A theory of interacting pomerons and hadronic reactions at high energies

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The problem of scattering of particles at ultrahigh energies is considered within the framework of the reggeon diagram technique. The problem reduces to the problem of self-interaction of the pomeron field. Application of strong-coupling methods and of an ϵ expansion allows one to derive the asymptotic behavior of the total cross section and of the amplitude of elastic diffraction scattering. The matching of the solutions obtained here with perturbation theory is investigated. The position of the bare pole is determined for small coupling constants. The solutions obtained here exhibit scale invariance with anomalous dimensions in the ξ,ρ representation, where ξ is the rapidity and ρ is the impact parameter.

INTRODUCTION

In high-energy hadron collisions the produced particles are emitted preferentially in the forward direction. The transverse components $p_\perp$ of their momenta are bounded by a quantity of the order of the particle masses $\mu \sim 300$ MeV/c. This means that the interaction between the hadrons occurs at relatively small impact parameters $\rho \gtrsim 1/\mu$. A mechanism for such an interaction is the multiperipheral particle generation, for which the simplest model has been proposed and studied in detail by Amati, Fubini, and Stanghellini[1]. In this mechanism the density $\rho = dN/dy$ of the produced particles in the space of rapidities $y = \ln(p_{\|}/\mu) \simeq \ln(E/\mu)$ is constant $\rho(y) = \rho_0$ for high energies in the physical region $0 \le y \le \xi$ and also does not depend on the rapidity $\xi = \ln(E_0/\mu)$ of the incident particle. Correspondingly, the average particle number, i.e., the average multiplicity, increases logarithmically with the energy: $N = \rho_0\xi$.

Physically, multiperipheral production corresponds to a process of independent particle production, without any correlations between the magnitudes of their momenta for large relative rapidities. The diffraction scattering amplitude in the multiperipheral model is determined by the Regge formula. It can be considered as a good zero-order approximation, but the interaction with large relative rapidities becomes more and more important with increasing energy. This interaction produces, in particular, nonuniformities of the density of the produced particles in rapidity space. Thus, two particles with rapidities y_1 and y_2 and with $y_1 - y_2 \gg 1$ can produce multiperipherally $(y_1 - y_2)\rho_0$ additional particles in the rapidity interval $y_2 \le y \le y_1$. As a result the particle density in this interval is doubled. It is also possible that these two particles absorb all other particles in the indicated interval of rapidities, and a hole is produced in the distribution density $\rho(y)$.

The corresponding corrections to the Regge formulas are described by the reggeon diagram technique of Gribov[2]. Historically, this technique grew out of an analytic continuation to nonintegral j of the t-channel unitarity condition[3] and the analysis of asymptotic behavior of Feynman diagrams in the s-channel by means of the Sudakov method[4]. The mentioned connection between reggeon diagrams with multiple production processes and the s-channel unitarity condition was established recently in the interesting paper of Abramovskiĭ, Gribov, and Kancheli[5].

The asymptotic behavior of the elastic scattering amplitude is determined by the contribution of the so-called "enhanced" and "unenhanced" (including "semi-enhanced") reggeon diagrams. Some of these are illustrated in Fig. 1. For ultrahigh energies, when the rapidity of the incident particle is large, $\xi = \ln(E/\mu) \gg 1$, the principal role is played only by the enhanced diagrams which contain powers of the quantity $r^2\xi$, where r is the coupling constant for the decay of one reggeon into two others. For $r^2\xi \gtrsim 1$ the contribution of an infinite number of different enhanced diagrams is important and the problem of summing these diagrams arises (this problem is in itself equivalent to a problem of interaction of a nonrelativistic quantized particle field[6]). This problem has been considered by Gribov and one of the present authors[6-8], where two versions for its solution were proposed, consistent with the t-channel reggeon unitarity conditions[3].

The first version (strong coupling) corresponds to a significant change of the trajectory and residue of the Pomeranchuk pole for $r^2\xi \gtrsim 1$ on account of pomeron interaction. In the second version (weak coupling) the effective vertex for the decay of a pomeron into two pomerons (the three-pomeron vertex) vanishes like the square of the reggeon momenta in the region $k_i^2 < r^2$ as $k_i^2 \to 0$. The corresponding corrections to the pomeron Green's functions and to the scattering amplitude in the low-momentum-transfer region turned out to be small as $\xi \to \infty$. Among these versions one was supposed to select the one which satisfies the s-channel unitarity conditions.

The weak-coupling version with a three-pomeron vertex falling off linearly as $k_i^2 \to 0$ seemed at that time to be the most attractive on account of its simplicity (theoretically, in the whole energy region it reduces to simple perturbation theory formulas). However, further development of the theory has shown that the weak coupling can be brought into agreement with s unitarity only at the price of a whole set of artificial additional selection rules for vanishing pomeron momenta. It turned out that one must forbid the scattering of pomerons on pomerons and the decay of a pomeron into three pomerons[6, 7], the emission of particles by pomerons[9], the production of resonances and particle jets on a pomeron[10], etc.

Experiment does not confirm these selection rules. The cross section for the production of resonances on protons at high energies exhibits no tendencies[11] to vanish at zero momentum transfer, i.e., as $-t \simeq p_\perp^2 \to 0$. The three-pomeron vertex $r(p_\perp^2)$ which can be derived[12,13] from the ISR[14] and NAL[11,15] data on inclusive spectra also turned out not to decrease with decreasing $p_\perp$ of the pomerons. The experimental data indicate more readily that as $p_\perp^2 \to 0$ it tends to the constant value $r \simeq 10^{-1}\alpha_p'^{1/2}$ (α_p' is the slope of the vacuum trajectory). Thus, experiment reveals no tendencies for the decrease, corresponding to the weak coupling hypothesis of the inelastic cross sections as $p_\perp^2 \to 0$.

All this gives serious reasons to revise in general the problem of pomeron interactions, dispensing with the weak coupling hypothesis.

The experience accumulated over the past few years in investigating analogous problems, particularly in the theory of phase transitions[16-18], shows that in such cases strong coupling occurs, accompanied by a screening phenomenon, i.e., the suppression of the interaction amplitudes on account of particle repulsion. It was possible to obtain a simple form for the solution of the problems of phase transitions on the basis of the hypothesis of scale-invariance[16,17] and Wilson's ϵ-expansion method[18].

The method of ϵ expansion is applied below to the pomeron interaction, and on its basis we have obtained a simple solution of the problem corresponding both to reggeon strong coupling for $r^2\xi \gtrsim 1$, and to the previously established[8] scale-invariance condition. In the region $r^2\xi \lesssim 1$ the solution joins smoothly with the perturbation theory series and exhibits physically reasonable properties. We have not been able to find a rigorous proof of the s-channel unitarity of this solution, but have convined ourselves that it does not involve those contradictions which, as we have seen, appeared in the weak coupling case.

We shall describe the pomeron interaction below by means of a Lagrangian for which the perturbation theory reproduces exactly the contribution of all the reggeon diagrams of Gribov. We apply the ϵ-expansion method to this Lagrangian and in the first part of the paper we consider the purely theoretical question: what are the consequences of the three-pomeron coupling in the region of strong pomeron coupling for $r^2\xi \gtrsim 1$?

One should not right away that the dimensionless constant r (we measure all quantities in units of α_p' or $\alpha_p'^{1/2}$) may turn out to be—and indeed seems to be—very small; from the experimental data on inclusive spectra[11,14,15] it follows that $r \simeq 1/10$. Therefore the region $r^2\xi \gtrsim 1$, i.e., $\xi = \ln(m_N E/\mu^2) \gtrsim 10^2$ is far beyond the limits of energies that will ever be reached. Nevertheless, for an analysis of the general situation it is very important to understand what happens as $\xi \to \infty$, i.e., what the theoretical asymptotic behavior of the solution is. The region of attainable energies where the situation may be cardinally different from the region $r^2\xi \gtrsim 1$ is briefly discussed in the last section.

1. THE REGGEON DIAGRAM TECHNIQUE AND PERTURBATION THEORY

The reggeon diagram technique has been formulated by Gribov[2] in the $\boldsymbol{\omega}$, $\mathbf{k}$ representation, where $\boldsymbol{\omega} + 1 = j$ is the complex angular momentum and $\mathbf{k}$ is the two-dimensional momentum transfer. This representation is convenient for analyzing the singularities in the j-plane but the general properties of a number of quantities are more easily formulated in the ρ,ξ representation obtained by means of a Fourier-Mellin transform

$$\int e^{ik\rho}\frac{d^2k}{2\pi}\int_\uparrow e^{\omega\xi}\frac{d\omega}{2\pi i}$$

(the integral with the vertical arrow denotes integration along a path parallel to the imaginary axis). The two-dimensional vector ρ has the meaning of an impact parameter, and the rapidity $\xi \simeq \ln(p_{\parallel}/\mu)$ of the particle corresponds to the imaginary "time" $t' = i\xi$.

At very large energy $s = \mu^2 e^\xi$ and small momentum transfers $-t = k^2 \sim 1/\xi$ the most important contribution comes from the so-called "enhanced" diagrams[2] Fig. 1a, constructed from reggeon propagator lines. They begin and end in the t channel in a one-pomeron line and are constructed from three-pomeron vertices. The partial wave $\varphi(j, t)$ of the t-channel is determined by the Green's function of the pomeron: $\varphi(j, t) = g_0^2 G(\omega, k^2)$, where g_0 is the coupling constant for the emission of a pomeron by a particle. The whole scattering amplitude is determined by the well-known Sommerfeld-Watson representation in the form

$$T(\xi, t) = \hat{\eta}_+ \operatorname{Im} T(\xi, t),$$

$$\frac{1}{s}\operatorname{Im} T(s, t) = \int_\uparrow e^{(j-1)\xi}\varphi(j, t)\frac{dj}{2\pi i} = \int e^{ik\rho} g_0{}^2 G(\xi, \rho)\frac{d^2\rho}{2\pi}, \tag{1.1}$$

$$\hat{\eta}_+ = i + \operatorname{tg}\frac{\pi}{2}\frac{\partial}{\partial\xi} \approx i + \frac{\pi}{2}\frac{\partial}{\partial\xi},$$

where $\hat{\eta}_+$ is a signature factor and

$$G(\xi, \rho) = \int_\uparrow e^{\omega\xi + ik\rho} G(\omega, k^2)\frac{d\omega}{2\pi i}\frac{d^2k}{2\pi} = \langle 0|T(\psi(\xi, \rho)\psi(0, 0))|0\rangle \tag{1.2}$$

is the pomeron Green's function in the ξ,ρ representation. In this representation the pomeron is described by the complex wave field

$$\psi(\xi, \rho) = \sum_k a_k e^{ik\rho + \omega\xi}$$

with a non-Hermitian Lagrangian of the form

$$\mathscr{L}(\xi, \rho) = \psi^+ \varepsilon_0(-\nabla_\rho^2)\psi + \frac{ir}{2}\psi^+\psi(\psi + \psi^+), \tag{1.3}$$

where r is the three-pomeron vertex and $\epsilon_0(k^2) = \Delta_0 + k^2$ is the bare pomeron spectrum; its slope $(d\epsilon_0/dk^2)_{k^2=0}$ is by definition taken to be unity, i.e., $k = a_0'^{1/2}p_\perp$, where $p_\perp$ is the transverse momentum in conventional units.

The Lagrangian (1.3) describes the motion of the

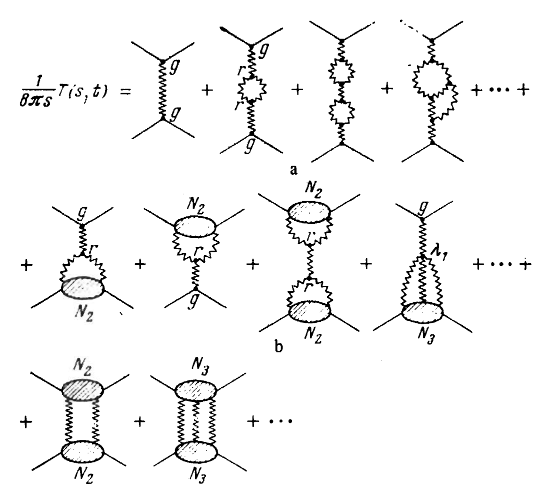

FIG. 1

pomeron in the sense that the perturbation theory derived from it for the cubic (anti-Hermitian) interaction $1/2ir\psi^+\psi(\psi + \psi^+)$ reproduces exactly the contributions of all enhanced Gribov reggeon diagrams to the Green's functions. Such quantum-mechanical quantities as the energy of the ground state or the ψ function have no physical interpretation in this problem. The corrections due to unenhanced diagrams can be taken into account by introducing correction terms into the definition of the partial wave, and will be discussed in the sequel.

The pomeron energy spectrum $\omega = -\epsilon(k^2)$ is determined by the poles of its exact Green's function

$$G(\omega, k^2)=1/[\omega+\varepsilon_0(k^2)-\Sigma(\omega, k^2)] \tag{1.4}$$

in the ω representation (it would be logical to call $E = -\omega = 1 - j$ the pomeron energy; here

$$\Sigma(\omega, k^2)=-\frac{1}{2!}\int_{\dagger}\frac{d\omega'}{2\pi i}\int\frac{d^2k'}{\pi}rG(\omega',\mathbf{k}'^2)G(\omega-\omega',(\mathbf{k}-\mathbf{k}')^2)\times\Gamma(\omega,\mathbf{k};\omega-\omega',\mathbf{k}-\mathbf{k}') \tag{1.5}$$

is the pomeron self-energy and Γ is the three-pomeron vertex part. The spectrum determines the position of the Pomeranchuk pole:

$$\alpha(-k^2)=1-\varepsilon(k^2). \tag{1.6}$$

The parameters r, $\epsilon_0(0) = \Delta_0$ of the Lagrangian have to be chosen such that there be no gap in the spectrum, i.e. that

$$1-\alpha(0)=\varepsilon(0)=0. \tag{1.7}$$

For this it is necessary that the bare spectrum have a gap $\Delta_0 = \epsilon_0(0) \neq 0$, depending on the constant r. The free Green's function, which is obtained when we set $r = 0$ in (1.4), (1.5), corresponds to the contribution of the "bare" Pomeranchuk pole, $G(\omega, k^2) = (\omega + \epsilon_0(k^2))^{-1}$, i.e., to the Regge scattering amplitude

$$\frac{1}{8\pi s}T^{(1)}(s,-k^2)\approx ig^2\exp[-\xi\varepsilon_0(k^2)]. \tag{1.8}$$

We show that at high energies, i.e., at $\xi \gg 1$ and at a small coupling constant r, perturbation theory corresponds to a series expansion in the parameter $r^2\xi$ and that the "bare" gap depends on r in the following manner

$$\Delta_0=\frac{r^2}{4}\ln\frac{r^2}{4}+O(r^2). \tag{1.9}$$

Indeed, the contribution of the simplest loop diagram to $\Sigma(\omega, k^2)$ is obtained by setting $G = G_0$ and $\Gamma = r$ in (1.5)

$\Sigma^{(1)}(\omega, k^2)$ = 

$$=-\frac{r^2}{2}\int\frac{d^2k'/\pi}{\omega+\varepsilon_0(k^2)+\varepsilon_0[(\mathbf{k}-\mathbf{k}')^2]}, \tag{1.10}$$

and the integration with respect to ω' is done by means of residues. For low frequencies and small momenta $\omega \sim k^2 \sim 1/\xi \ll 1$ and for $\Delta_0 \ll 1$; this integral is logarithmically large:

$$\Sigma^{(1)}(\omega, k^2)=-\frac{r^2}{2}\int_0^L\frac{dk'^2}{\omega+2\Delta_0+k^2/2+2k'^2}=-\frac{r^2}{4}\ln\frac{L}{\omega+2\Delta_0+k^2/2}+O\left(\frac{1}{\ln L}\right) \tag{1.11}$$

($L \sim \alpha'_0\mu^2 \sim 1$ is the cutoff radius, corresponding to the bound $p_\perp \lesssim \mu$ on the transverse momenta). This yields the pomeron spectrum in the first order of r^2:

$$\varepsilon^{(1)}(k)=\varepsilon_0-\Sigma^{(1)}(-\varepsilon_0(k),k)=\Delta_0+k^2+\frac{r^2}{4}\ln\frac{L}{\Delta_0-k^2/2}. \tag{1.12}$$

In order that there be no gap in the spectrum one must choose

$$\Delta_0+\frac{r^2}{4}\ln\frac{L}{\Delta_0}=0.$$

To logarithmic accuracy this yields the condition (1.9).

Substituting these values of $\epsilon_0(k^2)$ and $\Sigma \sim \Sigma^{(1)}$ into $G(\omega, k^2)$ we note that allowance for the interaction between the pomerons in first order of r^2 at $\omega \sim k^2$ leads to the appearance in $G^{-1}(\omega, k^2)$ of terms of order r^2/ω (apart from a logarithmic factor). In the ξ, ρ representation this yields terms of the order $r^2\xi$ (since $\omega\xi \sim 1$ in (1.2)). Corrections of the same order are also obtained in the three-pomeron vertex

$$\Gamma(\omega', k'; \omega-\omega', \mathbf{k}-\mathbf{k}')=r(1+O(r^2/\omega_m)),$$

by computing the contributions of the two simplest correction diagrams in first order of r^2; here ω_m is the largest among the quantities ω', ω, $\omega - \omega'$, k^2, k'^2, and $(\mathbf{k} - \mathbf{k}')^2$.

Higher-order diagrams contribute to $G^{-1}(\omega, k^2)$ in proportion to higher powers of the parameter r^2/ω[7, 8], or to $r^2\xi$ in the ξ representation. The powers of the same parameter r^2/ω_m also appear in the higher-order approximations to the quantity Γ. At $r^2\xi \gtrsim 1$ all these corrections add up to yield a contribution of order one to ωG^{-1} or Γ/r, and alter these quantities substantially (even at $r^2 \ll 1$). This is the region of strong coupling between the pomerons. The magnitude of the gap Δ_0 is determined by the value of $\Sigma(\omega, 0)$ for $-\omega = \epsilon_0 = \Delta_0 \simeq r^2\ln r^2$. Therefore the higher-order corrections to $\Sigma(\omega_0, 0)$, having the order $(r^2/\omega)^n \sim (1/\ln r^2)^n$ for $\ln(1/r^2) > 1$, are small and the magnitude of the bare gap which was found above does not change as $r^2 \to 0$ when all higher order terms are taken into account in G^{-1}.

Thus, as $r^2 \to 0$ the gap Δ_0 has a nonanalytic dependence on r. As can be seen from (1.9) the gap turns out to be negative, i.e., the bare Pomeranchuk pole $\alpha_0(0) = 1 - \Delta_0$ for $t = 0$ is situated to the right of 1. Without taking into account the interaction of the pomerons this would lead to a violation of the unitarity condition in the s channel (the Froissart theorem).

We calculate the scattering amplitude

$$T(\xi,t)\approx\left(i+\mathrm{tg}\frac{\pi}{2}\frac{\partial}{\partial\xi}\right)\,\mathrm{Im}\,T(\xi,t),$$

substituting into (1.2) the Green's function which we have determined in first order of r^2:

$$G^{(1)}(\omega,k^2)\approx 1\Big/\left[\omega+k^2+\frac{r^2}{4}\ln\frac{r^2/4}{\omega+k^2/2}\right]\approx\frac{1}{\omega+k^2}-\frac{r^2}{4(\omega+k^2)^2}\ln\frac{r^2/4}{\omega+k^2/2}. \tag{1.13}$$

Calculating the simple contour integral with respect to ω we obtain[1)]

$$\frac{1}{8\pi s}\,\mathrm{Im}\,T(\xi,t)=g^2\left\{1+\frac{r^2}{4}\xi\left[\ln\frac{1}{(4r)^2\gamma_0\xi}-\int_0^z(e^v-1)\frac{dv}{v}+\frac{e^z-1}{z}\right]\right\}, \tag{1.14}$$

where $z = k^2\xi/2$ and $\gamma_0 = 1.78$ is the Euler constant.

This amplitude corresponds to a slowly increasing total cross section

$$\sigma^{tot}=\frac{\mathrm{Im}\,T(\xi,0)}{s}=8\pi g^2\left\{1+\frac{r^2}{4}\xi\ln\frac{C}{r^2\xi/4}\right\}, \tag{1.15}$$

where $C = e/\gamma \simeq 1$. In the region of energies attainable at the present time, where $\xi = \ln(s/\mu^2) \sim 5$–$10$ for

$r^2 \sim 10^{-2}$, the corrections are small, of the order of 1–2%. Here they are considerably smaller than the corrections due to the unenhanced diagrams of Fig. 1, c, which are of the order[19] of several dozen per cent. Corrections of the same order of magnitude as (1.15), but that decrease σ^{tot} as ξ increases, come from virtual lines in the pomeron vertices and correspond to the semi-enhanced diagrams of Fig. 1, b. All these corrections, except those taken into account in (1.15) do not grow with ξ, and are therefore unimportant in the region $r^2\xi \to \infty$; they are discussed in the last section of this paper.

In order to verify the consistency of this theory it is important to obtain a theoretical asymptotic expression, due to all the approximations, of the amplitude as $\xi \to \infty$. In fact, for this it is necessary to obtain a solution for the Green's functions and vertex parts corresponding to the Lagrangian (1.3) in closed form. We begin solving this problem with an investigation of a simple model for it, different from the exact problem only in the assumption that the impact parameter space ρ is four-dimensional rather than two-dimensional.

2. THE FOUR-DIMENSIONAL MODEL AND THE SCREENING OF THE INTERACTION

If the space of impact parameters ρ in the interaction Lagrangian (1.3) is four-dimensional (this corresponds to a six-dimensional space-time for the original particles), the integrals for the reggeon diagrams will acquire logarithmic divergences as $L \to \infty$ and the theory exhibits its renormalizability. At $r^2 \ll 1$ but $r^2 l \sim 1$ the theory has an exact solution, which can be found explicitly by summing the contributions of the leading diagrams. Here

$$l = \int_{\sqrt{\omega_m}}^{1} \frac{d^4k}{\pi^2 k^4} = \int_{\omega_m}^{1} \frac{dk^2}{k^2} = \ln\frac{1}{\omega_m}, \tag{2.1}$$

ω_m is the largest to the quantities ω_i and k_i^2 which are important in the problem.

The first corrections of the renormalizable perturbation theory, corresponding to the contribution of the one-loop diagram of Fig. 1, a (or the diagram of Fig. 2, b) to the Green's function

$$G^{-1}(\omega, k^2) = \omega + k^2 - \frac{r^2}{8}\left(\omega + \frac{k^2}{2}\right)\ln\frac{1}{\omega + k^2/2} + O(r^4) \tag{2.2}$$

and for the three-pomeron vertex (Fig. 3, b, c)

$$\Gamma(\omega, k_1; \omega_2, k_2) = r\left(1 - \frac{r^2}{2}\ln\frac{1}{\omega_m}\right) \tag{2.3}$$

are of order $r^2 l$ in the four-dimensional theory. Perturbation theory is valid only for $r^2 l < 1$. In the region $r^2 l \gtrsim 1$ the solution has the structure

$$G(\omega, k^2) = \frac{\beta(l)}{\omega + k^2 R^2(l)}, \quad \Gamma(\omega_1, k_1^2; \omega_2, k_2^2) = \Gamma(l), \tag{2.4}$$

if $\omega_1 \approx \omega_2 \sim \omega = \omega_1 + \omega_2 \sim k_1^2 \sim k_2^2 \sim k^2 = (\mathbf{k}_1 + \mathbf{k}_2)^2$.

By means of the equations of the renormalization group one can show in a general way[20] that all quantities $\beta(l)$, $R^2(l)$, $\Gamma(l)$ are proportional to powers of the invariant charge, the latter having the form

$$\lambda(l) = \frac{\Gamma^2\beta^3}{R^4} = \frac{r^2}{1 + ar^2 l}, \tag{2.5}$$

i.e.,

$$\beta = (\lambda/r^2)^{a_1}, \quad R^2 = (\lambda/r^2)^{a_2}, \quad \Gamma = r(\lambda/r^2)^{a_3}.$$

The coefficient a and the exponents a_i are easily determined by comparing, at $r^2 l < 1$, these general formulas with perturbation theory (2.2), (2.3). This yields $a = 3/4$ and

$$\beta(l) = (1 + {}^3/_4 r^2 l)^{1/6}, \quad R^2(l) = (1 + {}^3/_4 r^2 l)^{1/12},$$
$$\Gamma(l) = r(1 + {}^3/_4 r^2 l)^{-2/3}. \tag{2.6}$$

In distinction from the perturbation-theory series this solution is also valid for $r^2 \sim 1$, when the invariant charge $\lambda(l)$ is small. In other words, at $r^2 < 1$ it is valid for any frequency, and at $r^2 \gtrsim 1$ it is valid only for very small ω_m, for $r^2 l \gg 1$, when $\lambda(l) = (4/3)l < 1$. In the asymptotic region which interests us $\omega_m \to 0$, i.e., $l \to \infty$, it is valid for arbitrary r^2.

We note that if the interaction were Hermitian, i.e., if $ir = r_1$ were real, the invariant charge $\lambda(l)$ would increase with l and for values of l close to $l_0 = (4/3)r_1^2$ the solution would become inapplicable. Apparently there are no physical solutions at all in this case, on account of the instability of the vacuum. In our case of anti-Hermitian interaction the reggeon repulsion stabilizes the vacuum and no contradictions appear.

A characteristic trait of the solution found here is the screening—a suppression of the pomeron amplitudes for $\omega_m \to 0$, $l \to \infty$ owing to the repulsion of the pomerons. Thus, as $l \to \infty$ the three-pomeron vertex Γ tends to zero in (2.6) like

$$\Gamma \approx r\left(\frac{4}{3r^2 l}\right)^{2/3} \to 0. \tag{2.7}$$

We calculate the vertex $\Gamma'(\omega_1, \mathbf{k}_1; \omega_2, \mathbf{k}_2)$ describing the emission of a particle by the pomeron (corresponding to the operator $\Gamma_0'\psi^+\psi$ for the emission of a particle by the pomeron). In first order in r^2 this vertex is described by the contribution of the single diagram b in Fig. 4, in place of the two analogous diagrams b and c in Fig. 3 for the vertex Γ. Accordingly, in this order it is determined by

$$\Gamma'(\omega_1, \mathbf{k}_1; \omega_2, \mathbf{k}_2) = \Gamma_0'(1 - {}^1/_4 r^2 l), \tag{2.8}$$

in which the correction term is twice as small as in (2.3). The screening factor for this term is obtained by means of the renormalization group in the form

$$\Gamma'(\omega_1, \mathbf{k}_1; \omega_2, \mathbf{k}_2) = \Gamma_0'(1 + {}^3/_4 r^2 l)^{-1/3}. \tag{2.9}$$

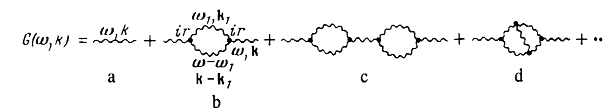

FIG. 2

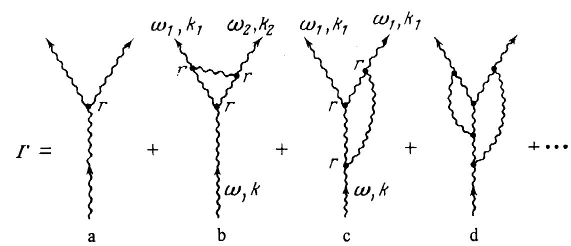

FIG. 3

3. THE ϵ EXPANSION AND SCALE INVARIANCE

In order to go over from this four-dimensional model to the realistic case of a two-dimensional ρ or k space, one must generalize the theory to an arbitrary noninteger dimension $d = 4 - \epsilon$. A detailed discussion of

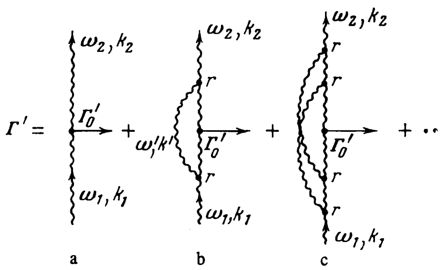

FIG. 4

such an analytic continuation from the integers d = 1, 2, 3, . . . (dimensions of the space) to noninteger values can be found in the papers of Wilson[18], who has proposed the method of ϵ expansion. The idea consists in retaining for nonintegral d the symmetry property: invariance under translations, dilations, and rotations of space. These requirements suffice in order to determine the value of the Feynman integral for noninteger d up to a common factor which renormalizes the coupling constant and is therefore inessential.

For example, the contribution of the simplest pomeron loop (Fig. 2, b) to the pomeron self-energy is defined as[2)]:

$$\Sigma^{(1)}(\xi, k^2) = -\frac{r^2}{2!}\int\frac{d\mathbf{q}}{N_d}\exp[-\xi q^2-\xi(\mathbf{k}-\mathbf{q})^2] = -\frac{r^2}{2}\int\frac{d\mathbf{q}_1}{N_d}\exp\left[-2\xi q_1^2-\xi\frac{k^2}{2}\right]$$
$$= -\frac{r^2}{2}\exp\left(-\xi\frac{k^2}{2}\right)(2\xi)^{-d/2}\int\frac{d\mathbf{q}^2}{N_d}\exp(-q_2^2) = -\frac{r^2}{2}\exp\left(-\xi\frac{k^2}{2}\right)(2\xi)^{-d/2}, \quad (3.1)$$

where we have successively performed the translation $\mathbf{q} = \mathbf{k}/2 + \mathbf{q}_1$ and the dilation $\mathbf{q}_1 = (2\xi)^{-d/2}\mathbf{q}_2$. Under the latter, by the definition of the dimension d of the space, the integral acquires a factor $(2\xi)^{-d/2}$. The remaining integral

$$\int\frac{d^d q_2}{N_d}\exp(-q_2^2)$$

leads to a renormalization of the constant r^2. Redefining N_d, i.e., the units in which the momenta are measured, one can normalize this integral as desired. We shall adhere to the natural normalization

$$N_d = \int e^{-q^2}d^d q = \pi^{d/2}\Gamma(d/2)$$

(for which the last equality in (3.1) is valid).

Carrying out a Mellin-Laplace transformation it is easy to calculate the singular part of the self-energy as ω, $k^2 \to 0$:

$$\Sigma_d^{(1)}(\omega, k^2) = \int_0^\infty e^{-\omega\xi}\Sigma^{(1)}(\xi, k^2)\,d\xi = -\frac{r^2\Gamma(1-d/2)}{2(1+d/2)}\left(\omega+\frac{k^2}{2}\right)^{d/2-1}. \quad (3.2)$$

For $d = 4 - \epsilon$, $\epsilon \to 0$, when $\Gamma(1 - d/2) \simeq -2/\epsilon$, this expression becomes

$$\Sigma_d^{(1)}(\omega, k^2) = \frac{r^2}{4}\left(\omega+\frac{k^2}{2}\right)\frac{(\omega+k^2/2)^{-\varepsilon/2}-1}{\varepsilon} + \text{Reg.} \quad (3.3)$$

It differs from the four-dimensional case by the substitution

$$l = \ln\frac{1}{\omega+k^2/2} \approx \ln\frac{1}{\omega_m} \to \frac{2}{\varepsilon}\left(\frac{1}{\omega_m^{\varepsilon/2}}-1\right), \quad (3.4)$$

where $\omega_m = \max(\omega, k^2)$. In other words, for $d \neq 4$ the logarithmic integral (2.1)

$$\int\frac{d^4k}{\pi^2 k^4} = \int_{\omega_m}^{1}\frac{dk^2}{k^2}$$

should everywhere be replaced by the power-law integral

$$\int\frac{d^4k}{\pi^{d/2}\Gamma(d/2)k^4} = \int_{\omega_m}^{1}\frac{dk^2}{k^{2+\varepsilon}}.$$

One can also show[18], that for all the other quantities one can use for $\epsilon \to 0$ the four-dimensional solutions of the preceding section, making in them the substitution (3.4).

Thus, the solution with dimensions $d = 4 - \epsilon$ differs from (2.5) and (2.6) only by the substitution (3.4). As $\omega = \omega_m \to 0$, i.e., as $l \simeq 2/\epsilon\omega^{\epsilon/2} \to \infty$ it will have the following power-law form:

$$\beta = (r^2/r_0^2)^{1/6}\omega_m^{-\varepsilon/12} + O(\varepsilon^2),$$
$$R^2 = (r^2/r_0^2)^{1/12}\omega_m^{-\varepsilon/24} + O(\varepsilon^2), \quad (3.5)$$
$$\Gamma = r(r^2/r_0^2)^{-2/3}\omega_m^{-\varepsilon/3} + O(\varepsilon^2);$$
$$\Gamma' = \Gamma_0'(r^2/r_0^2)^{-1/3}\omega_m^{\varepsilon/6} + O(\varepsilon^2), \quad (3.6)$$

where $r_0^2 = 2\epsilon/3$. From (2.4) it follows that

$$G^{-1}(\omega, k^2) = \frac{\omega+k^2R^2}{\beta} = \omega\omega_m^{\varepsilon/12}\left(\frac{r^2}{r_0^2}\right)^{-1/6}\left[1+\frac{k^2}{\omega\omega_m^{\varepsilon/24}}\left(\frac{r^2}{r_0^2}\right)^{1/12}\right],$$
$$\Gamma = r_0\left(\frac{r_0^2}{r^2}\right)^{1/16}\omega_m^{\varepsilon/3}. \quad (3.7)$$

This solution corresponds exactly to the one found earlier by Gribov and one of the authors[8] on the basis of an analysis of skeleton reggeon diagrams in the following general form:

$$G(\omega, k^2) = Z_0\omega^{-1-\eta}f(k^2R_0^2/\omega^\nu),$$
$$\Gamma(\omega_1, \mathbf{k}_1; \omega_2, \mathbf{k}_2) = Z_0^{-3/2}R_0^{d/2}\omega^\gamma F\left(\frac{k_1^2R_0^2}{\omega_1^\nu}, \frac{k_2^2R_0^2}{\omega_2^\nu}\ \frac{k^2R_0^2}{\omega^\nu}\ \frac{\omega_1}{\omega_2}\right) \quad (3.8)$$

where Z_0 and R_0^2 are scaling factors (dependent on r^2), η, γ, ν are universal numbers, independent of r, related by the scaling relation

$$2\gamma - 3\eta + \nu d/2 - 2 = 0, \quad (3.9)$$

and f and F are some universal functions of the indicated variables, with $\omega = \omega_1 + \omega_2$ and $\mathbf{k}_1 + \mathbf{k}_2$ in (3.8).

The relation (3.9) is a consequence of the requirement that all skeleton diagrams for G, Γ, and for the other quantities have the same homogeneity exponent (of the form γ or Γ, or $-1-\eta$ for G). It is easy to see, e.g., by comparing Γ with the contribution from the simplest triangular diagram (Fig. 3, b) that this requirement leads to the condition[8]

$$\Gamma^2G^3\omega k^d \sim 1, \quad (3.10)$$

which yields the equality (3.9) upon substituting (3.8) in the region $\omega \sim \omega_1 \sim \omega_2 \sim k^2 \sim k_1^2 \sim k_2^2 \to 0$. The terms which are singular with respect to the indicated variables in the right-hand sides of the Schwinger-Dyson equations for G^{-1} and Γ are all of the same order of magnitude and reproduce[8,17] the left-hand sides of these equations (for a definite form of the functions f and F, independent of the choice of r^2), and the regular terms together with the contribution of $\omega + k^2$ or r cancel in both sides of these equations.

It can be seen that the solution (3.7) has exactly the form (3.8), where as $\epsilon \to 0$ the exponents η, γ, and ν have the value

$$\eta = \varepsilon/12 + O(\varepsilon^2), \quad \gamma = \varepsilon/3 + O(\varepsilon^2), \quad \nu = 1 + \varepsilon/24 + O(\varepsilon^2), \quad (3.11)$$

and as $\epsilon \to 0$ (when d = 4)

$$Z_0 = (r^2/r_0^2)^{1/6} = (2r^2/2\varepsilon)^{1/6}, \quad R_0^2 = (r^2/r_0^2)^{1/12} = (3r^2/2\varepsilon)^{1/12},$$
$$f(x) = 1/(1+x), \quad F \approx (2\varepsilon/3)^{1/2} = \text{const.} \quad (3.12)$$

For such values of η, γ and ν the scaling relation

(3.9) is valid to first order in ϵ; it must further be valid in each successive order. The relation becomes particularly clear and intuitive in the ρ, ξ representation, where it reduces to the invariance condition for the n-point Green's functions with respect to scale transformations[17] of the pomeron field, of the form

$$\psi(\rho, \xi) \to \xi_0^{-\Delta}\psi(\rho\xi_0^{-\nu/2}, \xi/\xi_0) \tag{3.13}$$

with

$$\Delta = {}^1/_4\nu d - {}^1/_2\eta, \tag{3.14}$$

where ξ_0 is an arbitrary parameter. Choosing $\xi_0 = \xi$ we note that ψ has the dimension $\xi^{-\Delta}$ and the impact parameter ρ has the dimension $\xi^{-\nu/2}$. Scale invariance means that the n-point Green's functions have the dimension $\xi^{-n\Delta}$, in particular

$$G(\rho, \xi) = \langle 0|T(\psi(\rho, \xi)\psi^+(0, 0))|0\rangle = \xi^{-2\Delta}g_2(\rho/\xi^{\nu/2}),$$

$$\langle 0|T(\psi(\rho, \xi)\psi(\rho', \xi')\psi^+(0,0))|0\rangle = \frac{1}{\xi^{3\Delta}}g_3\left(\frac{\rho}{\xi^{\nu/2}}, \frac{\rho'}{(\xi')^{\nu/2}}, \frac{\xi}{\xi'}\right).$$

A transformation to the ω, k representation yields for G and Γ indeed exactly the value (3.8).

We shall show how the ϵ-expansion method allows one to determine not only the exponents η, γ, ν, $\Delta = 1 - \epsilon/4 + O(\epsilon^2)$ and all the other dimensionless parameters, but also the explicit form of the functions of type f and F which enter into the theory. In order to obtain the latter one must take into account the already mentioned universality of these functions, i.e., their independence of the constant r^2 of the "bare" interaction[18].

We pick this constant in the form $r^2(\epsilon) = a\epsilon^2 + b\epsilon^2 + \ldots$, in such a way that in each order in ϵ the solution obtained in perturbation theory should have the structure (3.8) (by means of the renormalization group method one can prove in general form[20] that such a function $r^2(\epsilon)$ exists indeed). In first order in ϵ one can consider $r^2 = a\epsilon$, and the space as four-dimensional. Then the perturbation theory corrections (2.2) and (2.3) can be regarded as the result of an expansion in powers of ϵ (i.e., a series in r^2) of the exponents of the powers in (3.8)

$$G(\omega, 0) = \omega + \frac{r^2}{8}\omega\ln\omega \approx \omega^{1+r^2/8+\ldots},$$

$$G^{-1}(0, k^2) = k^2 + \frac{k^2}{16}r^2\ln\frac{2}{k^2} \approx k^2\left(\frac{k^2}{2}\right)^{r^2/16+\ldots},$$

$$\Gamma(\omega_1, 0, \omega_2, 0) = r(1 + {}^1/_2 r^2\ln\omega) \approx r\omega^{r^2/2+\ldots},$$

i.e.,

$$\eta = \frac{r^2}{8} + O(r^4), \quad \gamma = \frac{r^2}{2} + O(r^4), \quad \frac{1+\eta}{\nu} = 1 + \frac{r^2}{16}, \quad \nu - 1 \approx \frac{r^2}{16},$$

where it has been taken into account that $G^{-1}(\omega, k^2)$ in (3.8) can be written in the form

$$G^{-1}(\omega, k^2) = Z_0^{-1}(k^2)^{(1+\eta)/\nu}f_1^{-1}\left(\frac{\omega^\nu}{k^2R_0^2}\right) \tag{3.15}$$

with $f_1(1/x) = (R_0^2x)^{(1+\eta)/\nu}f(x)$. Substituting these values into the scaling relation, we find that as $\epsilon = 4 - d \to 0$ it will be satisfied if

$$r^2 = r_0^2 = 2\varepsilon/3 + O(\varepsilon^2).$$

For η, γ, and ν this yields the values which were found above. The condition $r^2 = 2\epsilon/3$ can also be understood as the requirement that the invariant charge (2.5) for $l = 2(\omega^{-\epsilon/2} - 1)/\epsilon$

$$\lambda(l) = \left[\frac{1}{r^2} + \frac{3}{2\varepsilon}(\omega^{-\varepsilon/2} - 1)\right]^{-1} \tag{3.16}$$

should coincide with its asymptotic value as $\omega \to 0$.

The universal functions f(x) (with $x = R_0^2k^2/\omega^\nu$) and F in (3.8) are also easily constructed for small ϵ, considering their perturbation expansions (2.2) and (2.3) for $r^2 = 2\epsilon/3$ as power series in ϵ. Thus, for $G(\omega, k^2)$ we obtain from (2.2) up to ϵ^2

$$G^{-1}(\omega, k^2) \approx \omega^{1+\varepsilon/12}\left\{1 + \frac{k^2}{\omega^\nu} + \frac{\varepsilon}{12}\left(1 + \frac{k^2}{2\omega^\nu}\right)\ln\left(1 + \frac{k^2}{2\omega^\nu}\right)\right\}.$$

Comparing this with the general form (3.8) of G^{-1} for $r^2 = 2\epsilon/3$ and choosing at the same time

$$Z_0 = 1, \quad R_0 = 1 + {}^1/_{24}\varepsilon\ln 2 \tag{3.17}$$

so that $f(0) = 1$ and $f^{-1}(-1) = 0$, we obtain

$$f^{-1}(x) = 1 + x + \frac{\varepsilon}{12}\left[\left(1 + \frac{x}{2}\right)\times\ln(2+x) - (x+1)\ln 2\right]. \tag{3.18}$$

At $\epsilon = 0$ this function coincides with its zeroth approximation (3.12). In the same manner one can find the explicit form of the function F in (3.8).

Calculating the next approximations of the renormalizable perturbation theory in four-dimensional space one can find the corrections of order ϵ^2, ϵ^3, etc. for all the quantities. By analogy with the theory of phase transitions one may hope that even at $\epsilon = 2$ the succeeding terms turn out to be small compared to the first one, owing to numerically small coefficients like $\epsilon/12$ and $\epsilon/24$ in (3.11), (3.12), (3.18).

Let us analyze in more detail the matching up of the solutions (3.7)–(3.8) in the region of strong coupling, where ω, $k^2 \leq r^2$, to the perturbation-theory series, which converge for ω, $k^2 > r^2$.

The simplest case is that of the function $G(\omega, k^2)$ for $k^2 = 0$. In the weak and strong coupling regions we have (cf. (1.13), (3.7), (3.8))

$$[\omega G(\omega, 0)]^{-1} = \begin{cases} 1 - \dfrac{r^2}{4\omega}\ln\dfrac{4\omega}{r^2}, & \omega > r^2 \\ \left(\dfrac{v^2}{3}\dfrac{4\omega}{r^2}\right)^\eta, & \omega < r^2 \end{cases}, \tag{3.19}$$

where $\eta \simeq 1/6$ and $v^2 = r^2/r^2(4)$, with r(4) the three-pomeron coupling constant for d = 4 (it is convenient to choose the constant r dependent on the space dimension d). For $v^2 = 0.11$, i.e., for $(v^2/3)^{1/6} \simeq 0.52$, both curves (3.19) join smoothly in the region $\omega \sim r^2$ (Fig. 5, a).

A similar situation holds also for the vertex function $\Gamma(\omega_1, k_1, \omega_2, k_2)$. The first perturbation theory corrections, corresponding to the diagrams of Fig. 3, b, c, yield

$$\Gamma = r\left[1 - r^2\int\frac{d^2k'}{\pi}\frac{1}{[\omega_1 + k'^2 + (k_1 - k')^2][\omega + (k_1 - k')^2 + (k_2 + k')^2]} + (\omega_1 k_1 \rightleftharpoons \omega_2 k_2)\right]$$

or, if $k_1 = k_2 = 0$,

$$\Gamma = r\left[1 - \frac{r^2}{2\omega_2}\ln\frac{\omega}{\omega_1} - \frac{r^2}{2\omega_1}\ln\frac{\omega}{\omega_2}\right].$$

In the simplest case $\omega_1 = \omega_2 = \omega/2$ we obtain from this and from (3.8)

$$\frac{\Gamma}{r} = \begin{cases} 1 - \dfrac{2\ln 2}{\omega/r^2}, & \omega > r^2 \\ \left(\dfrac{4v^2}{3}\dfrac{\omega}{r^2}\right)^\gamma, & \omega \leqslant r^2 \end{cases},$$

where $\gamma \simeq 2/3$. At the same value $v^2 = r^2(2)/r^2(4)$

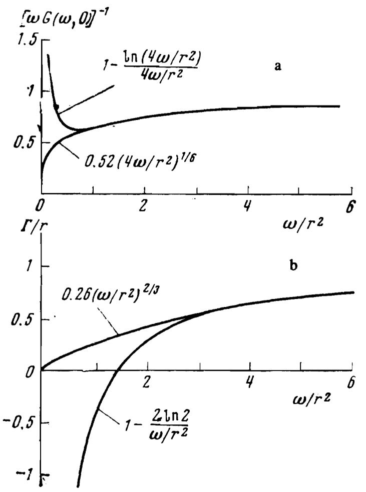

FIG. 5

$= r^2/r^2(4) \simeq 0.11$ (i.e., at $(4v^2/3)^{2/3} \simeq 0.23$), both functions join smoothly, as can be seen from Fig. 5, b.

4. THE ASYMPTOTIC BEHAVIOR OF DIFFRACTION SCATTERING

For ultrahigh energies, when $\xi = \ln(s/\mu^2) \gg 1$, the elastic scattering amplitude (1.1) is proportional to the pomeron Green's function in the ξ,**k** representation:

$$\frac{T(s,t)}{8\pi s} = g^2\left(i+\frac{\pi}{2}\frac{\partial}{\partial\xi}\right)G(\xi,k^2) \approx ig^2 G\left(\xi-\frac{i\pi}{2},k^2\right) \tag{4.1}$$

and can be expressed, according to (3.8), in terms of a function of the single variable $\tau = R_0^2 k^2 \xi^{\nu}$:

$$G(\xi,k^2) = \int_{\dagger} e^{\omega\xi} Z_0 f\left(\frac{k^2R^2}{\omega^{\nu}}\right)\frac{1}{2\pi i}\frac{d\omega}{\omega^{1+\eta}} = \frac{Z_0\xi^{\eta}}{\Gamma(1+\eta)}\varphi(\tau), \tag{4.2}$$

where

$$\varphi(\tau) = \Gamma(1+\eta)\int_{\dagger} e^{y} f\left(\frac{\tau}{y^{\nu}}\right)\frac{1}{2\pi i}\frac{dy}{y^{1+\eta}} \tag{4.3}$$

is some (real) function normalized so that $\varphi(0) = 1$. The most interesting feature of the amplitude is the growth of the corresponding total cross section:

$$\sigma^{tot}(\xi) = \frac{1}{s}\,\mathrm{Im}\,T(s,0) = \frac{8\pi g_0^2 Z_0}{\Gamma(1+\eta)}\xi^{\eta} \to \infty, \tag{4.4}$$

where the ϵ-estimate of the exponent η, as we have seen, yields

$$\eta = \varepsilon/12 + O(\varepsilon^2) \approx {}^1/_6.$$

The angular distribution of the scattering depends on the explicit form of the function $\varphi(\tau)$:

$$d\sigma_{el} = \left|\frac{T}{4\pi s}\right|^2 d^2k \approx \frac{[\sigma^{tot}(\xi)]^2}{64\pi^2}\varphi^2(k^2R^2\xi^{\nu})\,d^2k, \tag{4.5}$$

where $k^2 \simeq -\alpha' t$. Substituting into (4.3) the ϵ-expansion of the function $f(z)$ and of the exponents η and ν and calculating the contour integral in the usual manner (cf. footnote 1)

$$\int_{\dagger} e^{y} f\left(\frac{\tau}{y^{\nu}}\right)\frac{dy}{y^{1+\eta}} \approx \int_{\dagger} e^{y}\left\{\frac{1}{\tau+y} + \frac{\varepsilon}{12}\frac{(y+\tau/2)\ln(y+\tau/2) - {}^1/_2\tau\ln 2}{(\tau+y)^2}\right\}\frac{dy}{2\pi i},$$

we obtain

$$\varphi(\tau) = e^{-\tau}[1 + {}^1/_{12}\varepsilon\chi(\tau)], \tag{4.6}$$

where

$$\chi(\tau) = e^{\tau/2} - 1 + \frac{\tau}{2}\ln\frac{2}{\gamma_0} + \left(1-\frac{\tau}{2}\right)\int_0^{\tau/2}(e^{v}-1)\frac{dv}{v}$$

$$= \left(2+\ln\frac{2}{\gamma_0}\right)\frac{\tau}{2} - \sum_{n=2}\frac{\tau/2}{n(n-1)n!} \tag{4.7}$$

is some standard function of τ ($\gamma_0 = 1.78$), the graph of which is shown in Fig. 6. The same figure also shows the graph of the function $e^{\tau}\varphi(\tau)$ for $\epsilon = 2$, as well as the graph of the effective slope of the angular distribution

$$\beta_{eff}(\tau) = \frac{-\ln\varphi(\tau)}{\tau} = 1 - \frac{\varepsilon}{12\tau}\chi(\tau). \tag{4.8}$$

As can be seen, the corrections to the Regge law $\beta = 1$ are of the order of 20% in the interval $0 \le \tau \le 5$. As the energy increases the slope of the diffraction cone increases: $\beta_0 = \beta_{eff}(0)R_0^2\xi^{\nu}$, where $\nu \simeq 1 + \epsilon/24 \simeq 13/12$.

The total cross section for elastic scattering decreases as the energy increases:

$$\sigma_{el} = \int\left|\frac{T}{4\pi s}\right|^2 d\mathbf{k} = \mathrm{const}\cdot\xi^{-\alpha}, \tag{4.9}$$

where, for the exponent α, which in general equals

$$\alpha = {}^1/_2\nu d - 2\eta, \tag{4.10}$$

we obtain by means of the ϵ-expansion

$$\alpha = 2 - 7\varepsilon/12 \approx {}^5/_6.$$

We note that the value (4.9) of the elastic cross section

$$\frac{\sigma^{el}}{8\pi} = \frac{g_0^4}{2!}\int|G(\xi,k^2)|^2\frac{d\mathbf{k}}{\pi} = \int_{\dagger} e^{\omega\xi}A^{(2)}(\omega,k^2)\frac{d\omega}{2\pi i} \tag{4.11}$$

is determined by the contribution

$$A^{(2)}(\omega) = \frac{g_0^2}{2!}\int_{\dagger} G(\omega',k')G(\omega-\omega',k')\frac{dk'}{\pi}\frac{d\omega'}{2\pi i} \tag{4.12}$$

of the two-reggeon diagram of Fig. 7 for $d = 4 - \epsilon = 2$. It is convenient to estimate the contribution of diagrams of this sort by recognizing that in the integrals (4.11) and (4.12) the values $k'^2 \sim \omega'^{\nu}$, $\omega' \sim \omega \sim 1/\xi$ are important, and by replacing the differentials dk' by $k'^d \sim \omega^{\nu d/2}$, and $d\omega'$ by $\omega' \sim \omega$. Taking into account that $G \sim \omega^{-1-\eta}$, we obtain for this contribution the estimate $A^{(2)} \sim \omega^{\nu d/2 - 2\eta - 1}$. From this we again obtain the estimate (4.9) and (4.10):

$$\sigma^{el} \sim \omega A^{(2)} \sim \omega G^2\omega(k^2)^{d/2} \sim \omega^{\nu d/2 - 2\eta}.$$

5. SCATTERING AT ATTAINABLE ENERGIES

All these estimates are valid only in the theoretical region of strong coupling, where $r^2\ln(s/s_0) \gtrsim 1$, i.e., practically for unattainable energies $\ln(s/s_0) \sim 10^2$, if r^2 has a value close to 10^{-2} (cf.[12, 13]). In an article dedicated to the theory of particle production we shall consider processes of inclusive production taking into account the pomeron interaction and shall give theoretical arguments in favor of such a small value of r^2.

At $r^2 \sim 10^{-2}$ the pomeron interaction remains weak for all practically attainable energies, since $r^2\ln(s/s_0) \ll 1$ always. Regarding it as a small perturbation, we find the elastic scattering amplitude as a series in the powers of $r^2\xi$. Its t-channel partial wave

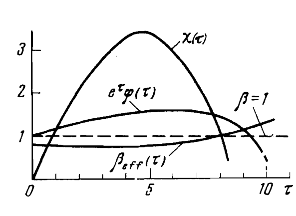

FIG. 6

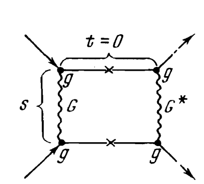

FIG. 7

$\phi(j, t) = g_A(\omega, k^2)\, g_B(\omega, k^2)\, G(\omega, k^2)$ was determined earlier (cf. (1.13)) in first order in r^2/ω by taking into account the main pole diagram and the simplest one-loop enhanced diagram (Fig. 1, a). In practice, corrections that are not small (of the order $r \ln \xi$) also come from semi-enhanced and unenhanced diagrams. The semi-enhanced diagrams of Fig. 1, b yield correction terms (corresponding to the pomeron loops indicated in Fig. 8) to the vertices $g_A = g_A(\omega, k^2)$ describing the emission of a pomeron by particles, of the following form:

$$g_A(\omega,k^2)=g_A^0-\frac{N_A r}{2\sqrt{\alpha'}}\int\frac{d^2k'}{\pi}\frac{1}{\omega+k^2+(\mathbf{k}-\mathbf{k}')^2} =g_A^0\left[1+\frac{\beta_A r}{2}\ln\left(\omega+\frac{k^2}{2}\right)\right], \qquad (5.1)$$

where in place of k' we have introduced the variable $q = k/2 + k'$ and the integration with respect to q^2 was cut off at $q^2 \sim 1$. The quantity $\beta_A = N_A/2\sqrt{\alpha'}g_A^0$ can be estimated by using for the two-reggeon emission vertex N_A the value $N_A \simeq C_A(g_A^0)^2$, corresponding to the eikonal approximation for $C_A = 1$. The constant C_A can be obtained[21] from the experimental data on diffraction scattering of particles; for protons, i.e., for A = p

$$C_p^2=1.3\pm0.1.$$

Thus, $\beta_A \simeq C_A g_A^0/2\sqrt{\alpha'}$ and we obtain for the partial wave amplitude of the t-channel, multiplying the vertices (5.1) and the Green's function (1.12):

$$\varphi(j,t)=g_A^0 g_B^0\left[1+\frac{\beta_A r}{2}\ln\left(\omega+\frac{k^2}{2}\right)\right]\left[1+\frac{\beta_B r}{2}\ln\left(\omega+\frac{k^2}{2}\right)\right] \times\left\{\frac{1}{\omega+k^2}+\frac{r^2}{4(\omega+k^2)^2}\ln\frac{\omega+k^2/2}{r^2/4}\right\}. \qquad (5.2)$$

The individual terms in this expression (which are of the order r and r^2) correspond to the pole pomeron contribution, the contribution of the one-loop enhanced diagram of Fig. 1, a and the contribution of the three "semi-enhanced" diagrams of Fig. 1, b. Substituting this expression into the Sommerfeld-Watson integral (1.1) and calculating it in the standard fashion (cf. footnote 1) we obtain for the total contribution of these diagrams $A' = \mathrm{Im}\, T'(s, t)/8\pi s$ to the imaginary part of the amplitude the following value

$$A'(\xi,k^2)=\tilde g_A(\xi,k^2)\tilde g_B(\xi,k^2)e^{-k^2\xi}\left\{1+\frac{r^2\xi}{4}\left[\ln\frac{4}{\gamma_0 r^2\xi} -\int_0^z(e^v-1)\frac{dv}{v}+\frac{e^z-1}{z}\right]-\frac{r^2}{2}\beta_A\beta_B\int_0^z e^{v+z}\frac{\ln(v/z)}{v+z}dv\right\}. \qquad (5.3)$$

where $z = k^2\xi/2$, and

$$\tilde g_A(\xi,k^2)=g_A^0\left\{1+\frac{r\beta_A}{2}\left[\ln\frac{1}{\gamma_0\xi}-\int_0^z(e^v-1)\frac{dv}{v}\right]\right\}. \qquad (5.4)$$

We recall that the amplitude $T/8\pi s$ itself can be obtained from here[6, 7] by replacing everywhere[3)] $\xi = \ln(E/\mu) = \ln(s/2m_N\mu)$ by $\xi - i\pi/2$.

If $\beta_A = \beta_B = 0$ in (5.4), then (5.3) yields the expression (1.14) obtained for the amplitude without taking into account the semi-enhanced diagrams. Their contribution does not involve the factor ξ, but is in practice not very small, since it is proportional to r rather than r^2, and also to the quantities β_A and β_B which are not small (for NN scattering, e.g.[24], $\beta_A = \beta_B = 2$). The same value (5.3)–(5.4) for the amplitude

$$T/8\pi\sqrt{s}=A(\xi-i\pi/2,\ k^2)$$

is easily obtained directly, developing the perturbation theory in ξ-space, starting from the Green's function of the free pomeron

$$G_0(\xi,\ k^2)=\exp\left[-(k^2+\Delta_0)\xi\right], \qquad (5.5)$$

where Δ_0 is the bare shift (1.9) of the Pomeranchuk pole (this method allows one to obtain a somewhat more accurate formula than (5.3) and (5.4), which also take into account the difference of the ranges of various pomeron vertices g_A, r, N_{AA}). The equations (5.3) and (5.4) for the contribution of the enhanced and semi-enhanced diagrams differ from the analogous expression obtained by Gribov and one of the present authors[7], under the assumption that the vertex r vanishes for vanishing pomeron momenta. In particular, at $k^2 = 0$ the corrections from these diagrams to (5.3 and (5.4) do not vanish (in distinction from[7]) and yield for the total cross section

$$\sigma^{tot}(\xi)=8\pi g_P^2\left(1+\frac{r\beta_A}{2}\ln\frac{1}{\gamma_0\xi}\right)\left(1+\frac{r\beta_B}{2}\ln\frac{1}{\gamma_0\xi}\right) \times\left[1+\frac{r^2}{4}\xi\ln\frac{4}{\gamma_0 r^2\xi}\right]. \qquad (5.6)$$

The first two factors, which are due to semi-enhanced diagrams, lead to a small decrease of σ^{tot} with the growth of the energy, and the last term leads to its increase. For very high energies, for $\xi \gg 1$, the last term, which leads to a logarithmic growth of σ^{tot}, is of fundamental importance (as we have seen, the logarithmic growth goes over in the strong coupling region, for $r^2\xi > 1$ into a growth of the form $\xi^{1/6}$). However, in the region of attainable energies $E \sim 10-10^3$ GeV, the effect of the first two terms prevails and σ^{tot} decreases weakly.

Taking into account the four-pomeron interactions $\lambda_{13}\varphi\varphi^{+3}/3!$ and $\lambda_{22}\varphi^2\varphi^{+2}/4$ leads to the diagrams of Fig. 4, which yield:

a) additional terms at the vertex (5.4), corresponding to the diagrams of Fig. 9, a, b:

$$\Delta g_a(\xi,0)=-\frac{N_A^{(3)}\lambda_{13}}{3\cdot 3!\,\alpha'\xi}$$

where $N_A = C_A^2 g_A^2$ is the vertex describing the emission of three pomerons by the particle A;

b) an addition to the pomeron Green's function $G(\xi, 0)$ of the form

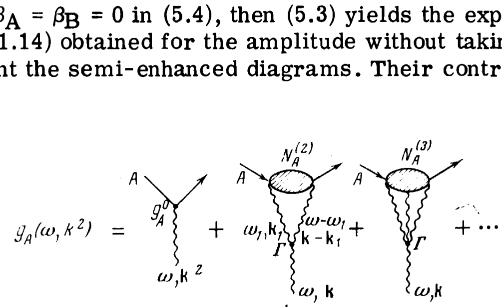

FIG. 8

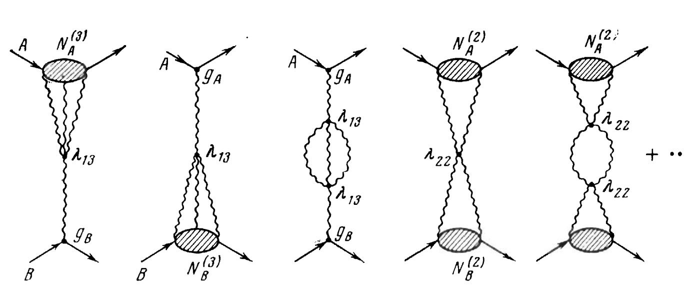

FIG. 9

$$\Delta G(\xi, 0) = \frac{\lambda_{13}^2}{3 \cdot 3!} \ln \gamma_0 \xi,$$

corresponding to the three-pomeron self-energy part of Fig. 9, c;

c) an addition to σ^{tot} of a term

$$\frac{\Delta\lambda\sigma^{tot}}{8\pi} = \frac{N_A N_B}{4\alpha'\xi}\left[1 - \left(1 + \frac{\lambda_{22}}{4}\ln\frac{\gamma_0\xi}{\xi_0}\right)^{-2}\right],$$

which determines the total contribution of all the diagrams of Fig. 9, d with any number of pomeron scatterings on pomerons in the t-channel. Here[22] $N_A = C_A^2 g_A^2$ and ξ_0 is a constant of order unity ($\xi_0 \simeq \xi(E \simeq 20\text{ GeV})$). The contribution of the first of the diagrams of Fig. 9, d is determined by the same expression with the square bracket replaced by $1/2\lambda_{22}\ln(\gamma_0\xi/\xi_0)$.

Figure 10 illustrates the cross section

$$\sigma^{tot'}(\xi) = 8\pi g_0^2 \left[\left(1 + \frac{r\beta_P}{2}\ln\frac{\xi_1}{\gamma_0\xi} - \frac{C_P^2 g_0^2 \lambda_{13}}{18\alpha'\xi}\right)^2 \left(1 + \frac{r^2\xi}{4}\ln\frac{4}{\gamma_0 r^2\xi} + \frac{\lambda_{13}}{18}\ln\gamma_0\xi\right) + \frac{C_P^2 g_0^2}{8\alpha'\xi}\lambda_{22}\ln\frac{\gamma_0\xi}{\xi_0}\right] \quad (5.7)$$

as a function of the laboratory energy E for pp interactions. In the computation it was assumed that $\xi_1 = \xi_0 = \xi(E = 20\text{ GeV})$ and $\xi = \ln E + R_0^2/\alpha'$, where $E = E_{lab}$ in GeV, $R_0^2/\alpha'_P \simeq 4.2$, $r = 1/12$, $C_P^2 = 1.3$, and $g_0^2 = 5.2$[19].

As can be seen, for small λ_{13} and λ_{22} the quantity $\sigma^{tot'}$ decreases on account of the growth of the two-pomeron corrections to the vertices $g(\xi, 0)$. Only for $\lambda_{12} = \lambda_{22} = 0.4$ this increase is compensated by the behavior of the three-pomeron terms. A rough estimate[12] of the constants λ_{13} and λ_{22} yields smaller values for them, of the order 0.2.

This result means, apparently, that the experimentally observed[23] growth of the pp-cross section in the region from 70 to 1400 GeV (by approximately 3 mb) is due to the contribution $\Delta A(\xi, k^2)$ of unenhanced diagrams of Fig. 1, c, which were not taken into account here. It can be represented approximately[24] in the form

$$\Delta A(\xi, k^2) = \sum_{n \geq 2}^{\infty} \frac{N_A^{(n)} N_B^{(n)} \exp(-k^2\xi/n)}{n \cdot n! \, (-\alpha_P'\xi)^{n-1}}, \quad (5.8)$$

where $A(\xi, k^2) = A' + \Delta A$, where A' is the contribution (5.3) of the enhanced diagrams. Here $N_A^{(n)}$, $N_B^{(n)}$ are the vertices describing the emission of n particles by the colliding particles A and B. For $N_A^{(2)} = C_A g_A^2$ [23, 24] with $C_A \simeq 1.3$ the contribution of the two-pomeron term is large, of the order 20–30%. A crude estimate of the other terms on the basis of the relation

$$N_A^{(n)} \approx C_A^{n-1} g_A^n, \quad (5.9)$$

which is difficult to justify theoretically, leads to the

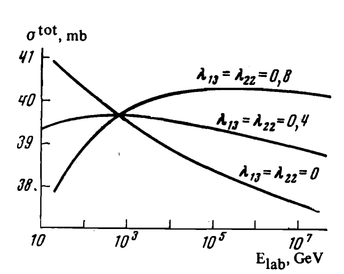

FIG. 10

"quasi-eikonal model[24]. In the framework of this model the total cross section

$$\sigma_0 + \Delta\sigma = 8\pi[g_0^2 + \Delta A(\xi, 0)] \quad (5.10)$$

turns out to be rising (even without taking the enhanced diagrams into account) on account of the "dying out" of the main two-pomeron contribution in (5.8) at $\xi \to \infty$. For the pp interaction it increases[19, 24] by about 2 mb in the indicated region.

As can be seen from Fig. 10, a similar behavior is exhibited by the total cross section

$$\sigma^{tot}(\xi) = \sigma^{tot'}(\xi) + 8\pi\Delta A(\xi, 0),$$

obtained by taking into account the contribution (5.7) of all the enhanced diagrams (for $\lambda_1 = \lambda_2 = 0.4$). A small change in the contribution of the higher-order branch points in (5.8) (e.g., a reduction in the contribution (5.9) of the three-pomeron term $\sim N_A^{(3)} N_B^{(3)}$ can give an even steeper increase of σ_{pp}^{tot} and lead to an enhancement of σ_{pp}^{tot} for an energy $E \simeq 2 \times 10^3$ GeV (ISR) by 3–5 mb, rather than the 2 mb of Eqs. (5.8)–(5.10), in agreement with the experimental data of[23].

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1) All integrals required for the calculation reduce to two basic types:

$$I_1 = \int_{\uparrow} \exp[(\omega+\omega_0)\xi] \frac{\ln(\omega+\omega_1)}{\omega+\omega_0}\frac{d\omega}{2\pi i} = \ln\frac{1}{\gamma_0\xi} - \int_0^{(\omega_0-\omega_1)\xi}(e^v - 1)\frac{dv}{v},$$

$$I_2 = \int_{\uparrow} \exp[(\omega+\omega_0)\xi] \frac{\ln^2(\omega+\omega_1)}{\omega+\omega_0}\frac{d\omega}{2\pi i} = I_1^2 - \frac{\pi^2}{6} - 2\int_0^1 \exp[(\omega_0-\omega_1)\xi(u+1)]\frac{du \ln u}{u+1}$$

and can be obtained by differentiating these identities with respect to ω_0 and ω_1.

2) The differentials $d^d q$, $d^d p_\perp$ of the vectors **q** and $\mathbf{p}_\perp$ etc. in a d-dimensional space will be abbreviated everywhere as dq, $dp_\perp$.

3) In conventional variables we replace $z = k^2\xi/2$ by $\lambda p_\perp^2/2$, where

$$\lambda = \left(\ln\frac{s}{2m_N\mu} - \frac{i\pi}{2}\right)\alpha_P'(0) = \left(\ln\left(\frac{s}{s_0}\right) - \frac{i\pi}{2}\right)\alpha_P' + R^2, \quad R^2 = \alpha'\ln\frac{s_0}{2m_N\mu^2}.$$

The data on the angular distribution in NN scattering for $s_0 = 2m_N E_0$, $E_0 = 1$ GeV yield [19] for R^2 a value close to 1.8 $(\text{GeV}/c)^{-2}$.

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