A simple model for the acceleration of ions by high-current relativistic electron beams

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A new model for ion acceleration in relativistic electron beams is proposed which considers explicitly the influence of the accelerated ions on the beam-front dynamics. The basic ideas are: Ions are produced within the beam front by electron and ion collisions with the gas particles creating a partial space charge neutralization which cancels the electrostatic stopping of the beam. The space charge fields in the accelerating beam front accelerate the produced ions, part of which reach a higher velocity than the front and pass the leading edge of the beam. These ions form a fast beam of high energy and small energy spread and produce further ions ahead of the beam edge. The electron beam-front dynamics is closely related to the ion-bunch relaxation and acceleration. The model is one dimensional, it neglects magnetic effects, and is maximally simplified. In spite of this, it explains qualitatively and partly quantitatively most experimental facts, such as the pressure dependence of the electron-beam velocity and ion current and the charge dependence of the ion energy and the ion current.

1. INTRODUCTION

Focusing of high-current relativistic electron beams that can be created by means of pulse discharges in field-emission diodes, and their transport over a long distance, are possible only if the space charge of the beam is neutralized. The latter can be realized by injecting the electron pulse into a drift tube filled with gas. As a result of the interaction of the fast electrons with the gas a plasma is produced and provides the required charge neutralization. As a consequence of the accompanying relaxation processes the front of the beam moves through the drift tube with a considerably smaller velocity than the injected electrons (cf., e.g.,[1]).

In experiments with such electron beams Graybill and Uglum[2], as well as Rander et al.[3], have discovered the interesting effect of ion acceleration, the ions reaching energies larger than the kinetic energy of the injected electrons. The main results of the experiments published so far are the following:

1) The energy of the ions is proportional to their charge under identical experimental conditions[1,2,4]. For hydrogen ions, two ion pulses have been observed in[3], with different energies and intensities.

2) The energy of the ions in the acceleration region depends weakly on the gas pressure[1,2].

3) The duration of the ion pulse (3–10 ns) is smaller by one order of magnitude than the duration of the electron pulse (40–80 ns)[2].

4) The ion pulse moves synchronously with the front of the electron beam[5].

5) As the energy of the electrons increases, so does the energy of the accelerated ions[3,5].

6) In the interval 0.1–1 Torr the ionic current decreases as the gas pressure increases[4].

7) The intensity of the ion pulse is considerably smaller for heavy gases (e.g., helium or nitrogen) than for hydrogen[2].

The picture of the acceleration phenomenon contains, however, several obscure points. Thus, there are contradictory data regarding the dependence of the ion energy on the electron current intensity[2,4]. Nevertheless, the results listed above seem to us to be sufficiently well established.

Two models have been devised for the explanation of the effect. In the simple electrostatic model proposed by Rostoker[6], and also by Rosinskiĭ, Rukhadze, and Rukhlin[7] it is assumed that the velocity of the electron beam which is decelerated by the space charge is determined solely by the degree of compensation of that charge on account of ionization of the gas through electron collisions. The potential well formed at the front of the beam captures the ions and they move along with the ionization front with the same speed. Although this model gives a correct estimate of the energy of the accelerated ions and describes some of the results indicated above; it is in patent contradiction to other experimental facts, particularly those indicated above under 1), 2) and 6).

In distinction from this mechanism, Putnam[6] in his "linear pinch model" assumes that the fundamental reason for the ion acceleration is the pinching of the beam produced by its own magnetic field. The focusing produced by the pinch-effect of the electron beam is a necessary condition for its passage through the tube, but does not compensate for the deceleration of the beam under the action of the space charge, which is not taken into account in[8]. As will be shown below, the dynamics of the beam front is determined in the first place by electrostatic effects of the space charge, which at the same time produce the conditions for the operation of the electrostatic mechanism of ion acceleration.

2. THE BASIC ASSUMPTIONS OF THE MODEL

Our model is a continuation and extension of the simple electrostatic model[6,7]. The main disadvantage of the latter is that the influence of the accelerated ions on the motion of the beam itself is in no way considered in it. Our model considers both the motion of the ions trapped in the potential well and their reaction on the dynamics of the beam front, including the ionization produced by these ions. Together with these extensions of the model we attempt to start from the simplest assumptions and neglect all secondary effects. The simplifications we use are the following:

a) the problem is solved only in the one-dimensional case. Problems related to the geometry of the beam (e.g., the variation of its diameter) and its focusing, are not considered;

b) only electrostatic forces are taken into account;

c) when account is taken of the motion of the ions in the region of the beam front, we make use only of easily calculated average values. No attempts at a self-consistent solution of the problem have been undertaken.

2.1. The Condition for Electrostatic Blocking of the Beam

When a high-current relativistic electron beam is injected into a field-free vacuum the space charge of the first electrons to enter the vacuum creates a longitudinal electrostatic field which decelerates the electrons which arrive later, and the beam is blocked electrostatically. In the one-dimensional case Poisson's equation yields for the deceleration length d_0 to which the beam can penetrate the vacuum without charge neutralization the value

$$d_0=\left(\frac{E_0\beta c}{2\pi je}\right)^{1/2}=\left[\frac{mc^3\beta(\gamma-1)}{2\pi je}\right]^{1/2} \tag{1}$$

Here it was assumed that the electron density n_e is constant in this volume; E_0 is the kinetic energy of the injected electrons, j is the current density, βc is the velocity of the electrons and $\gamma = (1-\beta^2)^{1/2}$. Equation (1) is in satisfactory agreement with the interpolation formula used in[7], where the variations of the velocity and density of the electrons in a layer of thickness

$$d_0\approx[mc^3/2\pi je]^{1/2}(\gamma^{2/3}-1)^{3/4} \tag{2}$$

were taken into account.

It was shown by Poukey and Rostoker[9] that the static solution of Poisson's equation is in fact unstable. The time-dependent solution leads to high-frequency oscillations of the depth of the potential well and of the deceleration length. This leads to an increase of the average depth of the potential well by a factor of two or three. For the moment we neglect this effect and use the simple estimate (1) for the subsequent discussion.

The one-dimensional model is highly simplified. In a realistic situation the beam has a finite radius R_s and moves in a conducting drift tube of radius R_d. Therefore the concepts of the one-dimensional model are applicable only for $d_0 < R_s$, i.e., for sufficiently large currents. This is also the condition for electrostatic blocking of the beam in the drift tube without charge neutralization.

If one assumes that the beam is homogeneous in the transverse direction and that the total current is J, it follows from (1) that

$$d_0=R_s\left[\frac{I_0}{2J}\beta(\gamma-1)\right]^{1/2}, \tag{3}$$

where $I_0 = mc^3/e \simeq 17$ kA. For $R_s = R_d$ the condition for electrostatic blocking of the beam takes the form

$$J\geqslant{}^1/_2I_0\beta(\gamma-1). \tag{4}$$

We assume that this condition is satisfied.

2.2. The Motion of the Ions in the Region of the Moving Beam Front

If the electron beam penetrates into the gas, ions are formed as a result of collisions. The ions neutralize the space charge and thus make it possible for the beam to advance with a velocity which is ultimately determined by the frequency of ionizing collisions. In the region of the front of the beam there exists a region of partial compensation of the space charge. This is the region which contains also the ions and the longitudinal electrostatic field produced by the uncompensated fraction of charge of the electrons, therefore that is the region where the process of ions acceleration occurs. The space charge distribution is modified in a complicated way by the moving ions; even in the one-dimensional case a self-consistent solution of this problem seems to be hopelessly difficult. Therefore we make some radical simplifications and assume the following:

a) a linear decrease of the potential $\Delta\varphi= E_0/e$ in the beam front region, i.e., in the region of partial charge neutralization;

b) homogeneous electron density n_e in the beam front region;

c) homogeneous ion density in the same region.

The ratio of the ion density to the electron density in the beam will be denoted by $\kappa = n_i/n_e$. The indicated partial compensation of the space charge leads to an increase of the deceleration length d. Since $n_e \sim j$ it follows from (1) that

$$d=d_0/\sqrt{1-\kappa}. \tag{5}$$

An ion moving in a layer of this thickness has the constant acceleration

$$g=zE_0/Md=z(\gamma-1)mc^2/Md,$$

where z is the charge number and M the mass of the ion.

We introduce the coordinate ξ moving with the speed of the beam front. Then the motion of the ion is determined by the equation:

$$\xi=\xi_0-wt+{}^1/_2gt^2, \tag{6}$$

where ξ_0 is the initial coordinate of the ion. Introducing the dimensionless quantities

$$\lambda=\xi/d,\quad 0\leqslant\lambda\leqslant1,$$

$$\alpha=\frac{w}{(2dg)^{1/2}},\quad (2dg)^{1/2}=c\left[\frac{2m}{M}z(\gamma-1)\right]^{1/2}, \tag{7}$$

$$\vartheta=\frac{t}{(2d/g)^{1/2}},\quad \left(\frac{2d}{g}\right)^{1/2}=\frac{d}{c}\left[\frac{2M}{mz(\gamma-1)}\right]^{1/2}.$$

we rewrite Eq. (5) in the form

$$\lambda=\lambda_0-2\alpha\vartheta+\vartheta^2. \tag{8}$$

The acceleration of the ion in the field of the space charge ends when the ion passes the points $\lambda = 0$ or $\lambda = 1$. In the first case the ion falls behind the front and in the second case it passes through the front. The time τ necessary for this in the first case (which is realized for all ions produced at the points $\lambda_0 \leq \alpha^2$) is

$$\tau_-=\alpha-(\alpha^2-\lambda_0)^{1/2}. \tag{9}$$

During this time the ion acquires the energy (in units of E_0)

$$\varepsilon_{i-}=z[\alpha-(\alpha^2-\lambda_0)^{1/2}]^2. \tag{10}$$

For all the ions which are produced at sites satisfying the condition $\alpha^2 \leq \lambda_0 \leq 1$ (the second case), we have

$$\tau_+ = \alpha + (\alpha^2 + 1 - \lambda_0)^{1/2},$$
$$\varepsilon_{i+} = z[\alpha + (\alpha^2 + 1 - \lambda_0)^{1/2}]^2. \tag{11}$$

It can be seen from (10) and (11) that the energies of the accelerated ions of both groups differ significantly. The second, fast, group of accelerated ions appears however only for front speeds $\alpha < 1$.

The average time which the ions spend in the layer of space charge depends on the velocity of the front. Assuming that all the sites λ_0 where the ions are produced are equally probable, we obtain the characteristic relaxation time of the ion cloud in the region of the beam front:

$$\mu = \int_0^1 \tau \, d\lambda_0. \tag{12}$$

Substituting (9) and (11) into (12) we easily obtain a function which describes the dependence of the relaxation time μ on the velocity α of the front. The graph of this function is represented in Fig. 1. Obviously, the relaxation time of the ion cloud influences substantially the dynamics of the beam front particularly at velocities where the relaxation time varies strongly as a function of the velocity.

2.3. The Motion of the Beam Front as a Function of the Ion Density

The ions which compensate the space charge of the beam electrons appear as a result of ionizing collisions of the electron beam with gas particles (with a cross section σ_e) and as a result of the collision of already accelerated ions with gas particles (cross section σ_i). The frequency of collisions determined by both processes equals

$$\nu = n_g(\overline{\sigma_e \beta c} + \varkappa \overline{\sigma_i v_i}), \tag{13}$$

where n_g is the density of gas particles and v_i is the velocity of the accelerated ions. In analogy with the old model we would obtain $w = \nu d$, but one must take into account that part of the accelerated ions moves ahead of the front and creates there an ion density n_{i^+} already before the arrival of the beam front, so that we obtain

$$w = \frac{\nu d}{1 - n_{i+}/n}. \tag{14}$$

Introducing the parameters

$$\varkappa_+ = n_{i+}/n_e, \quad B = \overline{\sigma_i v_i}/\overline{\sigma_e \beta c},$$

we obtain in dimensionless units (taking into account (5))

$$\alpha = \frac{\alpha_0}{\sqrt{1-\varkappa}\,(1-\varkappa_+)}(1 + B\varkappa). \tag{15}$$

Here $\alpha_0 = n_i \overline{\sigma_e \beta c} d_0 / \sqrt{2dg}$ corresponds to the beam front velocity in the old model, which did not take into account the ionizing action of the ions in motion.

The quantities κ and κ_+ depend on time. At the start of the electron injection both vanish. The equation for the dependence of κ on time and on the other parameters of the system is

$$\frac{d\varkappa}{dt} = \nu + \frac{w}{d}\varkappa_+ - \frac{\varkappa}{\mu\sqrt{2d/g}}. \tag{16}$$

Here the first term in the right-hand side corresponds to the formation of new ions by collisions, the second

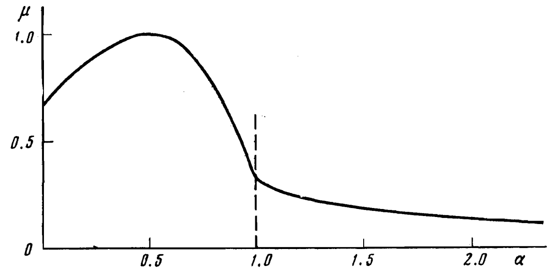

FIG. 1. The dependence of the time spent by the ions in the space charge layer on the beam front velocity.

term corresponds to the influx of ions when the beam penetrates into the region where there is already preionization, and the third term corresponds to the outflow of ions beyond the space-charge region, determined by the relaxation time μ. If one writes (16) in dimensionless form making use of the time scale $\vartheta_0 = \sqrt{2d_0/g_0}$, we obtain, with the help of (5), (7), (13)–(15) the equation

$$\vartheta_0 \frac{d\varkappa}{dt} \equiv \dot{\varkappa} = \left(2\alpha - \frac{\varkappa}{\mu}\right)\sqrt{1-\varkappa}. \tag{17}$$

Since our model is sufficiently crude, we have neglected in the derivation of Eq. (17) (as well as in the calculation of μ) the influence of the acceleration of the motion of the front, which is valid if $\vartheta_0 d\alpha/dt \equiv \dot{\alpha} \ll 1$.

For a complete determination of the front dynamics one also needs the expression for κ_+. The ion density ahead of the front of the beam is determined by the ionizing action of all the accelerated ions which have traversed the leading edge of the space charge before the arrival of the beam front at the given place. From these considerations it follows that

$$\varkappa_+ \approx C\alpha_0 \int_0^t \frac{\varkappa}{\vartheta_0} \zeta \, dt, \tag{18}$$

where

$$C = \sigma_{i+}\sqrt{2dg}/\overline{\sigma_e \beta c}, \tag{19}$$

and the parameter

$$\zeta = \left(1 - \frac{\alpha^2}{1 - \dot{\alpha}\sqrt{1-\varkappa}}\right)^{-1} \tag{20}$$

describes the fraction of accelerated ions which pass the leading edge of the beam front. If the right-hand side of the equation (20) becomes negative, we have by definition $\zeta = 0$.

The total number of the accelerated ions moving ahead of the beam front, N_{i^+} is related to κ_+ by the relation

$$N_{i+} = \frac{\pi R_e^2 n_e}{\sigma_{i+} n_g} \varkappa_+. \tag{21}$$

The maximal number of such ions can be estimated from the condition $\kappa_+ < 1$ since for $\kappa_+ \geq 1$ the beam would be decelerated.

The coupled system of equations (12), (15), (17) and (18) determines both the motion of the beam front, the intensity and the spectrum of the accelerated ions. In spite of its nonlinearity the system can be solved numerically with relative ease.

In the one-dimensional model there is no mechanism for the synchronization of the motion of the ions accelerated in the forward direction and the slower motion of the beam front. In the three-dimensional case such a mechanism can be imagined in such a way that the fast ions are decelerated by a growth of the potential in advance of the front (produced by the screening action of the drift tube), and the space charge of the cloud of accelerated ions pulls behind it the space charge of electrons situated at the front of the beam. Into our calculations we have not taken into account this effect.

The dynamical deepening of the potential well about which we have already talked above, can be easily taken into account in the numerical calculation. In view of the crudity of our model it suffices, making use of the results of ref.[9] to multiply E_0 by a deepening factor equal to 2–3, and to introduce the appropriate correction into the deceleration length d_0.

3. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTS

The numerical calculations have been carried out with a TPAI computer and partially with a Wang desk calculator for a wide range of parameters. As initial conditions for t = 0 we have $\alpha(0) = \alpha_0$, $\kappa(0) = 0$ and $\kappa_+(0) = 0$, hence we get from (15) and (17) directly $\dot{\kappa}(0) = 2\alpha_0$ and $\dot{\alpha}(0) = \alpha_0^2(1 + 2B)$.

Some of the results are shown in Figs. 2–7.[1] In Fig. 2 and 3 one can see the behavior of the beam front: it is accelerated until its velocity reaches the asymptotic limiting value, which for small α_0 (we recall that α_0 is proportional to the gas pressure) equals $\alpha = 1$, and for large α_0 increases approximately proportionally to α_0. This behavior can also be derived analytically from the system of equations. The boundary between these two regions becomes lower as B increases. In the case B = 1 this boundary is situated, e.g., at $\alpha_0 = 0.344$ and for B = 3 at $\alpha_0 = 0.187$. Figure 3 shows the characteristic formation of a plateau for low pressures, as observed experimentally.

As a function of time the dimensionless distance s (in units of $2d_0$) of the beam front from the injection point is given by the expression

$$s = \frac{1}{\vartheta_0}\int_0^t \alpha\, dt. \qquad (22)$$

The form of this function also reminds one of the experimental results (cf. Fig. 4). The maximum of the acceleration $\dot{\alpha}$ of the front at large values of α_0 almost coincides with t = 0. For small values of α_0 the acceleration is very weak at the beginning, but at a later time (at the bend of the graphs in Fig. 4) $\dot{\alpha}$ (t) exhibits a sharp maximum. During this short interval the condition $\dot{\alpha} \ll 1$ is not satisfied. In our opinion, however, such a short-time violation of one of the assumptions of the model does not influence strongly the qualitative shape of the functions shown in Fig. 2–4.

The behavior of the relative density κ of the ions at the beam front as a function of time is shown in Fig. 5. The asymptotic value of κ decreases as α_0 increases and lies in the interval $2/3 \geq \kappa_\infty \geq 1/2$.

For small pressures α remains smaller than one and the ions can be accelerated through the leading edge of the space charge. An exception is usually the time interval in which the front is strongly accelerated. Therefore, most of the time there appear two separate (in time) ion pulses of different energies. Taking into account the relations (11) one can compute the spectrum of the fast ions accelerated in the forward direction as well as its time development. The result of one of these calculations is shown in Fig. 6. A narrow peak of ions

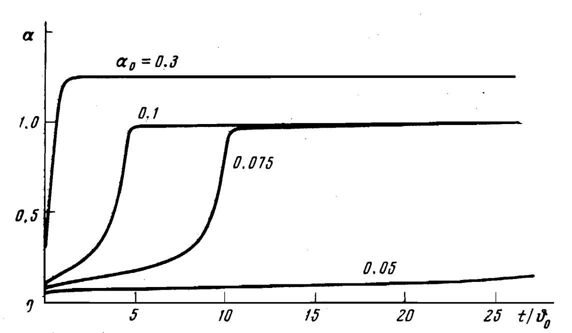

FIG. 2. The time variation of the beam front velocity for different values of α_0.

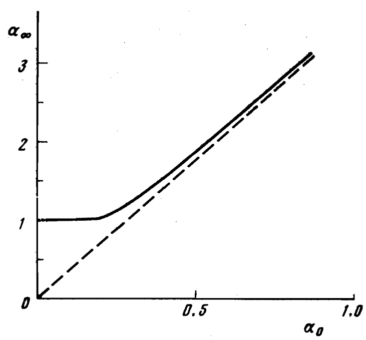

FIG. 3. The dependence of the asymptotic value of the beam front velocity on α_0. For large α_0 the asymptotic velocity approaches the dotted straight line ($\kappa = 1/2$).

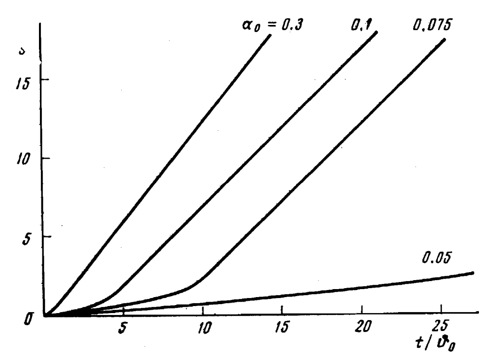

FIG. 4. The time dependence of the path traveled by the beam front for various values of α_0.

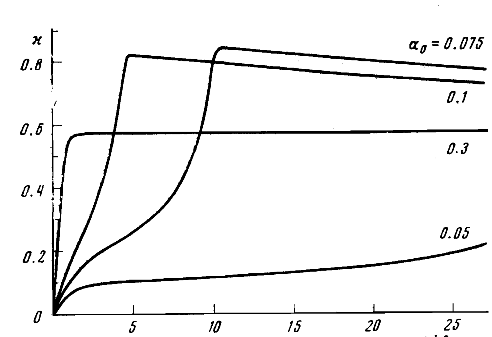

FIG. 5. The time behavior of the relative ion space charge κ at the beam front for various values of α_0.

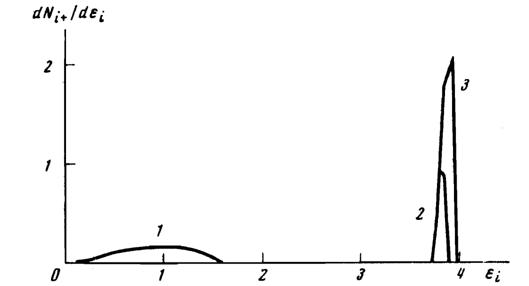

FIG. 6. The energy distribution of the ions which have been accelerated through the anterior boundary of the beam front, for the case $\alpha_0 = 0.16$: 1) $t = 0.9\ \vartheta_0$, the first pulse is formed in the initial stage during the slow motion of the pulse; 2) $t = 5\ \vartheta_0$ there appears a second pulse; 3) $t = 21\ \vartheta_0$, the second pulse is in the final stage.

with energies close to $4zE_0$ is obtained, as well as a wider peak at lower energies. The shape of the computed spectrum is also qualitatively similar to the experimentally observed one, but, as has already been said above, our model does not consider a mechanism of synchronization between the velocities of the accelerated ions and the velocity of the beam front. Therefore one cannot expect a quantitative agreement between the ion spectrum and the experimental data. The ions which go behind the beam front have in the majority of cases a lower energy; they have not been taken into account in Fig. 6.

As regards the total number of accelerated ions N_{i^+} (cf. Eq. (27)), it decreases with the increase of the density n_g of the gas particles and of the ionization cross section σ_{i^+}, in agreement with the experimental data. In particular, for multiply charged ions with their larger ionization cross sections one observes lower intensities of the ion pulses than for protons.

Finally, we carry out a quantitative comparison with the data of Rander et al.[3, 5]. Owing to the very large currents in these experiments the condition of electrostatic deceleration of the beam is better satisfied than in the experiments of Graybill and Uglum[2]. Taking into account the dynamical deepening of the potential well, as well as the known ionization cross sections, we have adjusted the parameters of the model so that a best fit is obtained with the experimental data. Comparing the measured beam front velocity in the plateau region with the calculated value we obtain for the deepening factor of the potential the value 2.0 ± 0.4, in good agreement with the expected magnitude. Estimates from the data in tables of the average ionization cross section of hydrogen yield ($Q = \sigma n_g$ for a pressure of 1 Torr):

$$\overline{Q_e\beta c} \approx 0.6\cdot 10^9\ \mathrm{Torr}^{-1}\mathrm{s}^{-1}$$

$$\overline{Q_i v_i} \approx 1.8\cdot 10^9\ \mathrm{Torr}^{-1}\mathrm{s}^{-1}$$

We therefore choose the value of the parameter $B = 3$. Since the ions ahead of the front have a larger energy than during the acceleration process, their cross section there is smaller and the parameter C must be smaller than B. We have adopted, as in earlier calculations, $C = B/2$. From a best fit of the calculated and experimental curves for given parameters B and C we obtain

$$\overline{Q_e\beta c} = 1\cdot 10^9\ \mathrm{Torr}^{-1}\mathrm{s}^{-1}$$

in fairly good agreement with the estimate from the data of tables. The result of an adjustment for a concrete experiment with $E_0 = 0.5$ MeV and $J = 160$ kA

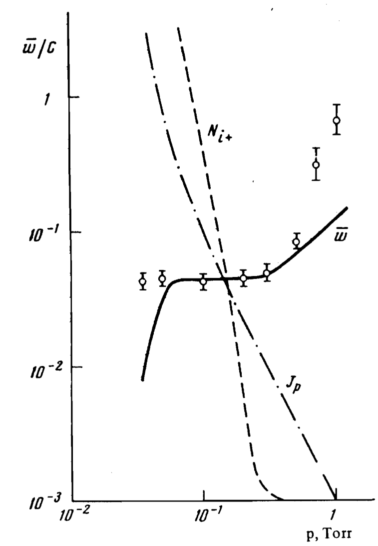

FIG. 7. The pressure dependence of the computed averages of the beam front velocity, $\bar{w}$ between the points $x = 12$ cm and $x = 54$ cm of the drift tube. The computed solid line has been adjusted to the experimental values (circles) obtained by Rander [5] for the case $E_0 = 0.5$ MeV, $J = 160$ kA. In addition are represented in a linear scale the computed total number of accelerated fast ions, N_{+i} and the proton current J_p according to the data of Yonas [4].

is shown in Fig. 7, where instead of the asymptotic value of the front velocity we have represented the average velocity between the points $x = 12$ cm and $x = 54$ cm of the drift tube. This is the reason for the strong decrease of the velocity for low pressures.

The measured dependence of the proton current on the pressure[4] agrees qualitatively with the behavior of the calculated quantity N_{i^+} (cf. Fig. 7). If one compares the maximally possible (in our model) number of accelerated ions with the one obtained experimentally, no contradiction arises in this case also. Indeed, the relation (21) yield in the conditions of the experiment represented in Fig. 7, for $p = 0.1$ Torr the value $N_{i^+} < 3 \times 10^{14}$. The experimental value is 10^{12}–10^{14} accelerated protons per pulse.

In conclusion one may say that the main features of the phenomena observed at low pressures (in the plateau region) are well described by our calculations, qualitatively and in part also quantitatively. The most important result consists in the fact that the idea of formation and acceleration of ions at the beamfront automatically yields a complete picture of the beam dynamics. It is impossible to understand the motion of the electron beam in the gas without taking into account the motion of the ions. This is clearly seen from the fact that according to our model, the velocity of the beam front in the plateau region does not depend either on the ionization cross sections or on the electron current, but only on the electron energy and the charge and mass of the ions. In the plateau region $\alpha \simeq 1$, i.e., $w = \sqrt{2gd} = c[2mz(\gamma-1)/M]^{1/2}$. One can propose an experiment in order to verify this assertion in a pure form. For this purpose it would be useful to compare the beam front velocities in hydrogen and deuterium, keeping the other conditions of the experiment identical. As regards the effect of appearance of fast accelerated ions, it is obviously not a side effect. This effect is inseparably related to the mechanism of propagation of the electron beam at low pressures.

Thus, the proposed model, in which in a crude form we have taken into account the motion of the ion in the space charge region at the front of the electron beam, is capable of explaining qualitatively, and in part quantitatively, the following experimentally observed phenomena: 1) the dependence of the beam front velocity on the time and range; 2) the dependence of the beam front velocity on the gas pressure in the drift tube, in particular the appearance of a plateau at low pressures; 3) the high energy of the accelerated ions, which can be several times larger than the energy of the primary electrons; 4) the proportionality of the ion energy to the ion charge; 5) the decrease of the intensity of the ion pulse as the pressure increases, and its disappearance for sufficiently high pressures; 6) the narrow energy spread of the ion pulse; 7) the independence of the maximal ion energy of the pressure in the plateau region; 8) the decrease of the intensity of the accelerated ions as the charge number increases. So far there is no explanation for the synchronization between the velocities of the ions which are accelerated in the forward direction and electron beam front which follows behind them. Therefore one must treat with care the conclusions about the energy spectrum of the accelerated ions. A good agreement of the calculations with the experimental data argues in favor of the fact that this deficiency of the one-dimensional model does not affect strongly the conclusions on the motion of the beam front.

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[1)]In the calculations illustrated in the figures we have used the parameters B = 3, C = 1.5.

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