Cyclotron resonance of electrons in Ge in a quantizing magnetic field in the case of inelastic scattering by acoustic phonons

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Results are presented of an experimental study of the linewidth of cyclotron resonance under strong quantization conditions on the scattering of electrons by acoustic phonons. The measurements were performed in the 2–0.4 mm wavelength range at temperatures between 10 and 1.4 °K. A number of singularities were observed in the temperature and frequency dependences of the cyclotron linewidth. These can be ascribed to the effect of inhomogeneous broadening due to nonparabolicity of the electron spectrum, which is renormalized as a result of interaction with acoustic phonons.

The present paper is devoted to experimental study of the line width of cyclotron resonance (CR) of electrons in Ge under the conditions of strong quantization (quantum cyclotron resonance) and acoustic scattering. The results that are obtained are explained by means of the theory of inhomogeneous line broadening of quantum cyclotron resonance.[1]

1. In a quantizing magnetic field $H(\alpha = \hbar\omega_c/kT > 1$, $\omega_c = eH/m^*c$, T is the temperature in energy units), the wave vector of the electron is of the order of l^{-1}, where $l = (c\hbar/eH)^{1/2}$ is the magnetic length. The wave vector of the phonons that interact effectively with electrons is also of the order l^{-1}, and their energy $\sim\hbar v_0/l$ (v_0 is the speed of sound). Thus, the energy of the characteristic phonon is regulated by the magnetic field.

In the state of thermodynamic equilibrium, at $\alpha > 1$, the electrons are found in the zeroth Landau band. Under the action of an electric field $E \perp H$ of frequency $\omega = \omega_c$, the electrons undergo a transition from the zero band to the first band. The high-frequency current that is developed in this case relaxes as a consequence of scattering. The relaxation is due to transitions inside the zero and first Landau bands (intraband transitions) and transitions from the first band to the zeroth (interband transitions). At $hv_0/l \ll \hbar\omega_c$ (this is satisfied in all known experiments on quantum cyclotron resonance), the interband scattering is elastic. So far as intraband scattering is concerned, we must distinguish two cases here: $\beta \equiv \hbar v_0/lT \ll 1$ and $\beta \gtrsim 1$. In the first case, the phonon energy is much less than the kinetic energy of motion of the electron along the magnetic field—the intraband scattering is elastic. In the second case, the intraband scattering becomes inelastic.

Formulas were obtained in [2] for the line half-width $\delta\omega$ at $\beta \ll 1$ and $\beta \gg 1$. Measurements were also carried out in the region $\beta \ll 1$, the results of which agreed well with the calculation. Experiments at $\beta \gg 1$ are much more difficult because they require significantly higher frequencies and lower temperatures. Therefore, experiments have been carried out to date only for $\beta \approx 1$.[3,4] Extrapolation of these results in the region $T \to 0$ leads to values of $\delta\omega$ which greatly exceed the half-width calculated in [2] for $\beta \gg 1$. Thus, the problem of the line width in the inelastic case remains unclear to the present time.

One of the authors [1] has constructed a theory of the width of the quantum cyclotron resonance line on the basis of the quantum kinetic equation. In particular, it has been shown that for acoustic scattering in the region $\beta \lesssim 1$, the resonance frequency of the electron generally turns out to be dependent on ϵ, thanks to renormalization of the electron spectrum as a consequence of interaction with phonons. The scattering leads to a shift in the electron energy by amounts (in units of $\hbar\omega_c$) $\Delta\epsilon_0(\epsilon)$ and $\Delta\epsilon_1(\epsilon)$ in the zeroth and first Landau bands, respectively. The corrections $\Delta\epsilon_0(\epsilon)$ and $\Delta\epsilon_1(\epsilon)$ are negative and have minima at $\epsilon = \epsilon^{(0)}$ and $\epsilon = \epsilon^{(1)}$, $\epsilon^{(0)} \sim \epsilon^{(1)} \sim \alpha^{-1}\beta$ and $\epsilon^{(0)} < \epsilon^{(1)}$ (the latter is connected with the different forms of the wave functions in the first and second bands). Therefore, for an electron with an unperturbed longitudinal energy ϵ, the resonance frequency $\omega_p(\epsilon)$ (partial frequency) turns out to be dependent on ϵ: $\omega_p(\epsilon) = \omega_c + \eta(\epsilon)$, where $\eta(\epsilon) = \omega_c[\Delta\epsilon_1(\epsilon) - \Delta\epsilon_0(\epsilon)]$ is the partial frequency shift. The absorption curve of electrons with a given ϵ has a maximum at the frequency $\omega_p(\epsilon)$ and a certain half-width $\delta\omega_p(\epsilon)$, which is determined by dissipative processes. The presence of a partial shift leads to the result that the frequency spectrum is smeared out over the interval $\delta\eta$ which is run through by the function $\eta(\epsilon)$ on variation of ϵ in the region near the mean value $\epsilon = \bar{\epsilon} \sim \alpha^{-1}$. If the $\eta(\epsilon)$ dependence is weak (the renormalized spectrum remains approximately parabolic), then $\delta\omega = \delta\omega_p(\bar{\epsilon})$. In the opposite case, the cyclotron resonance line is an envelope of partial lines which are continuously distributed in the interval $\delta\eta$, and the halfwidth of the line $\delta\omega$ turns out to be greater than $\delta\omega_p(\bar{\epsilon})$. Inhomogeneous broadening of the cyclotron resonance line arises due to the nonparabolic nature of the electron spectrum, which is renormalized thanks to the interaction with phonons. We emphasize that the inhomogeneous broadening is of the same order in the coupling constant as the dissipative line width. In this sense, it is not small, no matter how small the interaction.

In comparison with the previously published papers [2-4], the measurements in the present study were carried out over a much broader range of frequencies and temperatures. The maximum values of α and β are 20 and 1.4, respectively. In the region $\beta \sim 1$, a number of features were discovered in the dependence of $\delta\omega$ on the frequency of the radiation and the temperature which are naturally explained by the effect of inhomogeneous broadening.

2. Measurements of the cyclotron resonance line width were carried out on samples of pure n-Ge (the number of acceptor and donor impurities was $N_a + N_d < 10^{12}\,\mathrm{sm}^{-3}$) at frequencies $f = \omega_c/2\pi = 300$–750 GHz, and in the range of liquid-helium temperatures 1.4–10°K.

At such a small concentration of impurities and such high frequencies, we can assume that electron scattering takes place only from acoustic phonons.[5,6] Interelectronic interaction could be left out of account, since the concentration of electrons was small even at the highest temperatures (see below).

The concentration of free carriers that was necessary for indication of cyclotron resonance was produced at temperatures 1.4–6°K by ionization of donor centers by irradiation of the upper part of the cryostat ($T_{backg} = 300$°K). At higher T, the concentration was close to equilibrium. The principal measurements were based on the change of the static conductivity of the sample at cyclotron resonance.[7] It is well known that in pure semiconductors at low temperatures, the conductivity of the sample depends on the electric field. The absorption of microwave power under conditions of cyclotron resonance leads to a resonant change in the static conductivity of the sample, which is recorded by means of the change in the fixed bias of its contacts. The block diagram of the measurement apparatus is shown in Fig. 1.

Low-frequency-modulated microwave radiation (a backward-wave tube was used as the microwave source, see Fig. 1) was supplied via the quasioptical channel to the sample 1 of n-Ge, which was placed in the magnetic field of a superconducting solenoid 2. The sample was connected into a dc circuit. The change in the fixed bias that is developed on absorption of microwave radiation under cyclotron resonance conditions is recorded by the usual low-frequency circuit (the low-frequency amplifier V6-2, synchronous detector SD-1, recording instrument BPP). Recording of the line takes place on variation of the magnetic field about the resonance value.

As a control, some of the measurements were based on absorption. The radiation detector was an n-InSb detector 3, which was placed in the quasioptical channel behind the Ge sample, outside the magnetic field of the solenoid. To bring about a balance indication, the sample was illuminated by modulated interband light. The cyclotron resonance lines obtained by the two methods were identical.

The temperature of the sample was varied by pumping out helium vapor at $T < 4.2$°K. In the case of measurements above 4°K, the measurement channel, which was placed in the cryostat, was put in a jacket containing helium vapor under low pressure. The temperature was raised with a wire heater and the measurement was made with an Allen-Bradley 4 carbon resistor, which was pressed against the sample. Its dimensions were considerably smaller than the dimensions of the sample. The accuracy of the temperature measurement was no worse than 0.1°K.

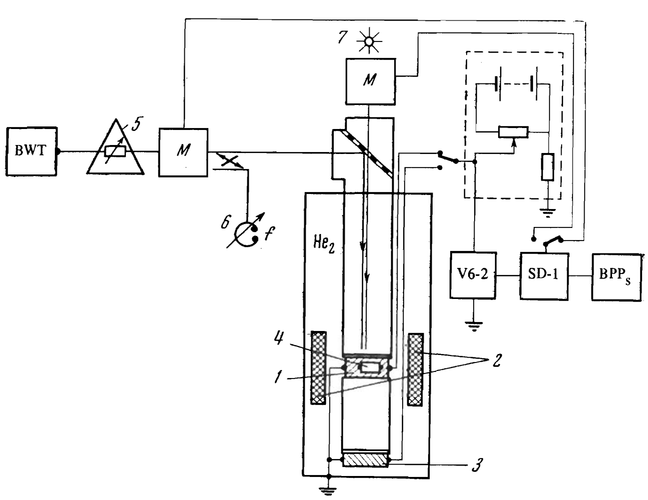

FIG. 1. Block diagram of the experimental apparatus: 1—investigated sample; 2—superconducting solenoid; 3—radiation detector; 4—carbon thermometer; 5—attenuator; 6—frequency meter; 7—illumination source; M—modulator, BWT—backward-wave tube; V6-2—low-frequency amplifier; SD-1—synchronous detector, BPP_S—recording instrument.

The samples had the shape of parallelepipeds of dimensions 5×5×1 mm, with ohmic contacts on the faces, before fusion of which the samples were polished and etched in boiling H_2O_2 solution.

The microwave power at all frequencies and the fixed bias did not exceed $\sim 10^{-6}$ W and 0.1 V/cm, respectively. Under these conditions, there were no changes in the width or shape of the cyclotron resonance line as a function of the value of the absorbed power or fixed bias.

The concentration of free carriers in the sample was determined from galvanomagnetic measurements. It varied in the limits $10^{8}-10^{10}$ cm^{-3}. Here the absorption coefficient of submillimeter radiation did not exceed ~0.1 at the highest temperatures. The cyclotron resonance was measured for electrons belonging to one of the minima of the Ge conduction band in a magnetic field parallel to the axis of rotation of the mass ellipsoid of this minimum. The values of α and β for the frequencies and temperatures used in the experiment are given in the Table.

3. The results of measurements of $\delta\omega(T)$ at f = const are shown in Fig. 2a. Similarly marked points correspond to the same frequency. The straight line I corresponds to $\delta\omega_{Cl} \sim T^{3/2}$. This dependence has been obtained repeatedly from measurements in the classical range $\alpha \ll 1$ (see, for example, [6]). The straight line II indicates the slope $\delta\omega \sim T^{1/2}$. For clarity, the curves

T, K	f, GHz: 131 α	131 β	337 α	337 β	428 α	428 β	500 α	500 β	746 α	746 β
1.4	4.4	0.72	11.6	1.13	17.2	1.39	20.0	1.43		
2	3.1	0.50	8.1	0.79	12.0	0.97	14.0	1.0	18.0	1.22
3	2.1	0.33	5.4	0.53	8.0	0.65	9.3	0.69	12.0	0.78
4.2	1.5	0.25	3.9	0.38	5.6	0.46	6.7	0.50	8.5	0.56

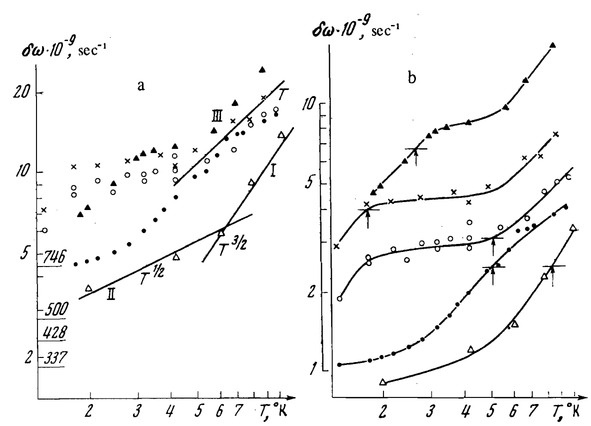

FIG. 2. a) Temperature dependences of $\delta\omega$ for frequencies f: Δ—131 GHz; ●—337 GHz; ○—428 GHz; ×—500 GHz; ▲—746 GHz; the ticks on the axis of ordinates give the values of $\delta\omega_{\infty}$ for the corresponding frequencies; b) the same $\delta\omega(T)$ dependences, along the vertical for clarity. Each curve has an arrow indicating the value $\delta\omega = 10^{10}$ sec^{-1}.

corresponding to the different frequencies in Fig. 2b have been shifted along the vertical. On each curve, the arrow shows the value $\delta\omega = 10^{10}$ sec^{-1}. It is seen that as the temperature is lowered, three regions are defined: high temperature—a narrowing of the line; intermediate temperature—a weakening of the temperature dependence; low temperature—further narrowing. We note that the region of low-temperature narrowing was not previously observed.

Another very important feature is the presence of the extremum of $\delta\omega$ in the frequency. It is clearly seen from Fig. 2a that at $T < 3^\circ$K and a given temperature, the line at the frequency 500 GHz was found to be broader than the lines corresponding both to lower and to higher frequencies.

Figure 3 shows three lines recorded at the same temperature ($T = 2^\circ$K) at frequencies of 337 GHz (a), 500 GHz (b) and 746 GHz (c). The values of α and β for the corresponding lines amounted to 8.1 and 0.8; 12 and 0.97; and 18 and 1.22. It is not difficult to establish that the line b is broader than the lines a and c. (We note for what follows that the lines a and c are symmetric and close to a Lorentzian line, while line b is nonsymmetric; it is broader on the high-H side.) The values of $\delta\omega$ given in Figs. 2 and 4 for both the symmetric and the nonsymmetric lines were obtained from the relation $\delta\omega = \omega H/2H_r$, where δH is the width of the line in oersteds at half-maximum of the cylotron line (see Fig. 3) and H_r is the resonance value of the magnetic field. Direct measurements were made of the $\delta\omega(f)$ dependence at T = const. The results of the measurements at $T = 4.2$, 3 and 1.8°K are shown in Fig. 4. It is seen that $\delta\omega(f)$ has a maximum and a minimum. To the left of the maximum, $\delta\omega$ increases relatively slowly with f, and then, close to the maximum, more quickly. The $\delta\omega(f)$ dependences for the other T have the same form. This N-shaped character of the $\delta\omega(f)$ dependence was also observed for the first time.

4. In this paragraph we shall formulate the basic results of the theory which are necessary for discussion of the experiment.

First, we make the following remark. In Ge, the ef-

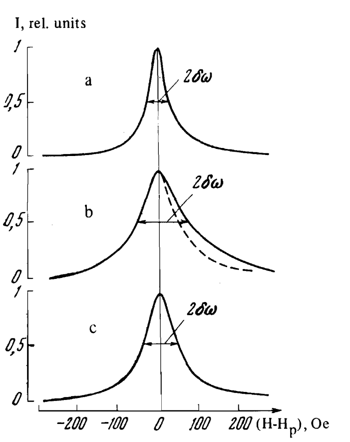

FIG. 3

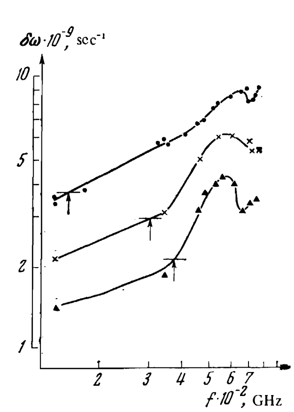

FIG. 4

FIG. 3. Cyclotron resonance lines at T = 2°K and frequencies f: a) 337 GHz; b) 500 GHz and c) 746 GHz. The dashed curve is symmetric to the left half of the cyclotron line b.

FIG. 4. Frequency dependences $\delta\omega$ at temperatures T: ●—4.2°K; ×—3°K; ▲—1.8°K. Curves corresponding to different temperatures are shifted vertically. The arrow on each curve indicates the value $\delta\omega = 5 \times 10^9$ sec^{-1}.

fective mass of the electrons is strongly anisotropic ($m_\parallel/m_\perp = 20$, $m_\parallel$ and $m_\perp$ are the "longitudinal" and "transverse" effective masses). In the isotropic case, the electrons interact only with longitudinal phonons. The mass anisotropy must be taken into account first of all in calculation of the final state density for scattering from longitudinal phonons. This leads to the appearance of the factor $(m_\parallel/m_\perp)^{1/2}$. In our case, this factor is quite large: $(m_\parallel/m_\perp)^{1/2} \approx 4.5$. It should also be taken into account that electrons with anisotropic mass also interact with transverse phonons. It can be shown that this circumstance does not change the character of the dependence of $\delta\omega$ on T and f. So far as the quantitative side of the picture is concerned, no calculation of $\delta\omega$ for $\alpha > 1$ with account of transverse phonons has as yet been carried out. We shall not raise the question of the exact quantitative comparison of theory and experiment, and shall not take the transverse phonons into account, assuming that their contribution can increase the scattering probability only insignificantly (as is the case at $\alpha \ll 1$ [8]). Then the theory developed in [1] can easily be generalized to the anisotropic case for chosen orientation of H. This generalization reduces simply to a change in the coupling between the longitudinal energy ϵ and the longitudinal wave vector k_z: in place of $\epsilon = \frac{1}{2}k_z^2$ we must now write $\epsilon = \frac{1}{2}m_\perp k_z^2/m_\parallel$ (ϵ and k_z are measured in units of $\hbar\omega_c$ and l^{-1}, respectively).

It is convenient to connect the constant of electron-phonon interaction C that enters in [1] with the momentum relaxation time τ_0 of electrons (on longitudinal phonons) with energy T at H = 0. Using the data of Dakhovskiĭ,[8] we find (V is the normalized volume)

$$\tau_0 = \frac{2VT^{1/2} m_\parallel^{1/2} m_\perp C}{\pi \hbar^5 v_0}. \quad (1)$$

The formulas written below were obtained by applying this generalization of the corresponding relations of [1] with Eq. (1). We have in mind pure Ge, so that the electrons are assumed to be nondegenerate.

The entire set of values of β can be broken up into three regions:

Region $\beta \ll 1$. The basic role is played by intraband transitions, which are elastic. The partial shift is equal to zero and inhomogeneous broadening is absent. The line width is due to purely dissipative processes. The half-width of the line is equal to

$$\delta\omega = \delta\omega_0 = \frac{1}{2}\alpha\tau_{cl}^{-1} \sim fT^{1/2}. \quad (2)$$

Region $\beta \gg 1$. At $\beta \gg 1$, intraband transitions do not play a role. Everything is determined by interband transitions with spontaneous emission of phonons. The value of η is only slightly dependent on ϵ; $\delta\eta = 0$.[1] The line half-width is equal to

$$\delta\omega = \delta\omega_\infty = \frac{1}{4}\left(\frac{\alpha}{2}\frac{m_\parallel}{m_\perp}\right)^{1/2} \beta\tau_{cl}^{-1}. \quad (3)$$

Region $\beta \sim 1$. Here as at $\beta \ll 1$, the principal role is played by intraband transitions. However, these transitions are inelastic in this region, and the quantum kinetic equation in [1] cannot be solved. It has not been possible to obtain explicit formulas, and we can deal only in estimates. At $\beta \sim 1$ the smearing interval reaches a maximum: $\delta\eta = \delta\eta_{max}$. The halfwidth of the partial line $\delta\omega_p$ is found to be of the same order as $\delta\eta_{max}$:

$$\delta\eta_{max} \approx \delta\omega_p \approx \alpha^{1/2}\delta\omega_\infty = 2^{-5/2}\alpha\tau_{cl}^{-1}.$$

Thus, there is inhomogeneous broadening comparable with the dissipative line width. One can assume very roughly that the half-width amounts to

$$\delta\omega=\delta\omega^*\approx 2\delta\eta_{max}\approx 2^{-1/2}\alpha\tau_{cl}^{-1} \quad (4)$$

at $\beta\sim 1$. (For the isotropic case, the formulas for $\delta\omega$ corresponding to $\beta\ll 1$ and $\beta\gg 1$ were first obtained in [2].)

In addition to the approximations noted above, it must be kept in mind that in our measurements the strong inequalities $\alpha\gg 1$ and $\beta\ll 1$ are not satisfied simultaneously; the condition $\beta\gg 1$ is also not achieved. We shall attempt to show only that firstly, the theory permits us to explain the observed regularities qualitatively and, secondly, it gives values for $\delta\omega$ that are sufficiently close to the experimental ones.

All quantities (α, β, H, T, $\delta\omega$) that refer to the regions $\beta\ll 1$, $\beta\gg 1$ and $\beta\sim 1$ are designated below by 0, ∞ and the asterisk, respectively.

For the classical relaxation time, we use the value determined from measurements of the cyclotron resonance width at $\alpha\ll 1$ [6]:

$$\tau_{cl}^{-1}=\delta\omega_{cl}/1.25=3.6\cdot 10^8\, T^{3/2}.$$

(This is an experimental value. It includes, of course, the contributions of both transverse and longitudinal phonons. To proceed consistently, it would be necessary to exclude the contribution of transverse phonons from this value in our approximation. In practice, this doesn't amount to anything, since even then only an approximate comparison would be possible.)

5. We now discuss the temperature dependence of the line width. In Fig. 2a, the points 6.25, 8.3 and 10.5°K for f = 131 GHz lie very close to the classical halfwidth of the line (line I). The points 2, 4.2 and 6.25°K lie on the line with slope $\delta\omega_0\sim T^{1/2}$, as they should according to Eq. (2) for $\delta\omega_0$. The transition from the dependence $\delta\omega\sim T^{1/2}$ to $\delta\omega\sim T^{3/2}$ takes place near 6°K. At the temperature 6.25°K, $\alpha=1$. The absolute values of $\delta\omega_0$ are found to be about 1.6 times greater than those calculated.

For high frequencies, in the region (III) of high T (>5°K) $\delta\omega$ falls off with decrease in T more rapidly than according to the law $\delta\omega\sim T^{1/2}$ (see Fig. 2a). It is natural to connect this with the fact that here $\beta\sim 1$. Actually, at $\beta\ll 1$, the change in $\delta\omega$ with decrease in T as $T^{1/2}$ is due to the decrease in the average number of scattering phonons, $N_{ph}=(e^{\beta}-1)^{-1}\sim T$ and the simultaneous increase in the average density of final states for the intraband transitions $\sim T^{-1/2}$. The volume of phase space of the phonons that are important in the intraband scattering decreases with the approach to unity,[1] and the effectiveness of the intraband scattering falls off rapidly.

With further decrease in T, a weakening of the temperature dependence sets in. The extent of this plateau is different at different frequencies. Thus, at f = 428 GHz, this region is 2–5°K, while for f = 746 GHz, it is 3–4°K. To explain the plateau, we note, first, that the weakening is observed in the region $\beta=\beta^*=1$ (see the Table; in accordance with the above designations of the quantities, we mark all quantities referring to the region of the plateau with an asterisk), i.e., where the inhomogeneous broadening reaches a maximum. In our view, it is the latter that leads to the appearance of the plateau.

In fact, with decrease in T at $\beta\approx 1$, along with a decrease in the contribution of the dissipative processes, a new factor arises—inhomogeneous line broadening. The presence of two oppositely directed tendencies naturally leads to a weakening of the temperature dependence. It is not difficult to establish the fact that estimation by Eq. (4) gives values for $\delta\omega^*$ that are quite close to those observed. At lower temperatures, the line is again narrowed, in our opinion because of the disappearance of the inhomogeneous broadening at $\beta>1$. In order to establish that such narrowing is possible in principle, we note that the ratio of $\delta\omega^*$ to $\delta\omega_\infty$ at ω_c = const amounts to

$$\frac{\delta\omega^*}{\delta\omega_\infty}\sim\left(\frac{m_\perp}{m_\parallel}\right)^{1/2}\left(\frac{\hbar\omega_c}{m_\perp v_0^2}\right)^{1/4} \quad (5)$$

(the temperature T is eliminated by means of the relation $\beta^*=\hbar v_0/lT^*=1$). It is then evident that at least for high enough ω_c, we have $\delta\omega_\infty<\delta\omega^*$, i.e., line narrowing should take place. Under our conditions, the ratio (5) is close to unity at the highest frequencies. Thus, our explanation of the narrowing does not contradict the relation (5).

6. We now turn to a discussion of the dependence of $\delta\omega$ on $\omega_c=eH/m_\perp c$. Like T, the magnetic field H affects the density of final states for interband and intraband transitions (T affects the density of states only for the intraband transitions) and the characteristic energy of the phonon. It is well known that for $N_{ph}\gg 1$, the square of the modulus of the matrix element for interaction with acoustic phonons does not depend on the wave vector of the phonon. Therefore, at $\beta\ll 1$ the field H affects only the density of states, which increases in proportion to H for intraband transitions. Thence $\delta\omega_c\sim f$. With the approach of β to unity, this dependence becomes weaker, since N_{ph} falls off more rapidly than $\omega_c^{-1/2}$. This is observed experimentally: it is seen from Fig. 4 that at f < 300 GHz, $\delta\omega$ increases sublinearly with f. A section with a more rapid increase of $\delta\omega$ is also observed. It can be explained by the effect of inhomogeneous broadening in exactly the same way as the presence of the temperature plateau (with the difference that now both tendencies "work" in the same direction). The subsequent narrowing is connected with cessation of intraband scattering. Here, on the one hand, the inhomogeneous broadening disappears and, on the other, the dissipative line broadening decreases, inasmuch as the density of final states for interband transitions is less than for intraband ones (by a factor of $\alpha^{1/2}$). It is not possible to separate these two factors without solution of the kinetic equation. However, the possibility of narrowing can be established in principle in the following way. We equate $\delta\omega^*$ and $\delta\omega_\infty$ at identical T and then determine β_∞. We get

$$\beta_\infty\sim\left(\frac{m_\perp}{m_\parallel}\right)^{1/2}\frac{\alpha^*}{\alpha_\infty^{1/2}}=\left(\frac{m_\perp}{m_\parallel}\right)^{1/2}\frac{T^{3/2}}{m_\perp v_0^2(\hbar\omega_{c\infty})^{1/2}}. \quad (6)$$

It is seen that for sufficiently large T, the value of β_∞ defined by the relation (6) can become large in comparison with unity. This means that the line can have the same width for $\beta\gg 1$ as at $\beta\sim 1$, if the change of β is realized by means of frequency at constant temperature. But, inasmuch as the halfwidth $\delta\omega\sim f$ at $\beta\gg 1$ (because of the increase in the density of final states for interband transitions with H), the half-width must necessarily pass through a minimum in the region $\beta\sim 1$.

We see that the observed dependence of $\delta\omega$ on f has a natural explanation.

The values of $\delta\omega_\infty$, computed from (3) for the frequencies 337, 428, 500 and 746 GHz are marked by short lines on the ordinate of Fig. 2a. The experimental values of $\delta\omega$ are always larger at the lowest T, and the difference decreases with increase in frequency. This is in order: the higher the frequency, the closer the limit $\beta = \beta_\infty \gg 1$ at a given temperature.

It was pointed out above that the line is found to be asymmetric at $\beta \sim 1$ (line b in Fig. 3). This can also be explained by means of the partial frequency shift. The existence of minima in the corrections $\Delta\epsilon_0(\epsilon)$ and $\Delta\epsilon_1(\epsilon)$, which are displaced relative to each other ($\epsilon^{(0)} < \epsilon^{(1)}$, see Par. 1) leads to the result that the $\eta(\epsilon)$ dependence has an oscillating character in the region $\epsilon \sim \beta/\alpha$ at $\beta \lesssim 1$: with increase in ϵ the function $\eta(\epsilon)$ first passes through a maximum (at $\epsilon \approx \epsilon^{(0)}$), then through a minimum (at $\epsilon \approx \epsilon^{(1)}$). The presence of extrema in $\eta(\epsilon)$ and, consequently, in the partial frequency $\omega_p(\epsilon)$ at certain values of ϵ means that the frequency distribution density of the electrons, which is proportional to $d\epsilon/d\omega_p$, is found to be large at these points. At $\beta \ll 1$, there are no electrons in the range $\epsilon \sim \beta/\alpha$, the line width has a purely dissipative origin, and the line is symmetric. With increase in β, at $\beta \sim \alpha\bar{\epsilon} \sim 1$, electrons appear near the extrema, initially in the region of the minimum of $\eta(\epsilon)$ and then in the region of the maximum. Here the line should at first become wider toward the low frequencies (high H), and then again become symmetric. This is exactly what happens in experiment.

Thus the dependences of $\delta\omega$ on the temperature and frequency obtained in the present work, and also the asymmetry of the line in the region of $\beta \sim 1$, can be explained completely by inhomogeneous broadening of the cyclotron resonance line and by the presence of two types of transitions (interband and intraband).

One can now understand why extrapolation of previous measurements to the low-temperature region led to values of $\delta\omega_\infty$ that appreciably exceeded the limiting calculated values of $\delta\omega$ at $\beta \gg 1$. First, the extrapolation was carried out from the region of the temperature plateau; the possibility of further narrowing of the line was not taken into consideration. Second, the mass anisotropy was not taken into account, which leads in $\delta\omega_\infty$ to the appearance of the factor $(m_\parallel/m_\perp)^{1/2} \approx 4.5$ and a change in the numerical coefficient. Thanks to this, $\delta\omega_\infty$ increases by a factor of about three.

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