Spatial dispersion in the vicinity of cyclotron resonance in a semiconductor located in a magnetic field

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It is demonstrated in the effective-mass approximation that spatial dispersion leads to deflection of electromagnetic waves in a semiconductor by an external magnetic field, provided that the wave frequency is close to the transition between two Landau levels of neighboring bands.

1. Recently, in an investigation of the propagation of a photon in vacuum in the presence of an external magnetic field, one of us [1,2] has noted a considerable distortion of the photon dispersion law near the threshold of the production of an electron-positron pair by the photon, and as a consequence, a deflection of the photon in a direction parallel to antiparallel to the magnetic field. The intensity of the external magnetic field, at which its influence becomes noticeable to some degree, is in vacuum of the order of $H_{cr} = (mc^2)^2/e\hbar c = 4 \times 10^{13}$ G, where m and e are the mass and charge of the electron and c is the velocity of light in vacuum. One can hope to encounter such a magnetic field only in cosmic objects (say in pulsars); it is presently not attainable in the laboratory. The reason for the appearance of so large a characteristic value is the large energy gap $2mc^2 = 1$ MeV between the sea of positrons and electrons. To the contrary, a semiconductor is characterized by a forbidden gap which is smaller by a factor of a million ($\mathscr{E}_g \sim 1$ eV). One can therefore expect the particular magnetorefraction effects of interest to us to become noticeable in semiconductors even in laboratory fields.

2. We study in this paper the dispersion of the photon in a semiconductor placed in a constant magnetic field **H**.

In the calculation of the dielectric constant, the effective-mass approximation and the lowest order of perturbation theory in the wave field are assumed. The dielectric constant is the result of (virtual or real) transitions of electrons from Landau levels in the valence band to Landau levels in the conduction band. The spins are not taken into account.

To avoid consideration of effects that do not interest us and are connected with polarization of the waves (such as birefringence), a cubic crystal is chosen, so that the dielectric constant contains only a single tensor $\epsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij}\epsilon(\omega, \mathbf{k})$ (the external field makes no noticeable contribution to the tensor structure ϵ_{ij} within the framework of the approximation, insofar as we are considering a transition that is already allowed in the absence of an external field (see also the Appendix)). What is important, however, is allowance for the spatial dispersion of the photon. The dielectric constant $\epsilon(\omega, |\mathbf{k}|, \mathbf{k}\cdot\mathbf{H})$ depends on the orientation of the photon momentum relative to the magnetic field, and this indeed is the source of the actual anisotropy of the problem. In this respect our analysis differs from the traditional method (see [3]) of taking spatial dispersion into account, which is appropriate in other magnetooptics problems [4], when the spatial dispersion contributes primarily to the tensor structure ϵ_{ij} formed as a result of the vector **k**. In our problem the anisotropy, which we shall show to manifest itself in a difference between the directions of the phase and group velocities of the wave, takes place also in the case of an isotropic tensor $\epsilon_{ij} = \delta_{ij}\epsilon$, something impossible without taking the spatial dispersion into account. If necessary, the tensor structure ϵ_{ij} can, of course, be taken into consideration, but then the conclusions of our paper will be applicable to each of the natural modes of the electromagnetic wave separately (cf. [1]), whereas in the present exposition the dispersion of the electromagnetic wave is indifferent to its (transverse) polarization.

The reason why the contribution of the spatial dispersion is appreciable in our problem is, as usual [3], that $\epsilon(\omega, |\mathbf{k}|, \mathbf{k}\cdot\mathbf{H})$ becomes infinite. It is well known [5] that the absorption coefficient of an electromagnetic wave has a reciprocal-square-root behavior when the wave frequency approaches the threshold of the particle-hole pair production at the Landau levels, and becomes infinite at this threshold (cyclotron resonance). Since the imaginary part of the square root is again a square root, the dielectric constant has a similar singular behavior by virtue of the optical theorem.

We investigate below the influence of cyclotron resonance on the solution of the electromagnetic-wave dispersion equations, paying particular attention to the specific contribution that is made by the spatial dispersion and that leads to a curious phenomenon—the deviation of the wave propagation direction parallel (antiparallel) to the external magnetic field. We study also the most favorable conditions for the possible observation of this effect.

3. The dispersion equation for normal transverse waves with frequency ω and wave vector **k** (here **b** is the unit vector of polarization)

$$\mathbf{E} = \mathbf{b}E_0 \cos(\omega t - \mathbf{kr}) \tag{1}$$

takes the form

$$\frac{\omega^2}{c^2} - k^2 + 4\pi\Pi(\omega, \mathbf{k}) = 0, \tag{2}$$

where the polarization operator Π is connected with the dielectric constant by the relation $\epsilon = 4\pi\Pi c^2/\omega^2 + 1$. The polarization operator is connected with the amplitude f for the forward scattering of the photon by an electron in the following manner:

$$\Pi = Nf(\mathbf{k}, \omega), \tag{3}$$

where N is the electron density. We take the imaginary part of relation (2) and, using the optical theorem, obtain

$$\operatorname{Im}\Pi = N|\mathbf{k}|\sigma_t/4\pi. \tag{4}$$

The total cross section σ_t is expressed in the following manner in terms of the square of the matrix element F_{12}:

$$\sigma_i=\frac{16\pi^2}{\hbar c^2}\frac{\omega^2}{|\mathbf{k}|E_0^2}\sum_{1,2}|F_{12}|^2\delta\left(\omega+\frac{\mathscr{E}_1}{\hbar}-\frac{\mathscr{E}_2}{\hbar}\right). \tag{5}$$

The energies in the valence band and in the conduction band are equal to, respectively,

$$\begin{aligned}\mathscr{E}_1&=\mathscr{E}_1^0-\hbar\omega_{c1}(n_1+{}^1/_2)-q_z^2/2m_1,\\ \mathscr{E}_2&=\mathscr{E}_2^0+\hbar\omega_{c2}(n_2+{}^1/_2)+q_z'^2/2m_2,\end{aligned} \tag{6}$$

where $\mathscr{E}^0_{(1,2)}$ are the energies of the edges of the bands prior to the application of the magnetic field, $n_{1,2}$ are the numbers of the Landau levels (subbands), $m_{1,2}$ are the effective masses in the bands, q_Z' is the component of the momentum of the hole (electron) along the external magnetic field **H**, and ω_c are the cyclotron frequencies, $\omega_{c1,2} = eH/m_{1,2}c$. The value of (5), neglecting the spatial dispersion, i.e., the dependence on r in (1), was calculated in [5]:

$$\sigma_i=\frac{Ve^2(\mathbf{pb})^2}{|\mathbf{k}|c^2m^2\hbar}\frac{eH}{c\hbar}\left(\frac{2m_1m_2}{m_1+m_2}\right)^{1/2}\sum_n\frac{\Theta(\omega-\omega_n)}{\sqrt{\omega\hbar-\omega_n\hbar}}. \tag{7}$$

Here the crystal volume V is the reciprocal of the electron density, $V = N^{-1}$, in accord with the normalization of the wave functions of the electrons to the crystal volume when the matrix element is calculated. The quantity **p** is the matrix element of the momentum in terms of the wave functions of the edge of the band $u_0(\mathbf{r})$ (the integral over the cell):

$$\mathbf{p}=\int_{V_0}u_{10}^*(\mathbf{r})\frac{\hbar\nabla}{i}u_{20}(\mathbf{r})\,d\mathbf{r} \tag{8}$$

and the resonant energy $\hbar\omega_n$ is

$$\hbar\omega_n=\mathscr{E}_g+(n+{}^1/_2)(\omega_{c1}+\omega_{c2})\hbar, \tag{9}$$

where $\mathscr{E}_g$ is the width of the gap in the absence of a magnetic field, $\mathscr{E}_g = \mathscr{E}_2^0 - \mathscr{E}_1^0$. In expression (7), $\Theta(\omega-\omega_n)$ is the step function equal to unity for positive arguments and to zero for negative arguments, thus reflecting the fact that the transition of the electron from the Landau subband numbered n in the valence band to the subband with the same number in the conduction band is possible only when the photon energy exceeds the threshold value (9). The photon energy in excess of the threshold value is consumed in the kinetic energy of the free motion of the electron and the hole in opposite directions along the magnetic field (see (6)) without a displacement of their mass center. The presence of a singular threshold behavior in (7) is connected with the peculiarities of the electron spectrum and the magnetic field, when the motion along **H** is free, but in the plane perpendicular to **H** is quantized with respect to one degree of freedom and degenerate with respect to the other. It can be assumed that the inverse square root dependence in (7) remains in force also in the case of deviation of the band (6) from parabolicity. The transition of the electron from one subband to the other forms the mechanism of photon absorption, for in this case the photon gets knocked out from the beam. At $\omega < \omega_n$, this process is blocked and the photons are not absorbed, i.e., we have a transparency region.

The contribution of each term of (7) to the polarization operator is

$$\Pi_n=\frac{1}{4\pi c^2}\frac{e^2(\mathbf{pb})^2}{m^2\hbar}\frac{eH}{c\hbar}\left(\frac{2m_1m_2}{m_1+m_2}\right)^{1/2}\frac{1}{\sqrt{\hbar\omega_n-\hbar\omega}}. \tag{10}$$

Indeed, (10) is an analytic function of ω with a cut along a part of the real axis at $\omega \geq \omega_n$.[1)] The discontinuity of this function on the cut is equal to double its imaginary part, and the latter coincides with the result of substituting in (4) one term of (7). When deciding on the common sign in (10), we took into account the circumstance [6] that, if real ω in (10) are taken to be the limits on the upper half-plane, we should have the imaginary part of ϵ positive at positive ω. Taking in (10) only one term from the entire sum over n, we aimed at considering subsequently only the values $\omega \sim \omega_n$ at which the given term predominates over the remaining terms and over the sum itself. For this reason we need not worry about resolving the polynomial uncertainty that results when the analytic function is reconstructed from its discontinuity on the cut[2)].

Substituting (10) in the dispersion equation (2), we see that in the vicinity of the cyclotron resonance $\omega \sim \omega_n$, k^2 should also become infinite because Π becomes infinite. This remark makes unwarranted, a posteriori, the neglect of the spatial dispersion in the calculation of Π, since we have just verified that the presence of cyclotron resonance distorts the dispersion law so strongly that **k** becomes much larger than ω/c. But a large value of **k** means a rapid spatial variation of the field of the wave (1), and allowance for the spatial dispersion becomes essential.

4. Spatial dispersion introduces in (10) the following modification (the calculations are given in the Appendix):

$$\Pi_{n_1n_2}=\frac{\beta\gamma}{4\pi c^2}I^2_{n_1n_2}\left(\frac{k_\perp^2}{2\gamma}\right)R^{-1/2}_{n_1n_2}(\omega,k_z^2). \tag{11}$$

Here

$$\begin{aligned}R_{n_1n_2}(\omega,k_z^2)&=\hbar\omega_{n_1n_2}-\hbar\omega+k_z^2\hbar^2/2(m_1+m_2),\\ \hbar\omega_{n_1n_2}&=\mathscr{E}_g+\hbar(n_1+{}^1/_2)\omega_{c1}+\hbar(n_2+{}^1/_2)\omega_{c2}\end{aligned} \tag{12}$$

and we use the following notation: $\gamma = eH/c\hbar = r_c^{-2}$ (r_c is the radius of the cyclotron orbit), $\beta = e^2(\mathbf{p}\cdot\mathbf{b})^2 \times (2m^*)^{1/2}(m^2\hbar)^{-1}$, and the reduced effective mass is $m^* = m_1m_2/(m_1 + m_2)$,

$$I_{n_1n_2}(\alpha)=\left(\frac{n_1!}{n_2!}\right)^{1/2}\exp\left(-\frac{\alpha}{2}\right)\alpha^{(n_1-n_2)/2}L_{n_1}^{n_2-n_1}(\alpha),$$

where $L_n^l(\alpha)$ is a Laguerre polynomial. In the foregoing formulas, $n_{1,2}$ are the numbers of the subbands between which the transition takes place, and generally speaking $n_1 \neq n_2$, i.e., when account is taken of the spatial dispersion, no selection rules appear with respect to the principal quantum number. $k_\perp$ and k_z are the components of the wave vector of the photon across and along **H**, respectively.

The vanishing of R provides us with a dispersion law for the particle-hole pair

$$\omega_{n_1n_2}+\frac{k_z^2\hbar}{2(m_1+m_2)}=\omega,$$

in which one can see the kinetic energy of the motion of the pair as a whole along the magnetic field. The independence of the pair energy $k_\perp$ is, in final analysis, the consequence of the degeneracy of the energies of the Landau levels relative to the position of the center of the orbit.

We shall henceforth deal with the vicinity of the first resonance $n_1 = n_2 = 0$. Substituting the corresponding expression for Π_{00} from (11) in (2), we obtain the dispersion equation of the normal transverse waves near the cyclotron resonance:

$$\omega^2-c^2k_\perp^2-c^2k_z^2+\beta\gamma\exp\left\{-\frac{k_\perp^2}{2\gamma}\right\}\left(\hbar\omega_{00}-\hbar\omega+\frac{k_z^2\hbar^2}{2(m_1+m_2)}\right)^{-1/2}=0. \tag{13}$$

The region of applicability of this equation is determined only from the requirement that we be much closer to the

chosen transition than to the neighboring transition (we have in mind the fact that the numerical coefficient in front of the reciprocal square root, as well as the exponential factor in (11), are common for all n_1 and n_2):

$$0<R_{00}(\omega,\ k_z^2)\ll\hbar\min\{\omega_{c1},\ \omega_{c2}\}. \tag{14}$$

The left-hand side of this inequality limits the region of transparency (reality of Π), the only region with which we shall deal henceforth, being interested in propagation and not in the absorption of the electromagnetic wave.

5. Equation (13) determines ω as a function of two variables, $k_\perp$ and k_z. At fixed k_z, the polarization part of Eq. (13) tends to infinity as ω approaches the threshold value $\omega_{00}+k_z^2\hbar/2(m_1+m_2)$. Therefore, the value of $k_\perp^2$ corresponding to this value of the frequency ω also tends to infinity by virtue of (13). By the same token, the $\omega(k_\perp^2)$ curve becomes gently sloping and the group velocity of the electromagnetic wave in a direction perpendicular to the external magnetic field $\partial\omega/\partial k$ tends to zero. Formally, by virtue of (13), we have

$$v_\perp^{gr}=\left(\frac{\partial\omega}{\partial k_\perp}\right)_{k_z}=2k_\perp c^2\frac{k^2(2\gamma)^{-1}-1}{-2\omega+c^2k^2\hbar(2R_{00})^{-1}}, \tag{15}$$

where $k^2=(\omega/c)^2-\mathbf{k}^2$ is the four-dimensional momentum squared. By virtue of condition (14), the first term in the denominator can be neglected:

$$v_\perp^{gr}=4k_\perp R_{00}\hbar^{-1}\left(\frac{1}{2\gamma}-\frac{1}{k^2}\right)=4k_\perp\frac{\beta^2\gamma^2}{\hbar c^4k^4}\exp\left\{-\frac{k_\perp^2}{\gamma}\right\}\left(\frac{1}{2\gamma}-\frac{1}{k^2}\right). \tag{16}$$

It is now clear that the group velocity (16) tends to zero as $\omega\to\omega_{00}+k_z^2\hbar/2(m_1+m_2)$, and by the same token as $k_\perp\to\infty$ when k_z is limited by virtue of (13).

We now consider the group velocity in the direction along the magnetic field. At fixed $k_\perp$, the solution of (13) $\omega(k_z^2)$ in the transparency region tends asymptotically (as $\omega\to\infty$ and $k_z^2\to\infty$) to the straight inclined line $\omega=\omega_{00}+k_z^2\hbar/2(m_1+m_2)$, and therefore the group velocity $\partial\omega/\partial k_z$ tends to $k_z\hbar/(m_1+m_2)$, increasing without limit. Formally we have

$$v_z^{gr}=\left(\frac{\partial\omega}{\partial k_z}\right)_{k_\perp}=k_zc^2\frac{2-k^2\hbar^2(2R_{00}(m_1+m_2))^{-1}}{2\omega-c^2k^2\hbar(2R_{00})^{-1}} \tag{17}$$

$$\cong k_z\left(\frac{\hbar}{m_1+m_2}-\frac{4R_{00}}{k^2\hbar}\right)\to\frac{k_z\hbar}{m_1+m_2}.$$

We have again neglected here the first term in the denominator, by virtue of (14). Thus, the group velocity along $\mathbf{H}$ tends to a finite limit at limited k_z, and tends formally to infinity if $k_\perp$ is limited. (The latter circumstance should not shock us, since it is the consequence of the assumed nonrelativistic description (6) of the charged particles, which in turn leads to a patently nonrelativistic dispersion law for the pair, $\omega=\omega_{00}+k_z^2\hbar/2(m_1+m_2)$.)

We now consider the refraction problem, when the electromagnetic wave is incident at an angle θ from vacuum on a semiconductor surface oriented parallel to the magnetic field (the angle between the direction of the wave front and the z axis is acute, $\pi/2-\theta$). In this case, on entering the medium, the component k_z is conserved (but $k_\perp$ is not). In vacuum, the relation $k_z=|\mathbf{k}|\sin\theta=(\omega/c)\sin\theta$ is satisfied. By virtue of the conservation of k_z and ω, we can substitute $k_z=(\omega/c)\sin\theta$ in (13), (16), and (17). Equating to zero the expression $R_{00}(\omega,\ (\omega/c)\sin\theta)$ and solving the quadratic equation, we obtain the resonant value of the frequency as a function of the angle of incidence:

$$\omega_{res}\approx\omega_{00}\left(1+\frac{\omega_{00}\hbar\sin^2\theta}{2(m_1+m_2)c^2}\right). \tag{18}$$

The second of the roots of the quadratic equation is large, and we shall not consider it. The second term in (18) is small in comparison with the first, and its higher powers have been discarded.

When ω approaches the threshold value (18) we have $k_z=(\omega_{res}/c)\sin\theta$, and $k_\perp^2\to\infty$ in accordance with (13). The group velocity (16) tends to zero, while (17) tends to the finite limit:

$$v_\perp^{gr}=\frac{\omega_{res}}{c}\sin\theta\frac{\hbar}{m_1+m_2}\approx10^{-5}c\sin\theta. \tag{19}$$

Thus, the electromagnetic wave is rotated in the direction of the magnetic field (or in the opposite direction if $\theta<0$), provided that the initial angle between its propagation direction (in vacuum) and the magnetic field is $(\pi/2)-\theta$.

Owing to spatial dispersion along the magnetic field (allowance for the dependence on k_z), the refracted ray therefore deviates away from the normal on going into the optically denser medium. As to the phase velocity $\mathbf{v}^{ph}=\omega\mathbf{k}/\mathbf{k}^2$, both its components tend to zero, but in such a way that the front-propagation angle $\theta_{ph}=\tan^{-1}(k_z/k_\perp)$ also tends to zero, i.e., the phase velocity tends in the limit to be orthogonal to the group velocity.

For the tangent of the angle of refraction (with respect to the group velocity) we have in accordance with (13), (15), and (17)

$$\operatorname{tg}\theta_{gr}'=\frac{v_z^{gr}}{v_\perp^{gr}}=\operatorname{tg}\theta_{ph}'\frac{1+\hbar^2\beta\gamma\exp\{-k_\perp^2/2\gamma\}(4c^2(m_1+m_2))^{-1}R_{00}^{-1/3}}{1+\beta\exp\{-k_\perp^2/2\gamma\}(2c^2)^{-1}R_{00}^{-1/3}}, \tag{20}$$

with $R_{00}\approx\hbar(\omega_{res}-\omega)$. The second term in the denominator of (20), if one traces its evolution, is due to the spatial dispersion across the field (allowance for the dependence on $k_\perp$ contained in the exponential of (13)). At sufficiently large values of the field, say $H=5\times10^4$ G, the exponential factor in (12) or (13) differs little from unity up to values $|k_\perp|\geq15\,\omega_{res}/c$ (we assume $\hbar\omega_{res}=1$ eV). The influence of the dependence on k_z comes into play, as we shall show presently, at much smaller $|k_\perp|$, i.e., at a larger distance between ω and ω_{res}. Therefore, discarding the second term in the denominator of (20) and replacing the exponential by unity in (13) we obtain (we put $\omega=\omega_{res}$ throughout, with the exception of the singular quantities)

$$\operatorname{tg}\theta_{gr}'=\frac{1+\gamma\beta\hbar^2(4c^2(m_1+m_2))^{-1}(\hbar(\omega_{res}-\omega))^{-1/3}}{[\cos^2\theta+\beta\gamma\omega_{res}^{-2}[\hbar(\omega_{res}-\omega)]^{-1/3}]^{1/2}}\sin\theta. \tag{21}$$

As cyclotron resonance is approached, $\omega\to\omega_{res}-0$, the refraction angle θ_{gr}' decreases (the usual effect of frequency dispersion) until the spatial dispersion with respect to k_z comes into play, after which it begins to increase. The minimum value of the refraction angle (21) is reached when the deviation from resonance is (in relative units)

$$\left(1-\frac{\omega}{\omega_{res}}\right)_{turn}\approx\left(\frac{5\beta\hbar^2\gamma}{4c^2(m_1+m_2)}\right)^{2/3}\frac{1}{\omega_{res}\hbar}. \tag{22}$$

Assuming $m_1=m_2=m^*$ and taking into account the relation $m^*=2\mathscr{E}_g m^2/(\mathbf{p}\cdot\mathbf{b})^2$, we obtain ultimately for that relative approach to the threshold, starting with which the decrease of the refraction angle gives way to an increase,

$$\left(1-\frac{\omega}{\omega_{res}}\right)_{turn}=\left(\frac{5}{4}\right)^{2/3}\frac{e^2(\mathscr{E}_gH)^{2/3}}{m^*c^2(\mathscr{E}_g+eH\hbar/m^*c)}. \tag{23}$$

At a magnetic field $H=5\times10^4$ G, at an effective mass

$m^* = 0.04m$, and at a forbidden-band width $\mathscr{E}_g = 1$ eV, the value of (23) is 10^{-4}. According to (21) we have $(\tan\theta'_{gr})_{turn} = 0.7\sin\theta$, and the second term in the denominator of (21) is equal to 3, which is much larger than $\cos^2\theta$, and by the same token our neglect of the latter in the derivation of (22) from (21) is justified by the last number. The value of the "transverse refractive index" $n_\perp$, defined as the proportionality coefficient in the formula $|k_\perp| = n_\perp\omega/c$, in the considered approach to the threshold, amounts to $n_\perp = \sqrt{\cos^2\theta + 3}$, which is much less than the value (see above) starting with which the spatial dispersion with respect to $k_\perp$ comes into play.

If we take a field that is 30 times stronger, $H = 1.5 \times 10^6$ G (in this case the cyclotron increment to the resonant energy is $eH\hbar/m^*c \approx 0.45$ eV), then the distance of the turning point (23) from the threshold increases by a factor $(30)^{2/3}/1.5$ and amounts to 7×10^{-4}. The maximum distance at a fixed $\mathscr{E}_g$ is reached at a magnetic field such that $eH\hbar/m^*c = 2\mathscr{E}_g$, i.e., at $\mathscr{E}_g \sim 1$ eV it is convenient to increase H by another four times, to 6×10^6 G. This, however, leads to only a negligible gain: $(1 - \omega/\omega_{res})_{turn}$ reaches 8.3×10^{-4}.

Increasing $\mathscr{E}_g$ simultaneously with H while satisfying the relation $eH\hbar/m^*c = 2\mathscr{E}_g$, we see that the value of (23) increases like $\mathscr{E}_g^{1/3}$. For example, for $\mathscr{E}_g = 10$ eV and $H = 6 \times 10^7$ G we obtain $(1 - \omega/\omega_{res})_{turn} = 1.7 \times 10^{-3}$, i.e., in energy units, the distance from the turning point to the resonance is on the order of 10^{-2} eV. The corresponding values of H are already at the limit of applicability of the effective-mass approximation, since the dimensions of the cyclotron orbit (more accurately, the period of the envelope of the wave function of the electron (A.5)) become comparable with the dimension of the unit cell. Further increase of (23) can be attained by choosing the effective mass. When the latter is decreased, it is possible to decrease the required value of H in proportion, and then (23) decreases in proportion to $(m^*)^{-1/3}$.

Thus, a material with the largest width of the forbidden band $\mathscr{E}_g$ and smallest effective mass m^* must be chosen in order to extend the energy region in which there is an appreciable specific effect of spatial dispersion in k_z, which consists of a deflection of the electromagnetic wave in a direction parallel (antiparallel) to the external magnetic field, and in order to make the width of this region much larger than the smearing of the edge of the band. At the same time, the magnetic field chosen should be $H \lesssim 2\mathscr{E}_g m^*c/e\hbar$. It must be borne in mind, however, that the values of the magnetic field for which our analysis can be strictly speaking valid are limited not only by the aforementioned equalization of the orbit radius with the lattice constant, but even prior to that by the fact that the cyclotron increment to the energy in the ground state, $(1/2)eH\hbar/m^*c$, exceeds those values for which the quadratic dependence of the electron (hole) energy on the quasimomentum remains in force (prior to the application of the magnetic field). In any case, the cyclotron increment must not exceed the width of the conduction band. Nonetheless, the foregoing considerations (see the text between formulas (9) and (10)) suggest that deviations from parabolicity are not critical for the qualitative aspect of the effect, and should lead only to a deviation of the dispersion of the pair from the quadratic (the radicand in (11)). In any case, it goes without saying that the fact that the existing theoretical analysis is limited to certain ranges of the field H cannot be the reason for refusing to study these values of H in the experiment.

From among the effects of spatial dispersion in a direction perpendicular to the external magnetic field, we point to the appearance of an infinite number of so-called new waves (see [3]). Indeed, owing to the periodicity of the exponential in expression (13), the latter has an infinite set of complex solutions $k_\perp^2(\omega, k_z^2)$ (cf. [1,2]).

In conclusion, the authors consider it their duty to note that the starting point of this research was a remark made to one of us by V. L. Ginzburg, that it is possible and advantageous to apply to solids the results obtained earlier [1,2] for vacuum (see Sec. 1 of this paper). The authors are also grateful to Yu. V. Kopaev and L. V. Keldysh for discussions during the initial and concluding stages of the work, respectively.

APPENDIX

We calculate here the contribution of the individual cyclotron term to the polarization operator (11) with allowance for spatial dispersion.

According to [3], in the isotropic case, the general formula for the dielectric constant is

$$\varepsilon(\omega, k) - 1 = \varepsilon_0 + \frac{16\pi}{\hbar V E_0^2}\sum_m \frac{|F_{12}^m|^2}{\omega - \omega_{12}^{(m)} + i0}. \qquad (A.1)$$

Here ϵ_0 are nonsingular terms that are immaterial to us at $\omega = \omega_{12}^{(m)}$, and we have written out in detail the last term of formula (12.8) of the monograph [3], which corresponds to the positive-frequency ($e^{i\omega t}$) part of the perturbing electromagnetic wave; $F_{12}^{(m)}$ is the matrix element of the transition of the electron from the valence band 1 to the conduction band 2, which depends on the set of the quantum numbers m; $\omega_{12}^{(m)}$ is the transition frequency; E_0 is the amplitude of the electric field of the wave (1), specified by the vector potential

$$\mathbf{A} = \frac{cE_0}{\omega}\mathbf{b}\sin(\omega t - \mathbf{kr}). \qquad (A.2)$$

The perturbing term in the Hamiltonian, due to the wave (A.2), is given in the lowest-order approximation by (cf. [5])

$$\mathscr{H}' = \frac{e}{m}\left(\hat{\mathbf{p}} + \frac{e\mathbf{A}_{ext}}{c}\right)\frac{\mathbf{A}(\mathbf{r}, t)}{c}. \qquad (A.3)$$

The arrangement of the factors in (A.3) is immaterial, inasmuch as the periodic parts of the wave functions of the edges of the different bands are orthogonal:

$$\int_{V_0} u_{10}^*(\mathbf{r})\, u_{20}(\mathbf{r})\, d\mathbf{r} = 0.$$

For the same reason, the vector potential of the external magnetic field $\mathbf{A}_{ext}$, which is a slow (linear) function of the spatial coordinates, actually drop out of (A.3). By the same token, the dielectric constant acquires a tensor contribution only in the form $p_i p_j$, where p_i is the component of the vector (8). For a cubic crystal this tensor is isotropic: $p_i p_j \sim \delta_{ij}$.

In (A.1), $F_{12}^{(m)}$ should be taken in the form of the matrix element (we are interested only in the positive-frequency part of (A.2))

$$F_{12}^{(m)} = \frac{eE_0}{2im\omega}\int \psi_{n_1}^*(\mathbf{r})\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}_{ext}\right)\mathbf{b}e^{-i\mathbf{kr}}\psi_{n_2}(\mathbf{r})\, d\mathbf{r}, \qquad (A.4)$$

where the wave functions ψ_{nj} are made up of periodic functions of the edge of the band number j multiplied by the solution of the Schrödinger equation (without spin) in

a magnetic field **H** directed along the z axis. The eigenfunctions of this equation are determined by three quantum numbers: principal n, free-motion momentum q_z, and momentum q_y with respect to which the energy (6) is degenerate:

$$\psi_{n,q_z,q_y}(\mathbf{r}) = \mathcal{N} \exp\left\{\frac{i}{\hbar}(yq_y + zq_z) - \frac{\chi^2}{2}\right\} H_n(\chi) u_{j0}(\mathbf{r}). \qquad (A.5)$$

Here $\chi = (x - x_0)\sqrt{\gamma}$, $x_0 = q_y/\gamma\hbar$ is the center of the orbit, H_n is a Hermite polynomial, and u_{j0} are the periodic parts of the wave functions of the edges of the bands, normalized to the volume of the unit cell. The wave functions (A.5) are normalized to unity over the volume of the crystal, and in this case $\mathcal{N} = (\gamma/\pi)^{1/4}/(L_y L_z 2^n n!)^{1/2}$, where L_y and L_z are the linear dimensions of the crystal. The integrals with respect to dy and dz in (A.4) lead to the momentum conservation laws, and the integral with respect to dx was calculated in[7]. We obtain (we put henceforth $\eta = (q' - q)/\hbar$)

$$F_{12}^{(m)} = F_{n_1 q_z' q_y'}^{n_2 q_z q_y} = \frac{eE_0(\mathbf{pb})}{2mi\omega} \delta_{k_y,\eta_y} \delta_{k_z,\eta_z} I_{n_1 n_2}\left(\frac{k_\perp^2}{2\gamma}\right) \exp\{-i\rho + i\lambda(n_2 - n_1)\}, \qquad (A.6)$$

where $I_{n_1 n_2}$ is the same expression as in (11), $\rho = k_x(2q_y + k_y)/2\gamma$, $\lambda = \tan^{-1}(k_x/k_y)$, and $\mathbf{p}\cdot\mathbf{b}$ is given by formula (8).

Substituting (A.6) in (A.1), summing over q_y', and making the substitution

$$\sum_{q_y/\hbar} \to \frac{L_y}{2\pi} \int_{-L_x\gamma/2}^{L_x\gamma/2} d\left(\frac{q_y}{\hbar}\right) = \frac{L_y L_x \gamma}{2\pi}, \qquad \sum_{q_z/\hbar} \to \frac{L_z}{2\pi} \int_{-\infty}^{\infty} d\left(\frac{q_z}{\hbar}\right), \qquad (A.7)$$

we obtain (we omit ϵ_0 as unnecessary)

$$\varepsilon \cong \frac{e^2\gamma(\mathbf{pb})^2}{\pi\hbar m^2\omega^2} \sum_{n_1,n_2} I_{n_1 n_2}^2 \sum_{q_z'/\hbar} \delta_{k_z,\eta_z} \left(\omega + \frac{\mathscr{E}_1}{\hbar} - \frac{\mathscr{E}_2}{\hbar} + i0\right)^{-1} d\left(\frac{q_z}{\hbar}\right). \qquad (A.8)$$

Here $\mathscr{E}_1$ and $\mathscr{E}_2$ are given by formulas (6). Introducing the notation

$$\xi = \frac{q_z + q_z'}{\hbar}, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \nu = \frac{m_1 m_2}{m_1 - m_2}$$

and summing over $q_z'/\hbar$ in (A.8), we can represent the integral in (A.8) in the form

$$J = -\frac{4\mu}{\hbar} \int_{-\infty}^{\infty} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)} \qquad (A.9)$$

where ξ_1 and ξ_2 are the roots of the quadratic trinomial ($\omega_{n_1 n_2}$ is given by (12)):

$$\xi^2 + 2\frac{\mu}{\nu} k_z \xi + k_z^2 - \frac{8\mu}{\hbar}(\omega - \omega_{n_1 n_2} + i0) = 0,$$

namely:

$$\xi_{1,2} = -\frac{\mu}{\nu} k_z \pm \frac{2}{\hbar}[2\mu(-R_{n_1 n_2}(\omega, k_z^2) + i0)]^{1/2}, \qquad (A.10)$$

where the expression for $R_{n_1 n_2}$ is the same as in (11). When the photon frequency exceeds the transition frequency, the root ξ_1 (the upper sign in (A.10)) is shifted by i0 into the upper half-plane, while ξ_2 is shifted into the lower half-plane. Closing the integration contour in (A.9) by an infinitely remote semicircle in the upper half-plane, we have

$$J = \pi\sqrt{2\mu}\left[\mathscr{E}_g + \hbar\omega_c\left(n_1 + \frac{1}{2}\right) + \hbar\omega_{c2}\left(n_2 + \frac{1}{2}\right) + \frac{k_z^2\hbar^2}{2(m_1 + m_2)} - \omega\hbar - i0\right]^{-1/2}. \qquad (A.11)$$

We have taken into account here the fact that $\sqrt{-z - i0} = i\sqrt{z + i0}$ if the z plane is cut from zero to the right. Substituting (A.11) in (A.8) and recalling the relation $(\epsilon - 1)\omega^2/4\pi c^2 = \Pi$, we obtain (11).

[1)] In fact, there is also a symmetrically-situated branch point $\omega = -\omega_n$. We do not need the corresponding terms in (10).

[2)] When (10) is summed over n, a divergence is produced. To renormalize the polarization operator, it suffices to subtract from it the divergent constant $\sum_n \omega_n^{-1/2}$ and to add an arbitrary polynomial in ω, with coefficients that are fixed by choosing the subtraction points.

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