Theory of the radiative width of a highly excited nucleus

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A microscopic theory of the electromagnetic radiation emitted by a highly excited nucleus is developed on the basis of the Landau theory of a Fermi liquid. Closed formulas are obtained for the mean radiative width and its mean square fluctuation from level to level. The temperatures of many nuclei are found from the observed widths. The relaxation (i.e., the thermal equilibrium establishment) time is estimated from the experimental data on the radiative-width fluctuations. The regions of applicability of the various types of relations between the relaxation time and the lifetime of the compound nucleus, as well as the relevant physical consequences, are discussed.

1. INTRODUCTION

That there will be a gradual unification of the mechanisms underlying the emission of electromagnetic radiation by a not too light nucleus as the excitation energy of the nucleus increases is practically beyond question. Roughly speaking, if an infinite number of the levels of the nucleus as a whole lie below the initial excitation energy, then the system will itself find and prefer the easiest effective way of emitting γ quanta. In particular, there is below the initial excited level an abundance of levels γ transitions to which satisfy the most favorable "selection rules," so that from this point of view the process is, in the limit under consideration, practically one with an infinite number of channels. Analysis shows (see also[1]) that the electric-dipole radiation due to collisions between the (proton and neutron) quasiparticles and the "wall" of the nucleus predominates. The ideas of the Landau theory of the Fermi fluid[2,3] allow us to compute in closed form the radiative width $\bar{\Gamma}_\gamma$ and its fluctuations from level to level.

Besides, in a specific sense of the word, spherical nuclei are rather exotic objects: the application to them of the Fermi-fluid concepts requires in each specific case certain precautions. The fact that the spherical configuration is stable is in itself an indication of the essential role played by the "residual interaction" between the quasiparticles, an interaction which blurs the Fermi level: it can be shown that in the scheme without interaction the sphere is absolutely unstable (see[4]). Furthermore, analysis of the data on the shell and magic oscillations in the masses of spherical nuclei allows the establishment of the macroscopically ordered structure that this residual interaction possesses in the space of the values of the orbital momentum l of the individual quasiparticles[5]. (We shall again touch upon specifically spherical nuclei when we compare below the theoretical results with the experimental data.) Nonspherical nuclei, on the other hand, are easy to investigate, it being apparently necessary to regard their shape as a perfectly natural consequence of the properties of the "normal," disordered nuclear phase, in which the quasiparticles situated near the limit of the Fermi distribution move, in the main, independently of each other.

However, for the theory of radiative widths expounded below the "shape effects," as such, are of no independent importance, and a special allowance for them is not necessary. Indeed, the equilibrium deformation α of a nonspherical nucleus is equal in order of magnitude to ρ_f^{-1} (i.e., $\alpha \sim \rho_f^{-1}$), where

$$\rho_f = k_f R \gg 1 \tag{1}$$

(k_f is the limiting momentum of the quasiparticle distribution and R is the radius of the nucleus) is an important dimensionless parameter that arises in the most diverse investigations that have as their aim the treatment of the nucleus as a macroscopic body. In view of the scalar nature of the quantity Γ_γ to be computed, only the squared deformation can enter, so that the relative magnitude $\alpha^2 \sim \rho_f^{-2} \ll 1$ of the corresponding corrections is negligibly small and falls outside the limits of accuracy of the theory. In other words, only the possibility of considering the quasiparticles individually is important for what follows, it still being possible in actually occurring deformations to treat the geometry of the motion of each of the quasiparticles as spherically symmetric.

2. THE RADIATIVE WIDTH OF A HIGHLY EXCITED NUCLEUS

We shall derive the expressions, referred to one quasiparticle, on the basis of the correspondence principle. As applied to radiation processes, this principle asserts total analogy between the formulas of the classical and quantum theories (see, for example,[6]). The classical intensity I (i.e., the energy emitted per unit time) needs only to be divided by $\epsilon = \hbar\omega$ to be converted into the quantity of real interest–the probability of emission of individual γ quanta. The only remaining difference consists in the following: the spectral component of the multipole moment, which varies according to a classical law, should, generally speaking, be replaced by the corresponding matrix element of its operator. They, however, coincide in the quasiclassical limit (see (1))[7]. Consequently, we can speak of a quasiparticle trajectory: it is in this case a straight line joining two opposite points on the surface of the nucleus, i.e., a chord.

The basic formula of the classical theory of the electric dipole radiation has the form

$$I = (2e_d^2/3c^3)\ddot{\mathbf{r}}^2, \tag{2}$$

where e_d is the charge of the radiating particle (the radiating quasiparticle, in the general case; see below) and $\ddot{\mathbf{r}}$ is its acceleration vector. Before proceeding to the spectral decomposition of I, let us note that in the thermal-equilibrium state the quasiparticle motion inside the nucleus does not vary its qualitative character in time. Therefore, let us formally carry out the Fourier expansion over an arbitrary, but sufficiently long interval of time t and then take the limit as $t \to \infty$ (see[8]):

$$\overline{\ddot{\mathbf{r}}^2}=\frac{1}{\pi t}\int_0^\infty\left|\int_0^t \mathbf{r}(t)e^{i\omega t}\,dt\right|^2\omega^4\,d\omega. \tag{3}$$

In view of the independence of the different chords traced by the quasiparticles in their wall-to-wall motion, we have

$$\left|\int_0^t \mathbf{r}e^{i\omega t}\,dt\right|^2=\overline{\left|\int_0^{t_1}\mathbf{r}e^{i\omega t}\,dt\right|^2}n\cong\overline{\left|\int_0^{t_1}\mathbf{r}(t)\,dt\right|^2}n, \tag{4}$$

where $t_1 = l/v_f$ is the time it takes to travel from one end of a chord to the other, $l = 2\sqrt{R^2-\rho^2}$ is the length of the chord, and n is the number of chords. Here we have taken into account the fact that in the region of the radiation energy spectrum of interest to us

$$\omega t_1\ll 1 \tag{5}$$

(we shall return to the criterion (5) later). Furthermore, here and below the quasiparticle velocity v is replaced everywhere by its limiting value v_f. The point is that because of the Pauli principle only those "elementary emitters" (i.e., quasiparticles) that are situated in the immediate neighborhood of the Fermi level play a role (see below).

The distribution of the chords over the impact parameters ρ is easily found from considerations of isotropy and homogeneity of nuclear matter:

$$w(\rho)\,d\rho=\frac{3}{R^3}\sqrt{R^2-\rho^2}\,\rho\,d\rho;\quad \int_0^R w(\rho)\,d\rho=1. \tag{6}$$

Averaging, in accordance with (4) and (6), the square of the integral over the radius vector, we also express the number n of quasiparticle-nuclear wall collision events in terms of the physical time t spent in them:

$$\overline{\left|\int_0^{t_1}\mathbf{r}(t)\,dt\right|^2}=\frac{4}{v_f^2}\overline{\rho^2(R^2-\rho^2)}=\frac{24R^4}{35v_f^2},$$
$$n=t\frac{v_f}{l}=\frac{2}{3}\frac{v_f}{R}t. \tag{7}$$

Taking into consideration the relations (2)–(7) and the relevant considerations, we obtain

$$f(\varepsilon)\,d\varepsilon=\frac{32}{105\pi}\frac{e_d^2}{\hbar^5c^3}\frac{R^3}{v_f}\varepsilon^3\,d\varepsilon. \tag{8}$$

This expression gives the probability per unit time of emission by one quasiparticle of a γ quantum in the interval $d\epsilon$ of its energy values.

According to the theory of the Fermi fluid[2,3], the mean occupation numbers of the individual quantum states are given by the standard Fermi distribution

$$\bar n(\varepsilon')=(e^{\varepsilon'/T}+1)^{-1}, \tag{9}$$

where ϵ' is the quasiparticle energy measured relative to the chemical potential and T is the temperature. On the other hand, the number of actual single-quasiparticle states in the volume $V = 4/3\pi R^3$ is equal to

$$d\bar N=\frac{V}{\pi^2\hbar^3}\frac{p^2}{d\varepsilon'/dp}d\varepsilon'\cong\frac{4R^3}{3\pi\hbar^3}\frac{p_f^2}{v_f}d\varepsilon' \tag{10}$$

($p_f = \hbar k_f$ is the limiting momentum in standard units), where allowance has been made for the additional spin doubling. In fact, even in the quasiclassical limiting case, (1) remains an important quantum effect due to the identity, the indistinguishability of identical fermions[7]: the above-described classical picture of the process is actually realizable only in the case of radiative transitions that are compatible with the Pauli principle. Therefore, the product of the expressions (8), (9), and (10) should be supplemented by the factor

$$1-\bar n(\varepsilon'-\varepsilon),$$

which determines the fraction of the transitions admissible by this principle. Then integration over the energies ϵ' of the radiating quasiparticles will reduce to

$$\int_{-\infty}^{\infty}\bar n(\varepsilon')[1-\bar n(\varepsilon'-\varepsilon)]\,d\varepsilon'=\frac{\varepsilon}{e^{\varepsilon/T}-1}. \tag{11}$$

Integration over the boson energies yields

$$\int_0^\infty\frac{\varepsilon^4\,d\varepsilon}{e^{\varepsilon/T}-1}=24\zeta(5)T^5,\quad \zeta(z)=\sum_{n=1}^{\infty}\frac{1}{n^z}, \tag{12}$$

where ζ is the Riemann zeta function. Finally, the γ-quantum emission probability per unit time, appropriately summed over the entire set of quasiparticles of the same sort will be given by

$$W=\frac{1024}{105\pi^2}\zeta(5)\frac{e_d^2m^{*2}}{\hbar^5c^3}R^6T^5, \tag{13}$$

where $m^* = p_f/v_f$ is the effective mass of the quasiparticle.

Above, as the coordinate origin convenient for the calculations, we used the geometrical center of the nucleus. However, the role of the total charge Ze of the whole system in processes induced by the oscillations of the radius vector of the individual nucleons (quasiparticles) is well known. Because of recoil, even the electrically neutral quasiparticles (i.e., the neutronic quasiparticles) will appear to emit radiation during their motion relative to the center of the nucleus. The corresponding, well-known, "charge-renormalization" formulas have the form

$$e_d^Z=\left(1-\frac{Z}{A}\right)e,\quad e_d^N=-\frac{Z}{A}e \tag{14}$$

(the "effective charges," (14), of the two components are correct only for processes induced by the oscillations of the electric dipole moment of the nuclear system (see, for example,[7])). Summing, with allowance for (14), the expressions (13) or the proton and neutron components of the nuclear matter, and multiplying them by $\hbar$ in order to convert them into the energy widths of interest to us, we finally obtain

$$\bar\Gamma_\gamma=\frac{1024}{105\pi^2}\zeta(5)\frac{e^2m^{*2}}{\hbar^7c^3}\left[1-2\frac{Z}{A}\left(1-\frac{Z}{A}\right)\right]R^6T^5 \tag{15}$$

(notice that the numerical factor $(1024/105\pi^2)\zeta(5) \cong 1$ is very close to unity). The law $\bar\Gamma_\gamma \propto T^5$ was given in the preceding paper (see[1], formula (4)), where it was motivated by semiphenomenological considerations.

To what extent can the result (15) be identified with the radiative widths of the individual resonance levels of a compound nucleus that is excited, say, in a reaction involving slow-neutron capture? It follows from its derivation that the formula (15) corresponds to a state in which at the temperature T the quasiparticles of the nuclear Fermi liquid are in thermal equilibrium with each other. On the other hand, the width Γ_γ of a specific level can, reasoning abstractly, be conceived to have been computed from some very complicated, unknown (to us) wave function of the corresponding state of the nucleus as a whole. According to the fundamental principles of statistics, the two approaches lead to results that coincide to within the values of the fluctuations (see, for example,[3]).

Let us now rewrite the condition (5) of applicability of the theory in a more concrete form. Owing to the thermal nature of the radiation, the inequality (5) is equivalent to the following inequality:

$$T\ll\varepsilon_0, \tag{16}$$

where

$$\varepsilon_0=\hbar v_f/2R\sim 5\ \text{MeV} \qquad (17)$$

is the characteristic energy corresponding to the reciprocal of the time it takes a quasiparticle to cross the nucleus along a diameter.

It is worth noting that in the opposite limiting case

$$T\gg\varepsilon_0, \qquad (18)$$

because of the oscillations of the exponent $e^{i\omega t}$ along the chord traced by the quasiparticle (see (3) and (4)), the energy distribution of the γ-quantum emission probability acquires the form of the well-known Planck black-body radiation spectrum[3]. The radiation width would, accordingly, become proportional to the cube of the temperature in the case of a sufficiently strict fulfilment of the condition (18). However, this "black-body radiation limit" defined by (18) is, in practice, hardly attainable in nuclear physics. At least the temperature of the compound nucleus should not exceed the nucleon binding energy, which is ~8 MeV–otherwise the neutrons would fly out of the nucleus "instantly," escaping the thermal-equilibrium establishment phase[1)].

3. THE MEAN-SQUARE FLUCTUATION IN THE RADIATION WIDTH. THE ROLE OF THE RELAXATION TIME

The direct, quantum-mechanical computation of the characteristics of the individual states of the nucleus is inexpedient and practically impossible. Furthermore, as applied to macroscopic bodies (see the criterion (1)), this, as a rule, borders on the theoretical impossibility[3]. Therefore, we are obliged here to treat the state of the occupation of the individual quantum states of the quasiparticles of the Fermi liquid as a randomly varying function of the time. We shall calculate the instantaneous "emissive power" $\tilde{\Gamma}_\gamma$ of the nucleus in a manner completely similar to the computations of the preceding section. The "one-component" variant of the corresponding formula can be represented in the form

$$\tilde{\Gamma}_\gamma=\frac{32}{105\pi}\frac{e_d^2}{\hbar^4c^3}\frac{R^3}{v_f}\int_0^\infty d\varepsilon_1\cdot\varepsilon_1^3\sum_{\varepsilon'}n(\varepsilon')[1-n(\varepsilon'-\varepsilon_1)]. \qquad (19)$$

Here we have, for simplicity and convenience, written the discrete sum $\Sigma_{\epsilon'}$ over the fermion states. In case of need the transition to integration can easily be accomplished with the aid of (10).

The "instantaneous," physically realizable values

$$n_{\varepsilon'}=0,\ 1 \qquad (20)$$

of the fermion occupation numbers differ from the mean occupation numbers (9). This circumstance is the obvious cause of fluctuations in Fermi systems. It is convenient to consider them with the aid of the simple relation (see[3])

$$\overline{\Delta n'\Delta n''}=\bar{n}'(1-\bar{n}')\delta_{\varepsilon',\varepsilon''}. \qquad (21)$$

Let us find the mean-square fluctuation of the expression (19)–the number of summations and integrations doubles upon squaring. One summation over the quasiparticle states is trivial owing to the presence of the δ symbol on the right-hand side of (21); the subsequent integration is elementary, although somewhat tedious. Adding, in accordance with (14), the squares of the fluctuations in the proton and neutron components and introducing the dimensionless variables $x_{1,2}=\epsilon_{1,2}/T$ in place of the γ-quantum energies, we obtain

$$\overline{(\Delta\tilde{\Gamma}_\gamma)^2}=\frac{8192J}{33075\pi^3}\frac{e^4m^{*3}}{\hbar^{12}c^6}\left[\frac{(1-Z/A)^4}{\rho_f^Z}+\frac{(Z/A)^4}{\rho_f^N}\right]R^{10}T^9,$$
$$J=\int_0^\infty\int_0^\infty\left\{\frac{x_1e^{x_1}\operatorname{cth}(x_1/2)}{(e^{x_1}-e^{x_2})(e^{x_1}-e^{-x_2})}+\frac{x_2e^{x_2}\operatorname{cth}(x_2/2)}{(e^{x_2}-e^{x_1})(e^{x_2}-e^{-x_1})}\right\}x_1^3x_2^3\,dx_1\,dx_2. \qquad (22)$$

The details of the integration over the boson energies are given in the Appendix. The final result has the form

$$J=\frac{848}{1575}\pi^8+576\sum_{n=1}^\infty\frac{1}{n^4}\sum_{k=n+1}^\infty\frac{k-n}{k^5}. \qquad (23)$$

Notice that the term with the double sum is about half percent of the value of the integral, so that in practice we can restrict ourselves to the consideration of only the first term on the right-hand side of (23).

The problems pertaining to the fluctuations are relatively subtle and require a more careful physical treatment. In particular, there is no reason to equate $\overline{(\Delta\tilde{\Gamma}_\gamma)^2}$ to the mean square $(\Delta\Gamma_\gamma)^2$ of the actually observable, physical fluctuation in the radiative widths of many close resonance levels. This becomes especially apparent when we consider the most important and interesting case in which thermal equilibrium in the nucleus is established long before the "decay" of the nucleus:

$$\Gamma\tau/\hbar\ll 1. \qquad (24)$$

Here τ is the relaxation time (see below) and Γ is the total width of the initial state of the nucleus. Taking into account the fact that this quasistationary state decays according to the law $e^{-\Gamma t/\hbar}$, we express the number ν of emitted quanta and its fluctuation in terms of the instantaneous emissive power $\tilde{\Gamma}_\gamma$:

$$\nu=\frac{1}{\hbar}\int_0^\infty\tilde{\Gamma}_\gamma(t)e^{-\Gamma t/\hbar}\,dt,$$
$$(\Delta\nu)^2=\frac{1}{\hbar^2}\int_0^\infty\int_0^\infty\Delta\tilde{\Gamma}_\gamma(t)\Delta\tilde{\Gamma}_\gamma(t')\exp\left[-\frac{\Gamma}{\hbar}(t+t')\right]dt\,dt', \qquad (25)$$
$$\Delta\tilde{\Gamma}_\gamma(t)=\tilde{\Gamma}_\gamma(t)-\bar{\Gamma}_\gamma.$$

Further, it is convenient to introduce the notation $t'=t+\tau$. According to the thermodynamic theory of nonequilibrium processes (and of the corresponding fluctuations in the thermal-equilibrium state; see, for example,[3]), the mean value of the time correlation of the fluctuations is given by the relation

$$\overline{\Delta\tilde{\Gamma}_\gamma(t)\Delta\tilde{\Gamma}_\gamma(t+\tau)}=\overline{(\Delta\tilde{\Gamma}_\gamma)^2}\exp(-|\tau|/\bar{\tau}), \qquad (26)$$

where $\bar{\tau}$ is the relaxation time (i.e., the thermal-equilibrium establishment time). With allowance for (24), the substitution of (26) into (25) yields

$$\overline{(\Delta\nu)^2}\cong\frac{\overline{(\Delta\tilde{\Gamma}_\gamma)^2}}{\hbar^2}\int_0^\infty dt\exp\left(-\frac{2\Gamma}{\hbar}t\right)\int_{-\infty}^\infty\exp\left(-\frac{|\tau|}{\bar{\tau}}\right)d\tau=\frac{\overline{(\Delta\tilde{\Gamma}_\gamma)^2}}{\hbar\Gamma}\bar{\tau}. \qquad (27)$$

Let us now consider the ensemble of the large number of close levels of a compound nucleus of radiative width Γ: owing to the fact that the levels decay according to the single law $e^{-\Gamma t/\hbar}$, the equilibrium in the ensemble (the equipopulation of the levels) is not destroyed in time. The number of γ quanta

$$\nu=\frac{1}{\hbar}\int_0^\infty\Gamma_\gamma e^{-\Gamma t/\hbar}\,dt=\frac{\Gamma_\gamma}{\Gamma}$$

has been preaveraged over a group consisting of many levels with practically the same Γ_γ. In the final averaging of the square of the fluctuations $(\Delta\nu)^2$ over the entire ensemble of the groups differing in their radiative widths Γ_γ, each group is taken into account with a weight proportional to the number of levels in it:

$$\overline{(\Delta\nu)^2}=\overline{(\Delta\Gamma_\gamma)^2}/\Gamma^2. \qquad (28)$$

Equating the right-hand sides of the formulas (27) and (28), we finally obtain

$$(\overline{\Delta\Gamma_\gamma})^2=(\Gamma\tau/\hbar)\,(\overline{\Delta\Gamma_\gamma})^2. \qquad (29)$$

(we shall no longer write the averaging sign over the relaxation time τ). A striking feature of the relation (29) consists in the following: It turns out that the fluctuations in the probability of decay of the compound nucleus via the radiative channel depend on the total decay probability Γ, including all the generally possible decay channels. The physical meaning of the formula (29) is simple: In the time picture the deviation of $\Delta\overline{\Gamma}_\gamma(t)$, the emissive power, from its mean value has time to average out to some extent provided the decaying exponential function varies sufficiently slowly (see the criterion (24)). The small factor $\Gamma\tau/\hbar$ on the right-hand side of (29) is precisely the quantity that determines the fraction of the physical, actually observable effect that remains after such a partial averaging.

4. COMPARISON WITH EXPERIMENT

With the aid of the formula (15) we determined the temperatures of compound nuclei from the observed radiative widths of their resonance levels[9, 10]. The results of such an analysis for two well-known regions of nonspherical nuclei are given in the table. We assumed in the computations that

$$R=1.2\cdot10^{-13}A^{1/3}\ [\text{cm}] \qquad (30)$$

and $m^* = m_n$, where m_n is the mass of the free nucleon. It is noteworthy that the temperature in the case of the actinide nuclei turns out consistently to be ~100 keV lower than the characteristic temperature for the lanthanide region. This may be due to both the decrease of the neutron attachment energy toward the end of the Mendeleev periodic table and the difference in the atomic weight A. A similar temperature decrease apparently occurs only within the nonspherical-lanthanide region.

Compound nucleus	E_{max}, MeV	$\overline{\Gamma}_\gamma$, 10^{-3} eV	T, MeV	Compound nucleus	E_{max}, MeV	$\overline{\Gamma}_\gamma$, 10^{-3} eV	T, MeV
Nonspherical lanthanides							
$_{62}Sm^{148}_{86}$	8.14	52	0.42	$_{67}Ho^{166}_{99}$	6.33	91	0.45
$_{62}Sm^{150}_{88}$	7.98	64	0.44	$_{68}Er^{167}_{99}$	6.44	97	0.46
$_{62}Sm^{153}_{91}$	5.89	71	0.45	$_{68}Er^{168}_{100}$	7.77	96	0.46
$_{63}Eu^{152}_{89}$	6.29	89	0.47	$_{69}Tm^{170}_{101}$	6.38	86	0.44
$_{63}Eu^{154}_{91}$	6.39	102	0.48	$_{70}Yb^{172}_{102}$	8.14	74	0.43
$_{64}Gd^{156}_{92}$	8.53	110	0.48	$_{70}Yb^{174}_{104}$	7.44	79	0.43
$_{64}Gd^{157}_{93}$	6.35	110	0.48	$_{72}Hf^{178}_{106}$	7.62	64	0.41
$_{64}Gd^{158}_{94}$	7.93	89	0.46	$_{73}Ta^{182}_{109}$	6.06	54	0.39
$_{64}Gd^{159}_{95}$	6.03	105	0.48	$_{74}W^{183}_{109}$	6.19	58	0.40
$_{65}Tb^{160}_{95}$	6.40	90	0.47	$_{74}W^{184}_{110}$	7.42	74	0.42
$_{66}Dy^{162}_{96}$	8.20	122	0.49	$_{74}W^{185}_{111}$	5.75	64	0.41
$_{66}Dy^{163}_{97}$	6.25	175	0.52	$_{74}W^{187}_{113}$	5.46	62	0.40
$_{66}Dy^{164}_{98}$	7.66	103	0.47	$_{75}Re^{186}_{111}$	6.24	55	0.39
$_{66}Dy^{165}_{99}$	5.64	166	0.51	$_{75}Re^{188}_{113}$	5.73	55	0.39
Nonspherical actinides							
$_{90}Th^{233}_{143}$	4.96	21	0.30	$_{94}Pu^{240}_{146}$	6.46	40	0.33
$_{91}Pa^{232}_{141}$	5.52	44	0.33	$_{94}Pu^{241}_{147}$	5.41	31	0.32
$_{91}Pa^{234}_{143}$	5.12	48	0.35	$_{94}Pu^{242}_{148}$	6.22	37	0.33
$_{92}U^{234}_{142}$	6.78	40	0.34	$_{95}Am^{242}_{147}$	5.48	42	0.33
$_{92}U^{235}_{143}$	5.27	25	0.31	$_{95}Am^{244}_{149}$	5.29	50	0.35
$_{92}U^{236}_{144}$	6.47	40	0.33	$_{96}Cm^{244}_{148}$	6.72	37	0.33
$_{92}U^{237}_{145}$	5.30	29	0.31	$_{96}Cm^{245}_{149}$	5.70	39	0.33
$_{92}U^{239}_{147}$	4.78	23	0.30	$_{96}Cm^{247}_{151}$	5.21	35	0.32
$_{93}Np^{238}_{145}$	5.43	34	0.32				

Spherical nuclei possess a number of unique features that must be taken into consideration (see the Introduction). However, the question of the applicability to them of the formula (15) is at present difficult to answer categorically. Indeed, nuclei of this sort apparently undergo a phase transition to the "normal," nonspherical state at temperatures

$$T\sim\Delta\varepsilon',$$

where $\Delta\epsilon'$ is some characteristic width of the diffuse zone of the Fermi distribution, a zone which owes its existence to the residual interaction. Meanwhile, the spectrum of the emitted quanta (it is given by the integrand on the left-hand side of (12)) is such that the energy averaged over it is equal to

$$\bar{\varepsilon}\cong5T$$

(see also[1], formulas (5) and (6)). Thus, many of the radiative transitions can, roughly speaking, elude that region of the statistical distribution of the quasiparticles where the distribution differs significantly from (9). Therefore, the attempts to apply the formula (15) also to spherical nuclei, though not rigorous, is nevertheless of some interest. It is natural to suppose that spherical nuclei have higher temperatures (and, consequently, relatively low entropies; see also[1]). Comparison with the data on the radiative widths apparently corroborates this trend. For example, for the compound nucleus $_{79}Au^{198}_{119}$ we obtain T = 0.45 MeV, in the case of $_{80}Hg^{202}_{122}$ we have T = 0.56 MeV, and, finally, T = 0.62 MeV for $_{81}Tl^{204}_{123}$.

The experimental study of radiative-width fluctuations became possible only recently as a result of an increase in the accuracy of their measurement, and comparison of the theoretical formulas with experiment meets for the present with certain practical difficulties. Let us discuss three specific nuclei, for which a selection of resonance levels with accurately measured radiative widths nevertheless allowed the estimation of the relaxation time τ from the formula (29) (see also (22) and (23)). Data on two gadolinium isotopes are given in[11]; we took into consideration only the levels for which the error in the radiative width is $\leq 10^{-2}$ eV. In the case of $_{64}Gd^{156}_{92}$ (10 levels) $\overline{\Gamma}_\gamma$ = 0.11 eV, $[\overline{(\Delta\Gamma_\gamma)^2}]^{1/2}$ = 0.016 eV, and $\hbar/\tau$ = 26 eV. For $_{64}Gd^{158}_{94}$ (11 levels) we have $\overline{\Gamma}_\gamma$ = 0.089 eV, $[\overline{(\Delta\Gamma_\gamma)^2}]^{1/2}$ = 0.0087 eV, and $\hbar/\tau$ = 52 eV. Let us also give the results of a similar analysis of the data on holmium[12]: $_{67}Ho^{166}_{99}$ (21 levels) $\overline{\Gamma}_\gamma$ = 0.091 eV, $[\overline{(\Delta\Gamma_\gamma)^2}]^{1/2}$ = 0.0099 eV, and $\hbar/\tau$ = 39 eV. Thus, as far as we can judge, $\hbar/\tau \sim 50$ eV and $\tau \sim 10^{-17}$ sec, which is a remarkably long time on the nuclear scale. We must, however, not forget that the longest of the relaxation times τ of the system enters into the thermodynamic theory (see formula (26)). The "particle equilibrium" at each moment of time was understood to have been established over the significantly shorter relaxation times[2].

5. DISCUSSION. IS THERE ENOUGH TIME FOR THE ESTABLISHMENT OF THERMAL EQUILIBRIUM IN A NUCLEUS?

The question of relaxation in nuclear matter is of considerable interest. Thus far, as far as we know, it has not been possible to estimate the characteristic time of this process on the basis of any direct analysis

of the experimental data. Therefore, the observed fluctuations in the radiative widths can be a valuable source of such information, and even the preliminary, tentative figures ($\tau \sim 10^{-17}$ sec; see the preceding section) need to be discussed. For the above-mentioned particular cases of resonance excitation by neutrons of energy $\lesssim 1$ keV, the condition (24) was satisfied with three orders of magnitude to spare–in other words, total thermal equilibrium was attained in the nucleus. The situation can, however, change when we go over to higher kinetic energies of the bombarding particles (see below).

It would, apparently, be somewhat naive to regard the time $t_1 = h/\epsilon_0 \sim 10^{-22}$ sec of transit of a quasiparticle through the nucleus as an estimate for the relaxation time. Indeed, for example, the system is not in the least drawn nearer to the state of thermal equilibrium by a coherent, reversible, purely elastic act of collision between a quasiparticle and the surface of the nucleus[3)]. On the other hand, interquasiparticle collisions appear to be quite an effective relaxation mechanism; it may well turn out to be the dominant mechanism.

For obvious reasons, the thermal-radiation data used in the present paper actually pertained to an extremely narrow part of the energy spectrum of the system. Let us now qualitatively consider how the real conditions under which the relaxation process proceeds in the nucleus should change upon further increase in the excitation energy of the nucleus. The neutronic width Γ_n increases first in proportion to the square root of the distance from the neutron-detachment threshold; then there sets in a phase of much more rapid exponential growth. It is well known that owing to this phenomenon the resonance levels merge, forming a continuous spectrum. But then whether the lifetime h/Γ of the compound nucleus will be long enough for the establishment of total thermal equilibrium in the nucleus may become doubtful, beginning from[4)]

$$\Gamma \cong \Gamma_n \geqslant 100 \text{ eV}.$$

In this connection, it is desirable to try and critically reinterpret the method, based on the neutron "evaporation" process, for determining nuclear temperatures. It is difficult for the present to judge how the fact that the state of the neutron-emitting nucleus is not a totally equilibrium state will influence such an analysis. Not much doubt has thus far been expressed about the evaporation temperatures probably because their order of magnitude is quite plausible (and, in so far as we can judge, indeed correct). However, as the experimental investigation of the reactions (n, n′) goes on, attention will have to be paid not only to the absolute figures, but also to the behavior of the relevant quantities. Of special interest, in particular, is the case when the temperature of the compound nucleus as a function of its excitation energy is an almost horizontal, nonmonotonic in detail, and often simply a decreasing function. Unfortunately, the authors of the corresponding publications give this remarkable circumstance comparatively little consideration (see, for example,[15]). We could have attempted to interpret the decrease of the temperature with increasing excitation energy as some giant random fluctuation, but it would have been difficult to conceive it as a phenomenon that would occur with any degree of consistency. On the average, however, negative specific heat is impossible for the nucleus. A state with negative specific heat is totally unstable, and cannot be realized in nature[3].

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APPENDIX

Let us expound in some detail the integration over the boson energies in the formula (22). The terms of the integral J are identical in form, but individually each of them contains a pole at $x_1 = x_2$. Therefore, it is sufficient to evaluate any of these integrals in the principal value sense:

$$J = 2\int_0^\infty dx_1 \cdot x_1^4 e^{x_1} \operatorname{cth}\frac{x_1}{2} \int_0^\infty \frac{x_2^3\, dx_2}{(e^{x_1}-e^{x_2})(e^{x_1}-e^{-x_2})}. \tag{A.1}$$

Let us transform the inner integral with the aid of the substitution $y = e^{-x_2}$, representing it as a derivative with respect to some parameter:

$$\int_0^\infty \frac{x_2^3\, dx_2}{(e^{x_1}-e^{x_2})(e^{x_1}-e^{-x_2})} = e^{-x}\int_0^1 \frac{(\ln y)^3\, dy}{(y-e^x)(y-e^{-x})} = e^{-x}\frac{\partial^3}{\partial \nu^3}\int_0^1 \frac{y^\nu\, dy}{(y-e^x)(y-e^{-x})}, \quad \nu \to 0 \tag{A.2}$$

($x = x_1$). In the decomposition

$$\int_0^1 \frac{y^\nu\, dy}{(y-e^x)(y-e^{-x})} = \frac{1}{2\operatorname{sh} x}\{J_\nu^+(x) - J_\nu^-(x)\}, \tag{A.3}$$

in which the integrand is expressed in partial fractions, the integrals

$$J_\nu^+(x) = \int_0^1 \frac{y^\nu\, dy}{y-e^x}, \quad J_\nu^-(x) = \int_0^1 \frac{y^\nu\, dy}{y-e^{-x}} \tag{A.4}$$

can conveniently be expressed as series. For this purpose, let us represent the fractions by the corresponding geometric progressions, and let us also take into consideration the formula

$$\sum_{n=-\infty}^{\infty} \frac{1}{n+\nu} = \pi \operatorname{ctg} \pi\nu.$$

We then have

$$J_\nu^+(x) = -\sum_{n=1}^\infty \frac{e^{-nx}}{n+\nu}, \qquad J_\nu^-(x) = -e^{-\nu x}\pi \operatorname{ctg} \pi\nu + \frac{1}{\nu} - \sum_{n=1}^\infty \frac{e^{-nx}}{n-\nu}. \tag{A.5}$$

Taking the limit in accordance with (A.2), we obtain

$$J = 192\int_0^\infty \left\{\frac{\xi^3}{3} - \frac{\pi^2}{3}\xi^6 - \frac{\pi^4}{90}\xi^4\right\}\frac{d\xi}{\operatorname{sh}^2\xi} + 6\sum_{n=1}^\infty \frac{1}{n^4}\int_0^\infty \frac{\xi^4 e^{-n\xi}}{\operatorname{sh}^2\frac{\xi}{2}}\, d\xi. \tag{A.6}$$

It is easy to see that the integral standing under the summation sign

$$\int_0^\infty \frac{\xi^4 e^{-n\xi}}{\operatorname{sh}^2\frac{\xi}{2}}\, d\xi = 4\int_0^1 \frac{(\ln y)^4 y^n}{(y-1)^2}\, dy = 4\frac{\partial^4}{\partial \nu^4}\int_0^1 \frac{y^{n+\nu}\, dy}{(y-e^x)(y-e^{-x})}, \quad \nu \to 0 \tag{A.7}$$

is essentially of the same type as (A.2), except that it is differentiated once more with respect to ν and that the additional passage to the limit $x \to 0$ will also be necessary. The subsequent simple computations, besides the passages to the limit, also include convenient redesignations of the indices of the double summation. As a result, we obtain the formula (27).

[1)] Notice that the radiation due to collisions between the quasiparticles would then become dominant only at $T \gg \sqrt{\rho f \epsilon_0}$.

[2)]From the data of the recent paper [13] we find that $\hbar/\tau \sim 20$ eV for the compound nuclei Th^{233} and U^{239}.

[3)]Besides, a characteristic time $\sim t_1$ is quite capable of playing an important role at the earliest stage of the development of the nuclear reaction. Here, however, we are discussing only the late, final phase of the thermal-equilibrium establishment process—see the end of the preceding section. In particular, it is extremely doubtful that there will, at such times, remain reasonable physical criteria for distinguishing the initial bombarding particle (or the corresponding quasiparticle).

[4)]It is possible that the so-called Erickson fluctuations in the nucleon-nucleus interaction cross sections [14] are also due to this circumstance.

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