Resonance scattering of a powerful electromagnetic pulse by a quantum system

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General expressions are obtained for the spectra of resonance Raman scattering and resonance fluorescence excited by a powerful Lorentz pulse. It is shown that for a short pulse the spectra consist of three and four lines, respectively. It is also shown that the well-known Weisskopf-Wigner formula for resonance scattering of an electromagnetic pulse with a broad spectrum is the first term in the expansion of an oscillating function of the field intensity in a power series.

The problem of resonant scattering of electromagnetic radiation was investigated by Weisskopf and Wigner immediately after the inception of quantum mechanics. Their results are very well known.[1] On incidence of an electromagnetic pulse with a broad spectrum on a narrow atomic level, photons chiefly of the characteristic atomic frequencies are observed in the scattered radiation. But if the atomic level has a large width, the spectrum of the scattered radiation is determined by the spectrum of the incident radiation. These classical results were reconsidered rather recently after the creation of lasers, when the problem of the scattering of very powerful electromagnetic pulses became a timely one. The results obtained, especially those of Rautian and Sobel'man,[2] pertain principally to scattering of narrow (in the limit, monoenergetic) fields, and their interpretation is based on a representation of the splitting of quantum levels in a monoenergetic external field.

Subsequently, the results of [2] have been repeatedly improved and generalized, in particular, to the case of a field with randomly varying parameters, and have also been applied to the consideration of several noncoherent processes (see [3-6] and the literature cited in them).

However, even in the case of weak fields, the question as to the role of the width of the spectrum of the incident coherent radiation is decisive. It is therefore of interest to clarify this question for strong fields, to which the present paper is devoted. The treatment is carried through for a pulse with a Lorentzian spectrum; however, the qualitative results that are obtained do not depend on the form of the spectrum.

1. TWO-LEVEL SYSTEM IN THE FIELD OF A LORENTZIAN PULSE

Let a quantum system (in what follows, we shall speak of an atom, for brevity) that was in state 1 at $t=-\infty$, interact with a field

$$\mathbf{E}(\mathbf{r}, t)=\mathbf{E}_0 e^{-\lambda|t|}\cos(\mathbf{kr}-\omega t), \tag{1}$$

and let there be a state 2 in the atom such that $\omega_{21}\sim\omega$. In this case, states 1 and 2 will be strongly disturbed at a sufficiently large E_0.

To find the wave function of the atom in the field, it is necessary to solve the set of equations

$$\begin{aligned}\dot{a}_1&=iV\exp\{-\lambda|t|-i\delta t\}a_2,\\ \dot{a}_2&=-\gamma_2 a_2+iV^*\exp\{-\lambda|t|+i\delta t\}a_1,\end{aligned} \tag{2}$$

where $a_{1,2}(t)$ are the probability amplitudes of finding the atom in states 1 and 2, $\delta\equiv\omega_{21}-\omega$, $V=\frac{1}{2}E_0 d_{12}$, d_{12} is the matrix element of the interaction of the atom, which can be regarded as real, $2\gamma_2$ is the probability of spontaneous decay of state 2, the quantity γ_1 is set equal to zero, i.e., it is assumed that level 1 is a ground or metastable state; in this study we use a system of units in which $c=\hbar=1$.

A solution of Eqs. (2) for $t>0$ is given in [7]. By considering this set of differential equations as a matrix equation, we obtained the following formulas for the matrix $\mathbf{U}$ of the fundamental solution in [7]: 1)

$$\mathbf{U}(t)=\begin{pmatrix} a_1^{(1)} & a_1^{(2)} \\ a_2^{(1)} & a_2^{(2)}\end{pmatrix} \tag{3}$$

$$\begin{aligned}
a_1^{(1)}(t\geqslant 0)&=B[J_{1-\nu}(\varkappa)J_\nu(\varkappa_t)+J_{\nu-1}(\varkappa)J_{-\nu}(\varkappa_t)],\\
a_2^{(1)}(t\geqslant 0)&=iBe^{i\delta t}[J_{1-\nu}(\varkappa)J_{\nu-1}(\varkappa_t)-J_{\nu-1}(\varkappa)J_{1-\nu}(\varkappa_t)],\\
a_1^{(2)}(t\geqslant 0)&=iB[J_\nu(\varkappa)J_{-\nu}(\varkappa_t)-J_{-\nu}(\varkappa)J_\nu(\varkappa_t)],\\
a_2^{(2)}(t\geqslant 0)&=Be^{i\delta t}[J_\nu(\varkappa)J_{1-\nu}(\varkappa_t)+J_{-\nu}(\varkappa)J_{\nu-1}(\varkappa_t)].
\end{aligned} \tag{4}$$

Here

$$\begin{aligned}&\nu=\frac{\lambda+\gamma_2+i\delta}{2\lambda},\quad \varkappa_t=\frac{V}{\lambda}e^{-\lambda|t|},\quad \varkappa=\varkappa_0=\frac{V}{\lambda},\\ &B=\frac{\pi\varkappa}{2\sin\pi\nu}\exp\left\{-\frac{1}{2}\lambda|t|-\frac{1}{2}(\gamma_2+i\delta)t\right\};\end{aligned} \tag{5}$$

J_ν is the Bessel function.

At $t=0$ the matrix $\mathbf{U}$ becomes unitary and has a simple physical meaning: if the values $a_1(0)$ and $a_2(0)$ are known, then

$$\mathbf{a}(t>0)=\mathbf{U}(t)\mathbf{a}(0),\quad \mathbf{a}=\begin{pmatrix}a_1\\ a_2\end{pmatrix}.$$

The latter circumstance is reflected in the notation of Eqs. (4): $a_k^i(t)$ is the probability amplitude of finding the atom in the state k at a time $t>0$ if it was in state i at $t=0$ with unit probability.

The fundamental solution of (3) for $t\leq 0$ can be obtained from (4) by changing the sign of the quantity λ:

$$\begin{aligned}
a_1^{(1)}(t\leqslant 0)&=B[J_\nu(\varkappa)J_{1-\nu}(\varkappa_t)+J_{-\nu}(\varkappa)J_{\nu-1}(\varkappa_t)],\\
a_2^{(1)}(t\leqslant 0)&=-iBe^{i\delta t}[J_\nu(\varkappa)J_{-\nu}(\varkappa_t)-J_{-\nu}(\varkappa)J_\nu(\varkappa_t)],\\
a_1^{(2)}(t\leqslant 0)&=-iB[J_{1-\nu}(\varkappa)J_{\nu-1}(\varkappa_t)-J_{\nu-1}(\varkappa)J_{1-\nu}(\varkappa_t)],\\
a_2^{(2)}(t\leqslant 0)&=Be^{i\delta t}[J_{\nu-1}(\varkappa)J_{-\nu}(\varkappa_t)+J_{1-\nu}(\varkappa)J_\nu(\varkappa_t)].
\end{aligned} \tag{6}$$

Formulas (4), (6) will be used below for construction of solutions to the set of inhomogeneous equations of the type (2).

The particular solution of (2) which satisfies the initial conditions $a_1(-\infty)=1$, $a_2(-\infty)=0$ can be obtained directly. It is easy to prove that at $t\leq 0$

$$a(t\leqslant 0)=\left(\frac{\varkappa}{2}\right)^{1-\nu}\Gamma(\nu)\exp\left\{\frac{(\lambda-\gamma_2-i\delta)t}{2}\right\}\begin{pmatrix} J_{\nu-1}(\varkappa_t) \\ ie^{i\delta t}J_\nu(\varkappa_t)\end{pmatrix}. \tag{7}$$

The solution for $t>0$ can be obtained with the help of (3)–(5):

$$a(t>0)=B\Gamma(\nu)\left(\frac{\varkappa}{2}\right)^{1-\nu} \times\begin{pmatrix} -\alpha_\nu(\varkappa)J_{-\nu}(\varkappa_t)+\beta_\nu(\varkappa)J_\nu(\varkappa_t) \\ ie^{i\delta t}[\alpha_\nu(\varkappa)J_{1-\nu}(\varkappa_t)+\beta_\nu(\varkappa)J_{\nu-1}(\varkappa_t)]\end{pmatrix}, \tag{8}$$

$$\alpha_\nu(\varkappa)=J_\nu^2(\varkappa)-J_{\nu-1}^2(\varkappa),\quad \beta_\nu(\varkappa)=J_{1-\nu}(\varkappa)J_{\nu-1}(\varkappa)+J_\nu(\varkappa)J_{-\nu}(\varkappa).$$

Formulas (7), (8) determine the behavior of the atom in the field at all t for selected initial conditions.

2. RESONANT RAMAN SCATTERING

We now consider the process shown in Fig. 1. The strong field (1), which is resonant at the 1–2 transition frequency, acts on an atom initially in state 1. A transition is possible from state 2 to state 3 with emission of a photon of frequency Ω.

The equations describing the kinetics of this system have the form

$$\begin{aligned} \dot A_1&=iV\exp\{-\lambda|t|-i\delta t\}A_2,\\ \dot A_2&=-\gamma_2A_2+iV\exp\{-\lambda|t|+i\delta t\}A_1,\\ \dot A_3&=-\gamma_3A_3+ive^{-i\Delta t}A_2.\end{aligned} \tag{9}$$

Here v is the matrix element of the interaction of the atom with a weak field of frequency Ω: $\Delta=\omega_{23}-\Omega$; the terms which describe the excitation of the atom by the weak field are omitted. In contrast to the previous section, the probability amplitudes which take the effect of the weak field into account are denoted here by capital letters. We shall denote by small letters the amplitudes found in the zeroth approximation in the weak field.

Equations (9) can be rewritten in the matrix form in the following way:

$$\dot{\mathbf{A}}=(\mathbf{L}+\mathbf{L}')\mathbf{A},$$

$$\mathbf{L}=\begin{pmatrix} 0 & iV\exp\{-\lambda|t|-i\delta t\} & 0\\ iV\exp\{-\lambda|t|+i\delta t\} & -\gamma_2 & 0\\ 0 & 0 & -\gamma_3\end{pmatrix}, \tag{10}$$

$$\mathbf{L}'=\begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & ive^{-i\Delta t} & 0\end{pmatrix}.$$

Considering the matrix $\mathbf{L}'$ as small in comparison with $\mathbf{L}$, we obtain in the first order of perturbation theory

$$\mathbf{A}(t)=\mathbf{A}_0(t)+\mathbf{Y}(t)\int\limits_{-\infty}^{t}\mathbf{Y}^{-1}(t')\mathbf{L}'(t')\mathbf{A}_0(t')dt', \tag{11}$$

where $\mathbf{Y}(t)$ is the fundamental solution of Eq. (10) for $\mathbf{L}'=0$ and $\mathbf{A}_0$ is the particular solution of this equation determined by the initial conditions.

Inasmuch as the matrix $\mathbf{L}$ has quasidiagonal form, we have

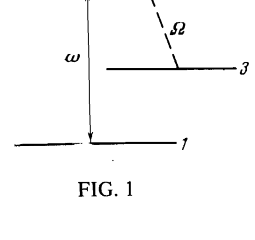

FIG. 1

$$\mathbf{Y}(t)=\begin{pmatrix} \mathbf{U}(t) & 0\\ 0 & \exp(-\gamma_3 t)\end{pmatrix},\qquad \mathbf{A}_0(t)=\begin{pmatrix} a_1(t)\\ a_2(t)\\ 0\end{pmatrix},$$

where $\mathrm{U}(t)$ is determined by Eq. (3) and the quantity $a_{1,2}(t)$ by Eqs. (7), (8). Substituting (3), (4), and (6) in (11), we obtain the following expression for the amplitude $A_3(t)$ after simple transformations:

$$A_3(t)=iv\int\limits_{-\infty}^{t}\exp\{-\gamma_3(t-t')-i\Delta t'\}a_2(t')dt'. \tag{12}$$

Generally speaking, the energy of the photon that is emitted in the transition $2\to 3$ is related to the energies of the photons emitted in other transitions of the incipient cascade (in the general case, the subsequent transitions can also be nonelectromagnetic). Inasmuch as such cascade transitions are not considered in the present study, we neglect the possibility of decay of level 3, i.e., we set $\gamma_3=0$ in (12). The results thus obtained are valid under the condition $\gamma_3\ll\gamma_2$, which holds in the majority of cases of practical interest.

With account of the assumption just made, the probability of emission of a photon of frequency $\Omega=\omega_{23}-\Delta$ is

$$W(\Delta)=|A_3(\infty)|^2=\left|v\int\limits_{-\infty}^{\infty}e^{-i\Delta t}a_2(t)\,dt\right|^2.$$

Substituting Eqs. (7), (8) under the integral, and using Eqs. (A.1), (A.3), and (A.4), we obtain

$$W(\Delta)=\Big|\frac{v}{\lambda}\Gamma(\nu)\left(\frac{\varkappa}{2}\right)^{1-\nu}\{\varkappa^\mu[(\nu-\mu-1)J_\nu(\varkappa)s_{-\mu-1,\nu-1}(\varkappa) \tag{13}$$
$$-J_{\nu-1}(\varkappa)s_{-\mu,\nu}(\varkappa)]+\varkappa^{1-\mu}[(\mu-\nu-1)J_\nu(\varkappa)s_{\mu-2,\nu}(\varkappa)+J_{\nu-1}(\varkappa)s_{\mu-1,\nu-1}(\varkappa)]\}\Big|^2,$$

where $\mu=(\lambda+\gamma_2+2i\Delta-i\delta)/2\lambda$, $s_{\mu,\nu}$ are the Lommel functions.

Expression (13), which determines the spectrum of the emitted photons, is rather complicated. It is therefore sensible to consider some limiting cases.

First of all, we consider the weak-field limit: $V\ll\lambda$. Expanding the Bessel and Lommel functions in series, we obtain

$$W(\Delta)=\frac{4\lambda^2(vV)^2}{(\gamma_2^2+\Delta^2)[\lambda^2+(\delta-\Delta)^2]^2}. \tag{14}$$

Equation (14) represents the result of ordinary perturbation theory for the $1\to 2\to 3$ transition probability [8]. However, it is characteristic that this result is obtained without limitations on the values of γ_2 and δ only upon the assumption of smallness of the interaction in comparison with the spectral width of the exciting radiation. This becomes understandable if we take into account that departures from perturbation theory develop only at long times after the beginning of the interaction. But if the field has a large spectral width, then its amplitude decays before the departures from perturbation theory become significant.

We now consider the case in which the excitation of the atom is produced by wide lines: $\lambda\gg\gamma_2$, δ. In this limit, $\nu\approx\frac{1}{2}$ and we can use the relation (A.5) for the Lommel functions. Also using the known relations for the confluent hypergeometric functions, we can obtain

$$W(\Delta)=\frac{1}{4}\left(\frac{v}{\Delta}\right)^2\Big|e^{i\varkappa}{}_1F_1\left(1;1-i\frac{\Delta}{\lambda};-i\varkappa\right)$$
$$-e^{-i\varkappa}{}_1F_1\left(1;1-i\frac{\Delta}{\lambda};i\varkappa\right)+e^{-i\varkappa}{}_1F_1\left(1;1+i\frac{\Delta}{\lambda};-i\varkappa\right) \tag{15}$$
$$-e^{i\varkappa}{}_1F_1\left(1;1+i\frac{\Delta}{\lambda};i\varkappa\right)\Big|^2.$$

The spectrum of the central part of the Raman scattering line ($\Delta \ll \lambda$) has the form

$$W(\Delta)=\frac{v^2}{\Delta^2}\sin^2\frac{2V}{\lambda}. \qquad (16)$$

Comparing (16) and (14), we see that for $V \ll \lambda$, Eq. (16) transforms into the formula of perturbation theory if we set $\gamma_2 = \delta = 0$ and $\Delta \ll \lambda$ in the latter.

It is interesting to note the singularities which arise in the case of a strong field ($V \gtrsim \lambda$), when the scattering probability becomes an oscillating function of the parameter V/λ. This behavior of the quantity $W(\Delta)$ is explained in the following fashion (see [9]). In the limit considered ($\gamma_2 \ll \lambda$) the exciting pulse passes through a region occupied by an atom during a time that is much less than the lifetime of level 2. Therefore, during the action of the pulse, the atom does not radiate at the frequency ω_{23}, and the subsequent radiation at this frequency is determined by which of the states 1 and 2 is occupied by the atom after departure of the pulse. A photon of the combination frequency is not emitted in the first case, but is one emitted in the second. In turn, as is well known, in a strong external field the two-level system oscillates in time between the two states with a frequency ~V. Therefore, if the action of the field is limited in time to an interval $\sim 1/\lambda$, then the probability of observing the system in one of the two states after the action of the field is an oscillating function of the parameter V/λ.

The considerations that have been given are directly confirmed by simple analysis of Eq. (8). The probability of finding the atom in state 2 after departure of the pulse, at $t \gg 1/\lambda$, is easily obtained from (8) for $\nu = 1/2$:

$$|a_2(t \gg 1/\lambda)|^2 \approx e^{-\gamma_2 t}\sin^2(2V/\lambda).$$

The oscillating factor here is the same as in (16).

Expression (16) has a singularity at $\Delta = 0$, connected with neglect of the quantity γ_2. The obvious way of avoiding this singularity lies in replacing (16) by the expression

$$W(\Delta)=\frac{v^2}{\Delta^2+\gamma_2^2}\sin^2\frac{2V}{\lambda}. \qquad (17)$$

In such a form, Eq. (17) is the direct generalization of the Weisskopf-Wigner results for resonant scattering of an intense pulse with a broad spectrum at a quasi-stationary level.

We now consider the wing of the Raman scattering line, assuming that the external field is strong ($\Delta \gg \lambda$, $V \gg \lambda$). If $\Delta \gtrsim V$ also, we can use the known asymptotic form for the confluent hypergeometric function:[10]

$$ {}_1F_1(a;c;c\xi) =(1-\xi)^{-a}\left[1-\frac{a(a+1)}{2c}\left(\frac{\xi}{1-\xi}\right)^2+O(|c|^{-2})\right], \quad |\xi|<1.$$

Substituting this expression in (15), we get

$$W(\Delta)=v^2V^2\left(\frac{\lambda}{\Delta}\right)^2\left|\frac{e^{i\varkappa}}{(V-\Delta)^3}+\frac{e^{-i\varkappa}}{(V+\Delta)^3}\right|^2. \qquad (18)$$

As is easy to see, Eq. (18) has two sharp maxima at $\Delta = \pm V$. The atom emits at just these frequencies on excitation by a monochromatic field of high intensity.[2] The radiation in this case is of the same nature. The strong field splits level 2 into two sublevels separated from each other by an amount ~2V and emission of a

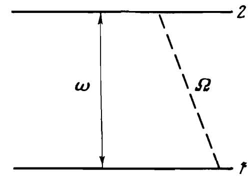

FIG. 2

photon of frequency $\Omega = \omega_{23} \pm V$ takes place from these sublevels with population of level 3. After damping of the field, the emission at frequencies $\Omega = \omega_{23} \pm V$ ceases, leaving only radiation at the unperturbed atomic frequency about which we spoke above.

The case of scattering of a narrow line $\lambda \ll \gamma_2$ was considered in many earlier studies [2-5]. Here emission arises only at the frequencies $\omega_{23} \pm V$.

3. RESONANT FLUORESCENCE

We now consider the process represented in Fig. 2. Photons of a weak field of frequency Ω are emitted in the same transition on which the field acts. The equations describing the kinetics of the system have the form

$$\dot{A}_1=iVe^{-\lambda|t|-i\delta t}A_2+ive^{-i\Delta t}A_2,$$
$$\dot{A}_2=-\gamma_2A_2+iVe^{-\lambda|t|+i\delta t}A_1,$$

where $\Delta = \omega_{21} - \Omega$. The component corresponding to absorption of the weak field is omitted in these equations, inasmuch as the processes of absorption and scattering of the weak field are not considered.

Proceeding as in the previous section, we get for the amplitude A_1 in the first approximation in the weak field

$$A_1(t)=a_1(t)+iv\int_{-\infty}^{t}[a_1^{(1)}(t)a_2^{(2)}(t') -a_1^{(2)}(t)a_2^{(1)}(t')]a_2(t')\exp\{\gamma_2t'-i\Delta t'\}dt'.$$

The spectrum of emitted photons is obviously determined by the equation

$$W(\Delta)=\left|v\int_{-\infty}^{\infty}[a_1^{(1)}(\infty)a_2^{(2)}(t) -a_1^{(2)}(\infty)a_2^{(1)}(t)]a_2(t)\exp\{\gamma_2t-i\Delta t\}dt\right|^2. \qquad (19)$$

Substituting Eqs. (4), (6)–(8) in (19), we get

$$W(\Delta)=\left|\frac{\pi v}{\lambda}\frac{\Gamma^2(\nu)}{\sin\pi\nu}\left(\frac{\varkappa}{2}\right)^{3-2\nu}\{\alpha_\nu(\varkappa)[I_{1-\nu,\nu-1}(i\varepsilon,\varkappa) -I_{-\nu,\nu}(-i\varepsilon,\varkappa)]+\beta_\nu(\varkappa)[I_{\nu-1,\nu-1}(i\varepsilon,\varkappa)+I_{\nu,\nu}(-i\varepsilon,\varkappa)]\}\right|^2, \qquad (20)$$

$$I_{pq}(\alpha,\varkappa)=\int_0^1 x^\alpha J_p(\varkappa x)J_q(\varkappa x)\,dx, \quad \varepsilon=(\Delta-\delta)/\lambda.$$

The integral I_{pq} with real first argument was encountered previously in [7] in a study of stimulated radiation and absorption of an exponentially damped pulse. The expansion of this integral in a power series in the second argument can easily be obtained by power-series expansion of the product of the two Bessel functions with subsequent term-by-term integration:

$$I_{pq}(a,\varkappa) =\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(p+q+2k+1)(\varkappa/2)^{p+q+2k}}{k!\Gamma(p+q+k+1)\Gamma(p+k+1)\Gamma(q+k+1)(p+q+2k+1+a)}. \qquad (21)$$

Using this expression, we can obtain Eq. (14) for the spectrum of resonance fluorescence excited by a weak field, in which now $\Delta=\omega_{21}-\Omega$.

We now consider the same limiting cases as in Sec. 2. If the excitation is produced by a wide line ($\lambda\gg\gamma_2$, δ; $\nu\sim 1/2$), then, proceeding in a fashion similar to that of Sec. 2, we can obtain

$$W(\Delta)=\left|\frac{v}{4\Delta}\left[{}_1F_1\left(1;1-i\frac{\Delta}{\lambda};-2i\varkappa\right)-{}_1F_1\left(1;1-i\frac{\Delta}{\lambda};2i\varkappa\right)\right.\right.$$
$$\left.\left.+{}_1F_1\left(1;1+i\frac{\Delta}{\lambda};-2i\varkappa\right)-{}_1F_1\left(1;1+i\frac{\Delta}{\lambda};2i\varkappa\right)\right]\right|^2. \quad (22)$$

For the intensity of fluorescence at frequencies lying in the center of the line ($\Delta\ll\lambda$), we can easily obtain the relation (17). The physical nature of the oscillations of $W(\Delta)$ in their dependence on V/λ is the same as in Sec. 2.

On the wings of the line ($\Delta\gg\lambda$), using the asymptotic form of the function ${}_1F_1$ written out in Sec. 2, we get

$$W(\Delta)=4v^2V^2\left(\frac{\lambda}{\Delta}\right)^2\left|\frac{1}{(2V+\Delta)^3}+\frac{1}{(2V-\Delta)^3}\right|^2. \quad (23)$$

The maxima at the frequencies $\omega=\omega_{21}\pm 2V$, in correspondence with what was said in Sec. 2, are formed when the strong field splits each level into two sublevels and the atom emits photons of frequencies $\Omega=\omega_{21}\pm 2V$ in transitions from each sublevel of level 2 to each sublevel of level 1. The transitions with emission of photons of frequency ω are found in the center of the line and do not differ from the photons of frequency ω_{21} in the considered approximation ($\lambda\gg\delta$).

We note that resonant fluorescence was studied in a recent paper [11] for a rectangular pulse; the discussion was limited to the case of zero detuning.

Thus, in Raman scattering of intense radiation with a broad spectrum, three maxima appear at the frequencies $\Omega\approx\omega_{23}$, $\omega_{23}\pm V$. The first of these lines corresponds to the characteristic atomic frequency and lasts for a time $\sim 1/\gamma_2$, and its intensity is an oscillating function of the parameter V/λ. The radiation at frequencies $\omega_{23}\pm V$ is connected with the splitting of the atomic levels by the fields; its duration is determined by the duration $\sim 1/\lambda$ of the exciting pulse.

The resonant-fluorescence spectrum excited by the strong field contains four maxima: at the frequencies ω_{21}, ω, $\omega\pm 2V$. The photons of frequency ω, $\omega\pm 2V$ are emitted during a time $\sim 1/\lambda$, and at $\lambda\ll\gamma_2$ the maximum at the frequency ω is twice as high as the lateral maxima.[2] The photons at the frequency ω_{21} are emitted during a time $\sim 1/\gamma_2$ and at $\lambda\gg\gamma_2$ the number of these photons is an oscillating function of the parameter V/λ.

To appraise the possibility of experimental observation of oscillations of the intensity of the central line of Raman or unshifted scattering, we shall have in mind a molecular gas (dipole moment ~1 debye, scattering cross section $\sim 10^{-17}$ cm^2). Narrow lines are observed in such systems—for example, in molecular iodine on excitation by the second harmonic of a neodymium laser[12]; the excited states have a lifetime of $\sim 10^{-7}$ sec [13].

The period of the oscillations is determined by the change in the pump field intensity by an amount $\Delta E_0 \sim 10^2$ V/cm if the pulse length $\sim 10^{-8}$ sec. Therefore, for a "carrier" pump field intensity $\sim 10^3$ V/cm at a focal area of ~ 0.1 cm^2, the necessary energy of radiation of the laser in the pulse is ~0.1 J. In this case, $\sim 10^{12}$ photons will be scattered in one pulse at a pressure of ~1 Torr.

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APPENDIX

We present here some relations for the Lommel functions used in the body of the text.

A Lommel function is defined by the integral representation

$$s_{\mu,\nu}(z)=\frac{\pi}{2\sin\pi\nu}\left[J_\nu(z)\int_0^z x^\mu J_{-\nu}(x)\,dx-J_{-\nu}(z)\int_0^z x^\mu J_\nu(x)\,dx\right], \quad (A.1)$$

and the function $S_{\mu\nu}(z)$ is related to it by

$$S_{\mu\nu}(z)=s_{\mu\nu}(z)+\frac{2^{\mu-1}}{\sin\pi\nu}\Gamma\left(\frac{\mu-\nu+1}{2}\right)\Gamma\left(\frac{\mu+\nu+1}{2}\right)$$
$$\cdot\left[J_{-\nu}(z)\cos\frac{\mu-\nu}{2}\pi-J_\nu(z)\cos\frac{\mu+\nu}{2}\pi\right]. \quad (A.2)$$

Integrating (A.1) once by parts and using the relations

$$\frac{dJ_\nu(z)}{dz}=\pm\frac{\nu}{z}J_\nu(z)\mp J_{\nu\pm1}(z),$$

we obtain

$$(\mu+1-\nu)s_{\mu\nu}(z)=\frac{\pi}{2\sin\pi\nu}\left[J_\nu(z)\int_0^z x^{\mu+1}J_{1-\nu}(x)\,dx+J_{-\nu}(z)\int_0^z x^{\mu+1}J_{\nu-1}(x)\,dx\right]. \quad (A.3)$$

A known indefinite integral for Bessel functions is [6]

$$\int x^\mu J_\nu(x)\,dx=(\mu+\nu-1)xJ_\nu(x)s_{\mu-1,\nu-1}(x)-xJ_\nu(x)s_{\mu,\nu}(x).$$

Applying (A.2) and the formula

$$J_\nu(z)J_{1-\nu}(z)+J_{\nu-1}(z)J_{-\nu}(z)=\frac{2}{\pi z}\sin\pi\nu$$

and taking account of the behavior of $s_{\mu\nu}$ as $z\to 0$:

$$s_{\mu\nu}(z)\sim z^{\mu+1}/[(\mu+1)^2-\nu^2],$$

we obtain

$$\int_0^z x^\mu J_\nu(x)\,dx=(\mu+\nu-1)zJ_\nu(z)s_{\mu-1,\nu-1}(z)-zJ_{\nu-1}(z)s_{\mu,\nu}(z),$$
$$\mathrm{Re}\,\mu>-1. \quad (A.4)$$

When the second index of the Lommel functions is half-integer, these functions can be related to the confluent hypergeometric function, since Bessel functions in the integral representation of (A.1) can be expressed in terms of elementary functions:

$$s_{\mu,\pm1/2}(z)=\frac{iz^\mu}{2\mu+1}\left[{}_1F_1\left(1;\mu+\frac{3}{2};-iz\right)+{}_1F_1\left(1;\mu+\frac{3}{2};iz\right)\right]. \quad (A.5)$$

1)There is an error in [7]; in all the formulas of that paper one must change the common sign of the quantities $a_1^{(2)}$ and $a_2^{(1)}$.

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