 3 by the solid lines. After the passage of the beam, the split atomic beams travel in parallel to the incident beam. The divergence of the beams is then proportional to the thickness of the light beam and becomes small at small thickness. Neverthe-

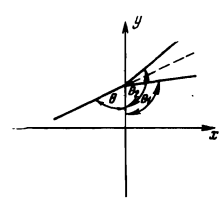

FIG. 2

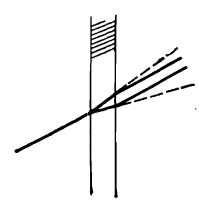
FIG. 3

less, even in this case it is possible to observe birefringence if we use a beam in which the intensity is modulated at a frequency equal to the reciprocal time of flight of the atoms through the beam. It is then possible to have for certain atoms a situation wherein the atoms on the front boundary are acted upon by the maximum field, and the field at the rear boundary vanishes and the atoms move along straight-line trajectories (dashed lines). In other words, for these atoms the effect due to a thin beam is the same as in the case of half-space.

4. SINGULARITY OF THE RESONANCE FLUORESCENCE IN AN INHOMOGENEOUS FIELD

We note the following singularity of resonance fluorescence in a strong inhomogeneous field. As is well known, in the case of resonance fluorescence the emission spectrum of the atom consists of a coherent part and an incoherent part. The coherent component has a zero emission line width in an external monochromatic field and is determined by the average dipole moment

$$p(\mathbf{r}t)=\int d^3v p(\mathbf{r}vt). \tag{19}$$

The incoherent component has a finite emission-line width equal to $\gamma/2$ and is determined by the fluctuations of the dipole moment (for details see, e.g., [5]). In the case of a strong homogeneous field at exact resonance we have $p \sim \hbar\gamma/E$, and the coherent component becomes very small.

In the inhomogeneous case, at not too high a velocity of the atom relative to the standing wave, when the condition

$$Mv^2\hbar\gamma \ll (dE)^2, \tag{20}$$

is satisfied, the value of the average dipole moment can increase strongly. Assume that the atom moves in the field of a standing wave and the criterion (16) is satisfied. We can then use formula (17), from which we obtain in the first-order approximation in the field

$$p(\mathbf{r})=d^2E(\mathbf{r})/Mv_0^2. \tag{21}$$

Thus, the fraction of the coherent part of the energy emitted by the atom is of the order of $(dE/Mv^2)^2$. When the potential energy becomes comparable with the kinetic energy, then the coherent and incoherent components of the emission become of the same order of magnitude.

5. NONRESONANT CASE

To emphasize the difference between the resonant and nonresonant cases, let us consider the problem of Sec. 3 for large detunings $\hbar\Delta \gtrsim dE$. We assume that when the atom moves the field changes in quasistationary manner:

$$v_x dE/dx \ll \Delta E(x). \tag{22}$$

In this case the initial equations (11)–(13) can be solved by expansion in the small parameter $1/\Delta$.[4] Neglecting relaxation, we obtain from (12) approximately

$$p=\frac{d^2}{\Delta}\left[\frac{E(x)q}{\hbar}-\frac{i}{2M}\frac{dE(x)}{dx}\frac{\partial f}{\partial v_x}\right]+\frac{id^2}{\hbar\Delta^2}v_x\frac{\partial}{\partial x}(E(x)q). \tag{23}$$

The last two terms in this expression are small, but they are the only ones that contribute to Eq. (13) for q. When (23) is substituted in (11), it suffices to retain only the first term. We finally obtain the following equations for f and q:

$$\begin{aligned}\chi(x)v_x\frac{\partial}{\partial x}(\chi(x)q)&=\frac{d^2}{\hbar\Delta M}\frac{dE^2(x)}{dx}\frac{\partial f}{\partial v_x},\\ v_x\frac{\partial f}{\partial x}&=\frac{d^2}{\hbar\Delta M}\frac{dE^2(x)}{dx}\frac{\partial q}{\partial v_x}\end{aligned} \tag{24}$$

where $\chi(x) = (1 + 4d^2E^2(x)/\hbar^2\Delta^2)^{1/2}$ is a parameter that determines the magnitude of the Stark splitting.

If the distribution function of the incident atoms has the same form as in Sec. 3, then f at $x > 0$ takes the form

$$f=v_{0x}\left[A_+\delta\left(v_x^2-v_{0x}^2-\frac{\hbar\Delta}{M}(\chi-1)\right)+A_-\delta\left(v_x^2-v_{0x}^2\mp\frac{\hbar\Delta}{M}(\chi-1)\right)\right] \tag{25}$$

with $A_+ + A_- = A$.

In the nonresonant case we also have two trajectories in accordance with the two possible terms of the atom in the external field $\pm\hbar\Delta\chi(x)$. Of course, the magnitude of the splitting of the trajectories and the shifts of the terms under the conditions of inequality (3) and (5) can be obtained from the simpler Schrödinger equation for a two-level atom. We do not know here, however, what fraction of the atoms goes to each trajectory. To answer this question it is necessary to solve the kinetic equations (11)–(13) and take the boundary conditions into account. If the incident atoms are in the ground state, then $q = -f_0$ and in this case the atoms move only on one trajectory, namely the one determined by the term adjacent to the ground state when the field is turned off adiabatically. If the incident atoms are in a mixed state, then $q_0 = -Wf_0$ and the atoms in the region $x > 0$ fill both trajectories with weights $A_\pm = A(1 \pm W)/2$. In order for the atoms to be incident in the mixed state, it suffices to apply a weak resonant field $E_1 \ll E$ such that $dE_1 \sim \hbar\gamma$.

Notice should be taken of the fundamental difference between the resonant and nonresonant scattering. In the former case the atoms in the incident beam are all in identical states, and we are dealing with birefringence. In the nonresonant case, two trajectories appear only for atoms that are incident in the mixed state. We have here simply the selection, used in masers, of the excited and unexcited atoms.

6. HEATING OF ATOMS BY A NONMONOCHROMATIC FIELD

In a nonstationary field with a broad frequency spectrum, the atoms experience stochastic acceleration (heating). This effect was estimated earlier[4] for the case of strong fields and large detunings $\hbar\Delta \sim dE$, when the average force acting on the atom is strong enough. We shall show here that at $\Delta = 0$ the atoms also become heated by the fluctuating gradient force in a nonmonochromatic field.

We consider the field of a standing wave with random phase $\varphi(t)$

$$E(xt)=E_0\cos(kx+\varphi(t)) \tag{26}$$

and with a phase correlator in the form

$$\langle e^{i(\varphi(t)-\varphi(0))}\rangle = e^{-\Gamma t}, \tag{27}$$

so that on the average this field is at resonance with the transition frequency.

If $kdE \ll vM\Gamma$, then we can use perturbation theory. As a result we obtain for the slowly varying part of

the distribution function of the atoms $f_0(vt) = \langle f(xvt) \rangle$ the Fokker-Planck equation

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial v}\left(D(v) \frac{\partial f_0}{\partial v} \right), \quad D(v) = \left(\frac{dE}{\sqrt{2}M} \right)^2 \frac{\Gamma}{v^2 + (\Gamma/k)^2} \tag{28}$$

At low velocities ($kv < \Gamma$) the kinetic energy of the atoms increases in proportion to t, and at $kv > \Gamma$ we have $v^2 \sim t^{1/2}$.

We note that in the resonant case the diffusion coefficient D is, generally speaking, larger than in the nonresonant case. Both coefficients become of the same order only if in the nonresonant case the atom has a maximum polarizability α, i.e., when $\hbar\Delta \sim dE$ (see Fig. 1).

7. CONCLUSION

Thus, in a strong quasistationary field the atom has, generally speaking, two trajectories. If the atom was in the ground state at the initial instant of time, then it it has one trajectory in the resonant case. In the case of exact resonance it is necessary to take both trajectories into account.

From the quantum point of view it can be stated that at $\Delta = 0$ the average gradient force vanishes, and account must be taken of the fluctuations of this force. A correlation exists in this case between the fluctuation of the dipole moment and the motion of the atom in a strong inhomogeneous field. As a result, the average dipole moment becomes different from zero at exact resonance (formula (21)). This circumstance leads to birefringence at the atomic beam in the optical field and to the appearance of a coherent component in the resonant fluorescence.

In the present article we have taken into account only the fluctuations of the gradient force. If we do not neglect the relaxation, then the atom is also acted upon by a force due to retardation[4]. It appears that near resonance this force also undergoes strong fluctuations.

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