Polarization effects of the self-action of a light beam in a resonant medium

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The effect of radiation polarization on resonant self-focusing and self-defocusing in a three-level atomic system is investigated theoretically. A strong dependence of the self-action of the light beam on the frequency and polarization of the radiation is observed for frequencies lying between two of the atomic frequencies. It is shown that on the basis of polarization measurements it is possible to trace the dynamics of self-action of a light pulse in a resonant medium.

It is known that many interesting nonlinear effects are produced when high-power laser radiation acts on a resonant medium. A large group of such effects is connected with multiphoton processes such as three-photon scattering[1,2], four-photon parametric interaction[3-5], etc.

Another class of nonlinear resonant effects is connected with self-action processes. At relatively large deviations from resonance, and at negligibly small absorption, these self-action phenomena are the result of the refractive-index nonlinearity that depends on the field intensity, and lead to self-focusing or self-defocusing.

Resonant self-focusing was first observed in potassium vapor[6]. In[7] they investigated resonant self-focusing and self-defocusing, using a source in the form of a high-power pulsed continuously-tunable parametric light generator, so that the dispersion of the nonlinear susceptibility could be traced in the entire frequency region $\omega > \omega_{01}$, $\omega_{01} > \omega > \omega_{02}$ and $\omega < \omega_{02}$, where ω_{01} and ω_{02} are the frequencies of the transitions $4S_{1/2} \rightarrow 4P_{3/2}$ and $4S_{1/2} - 4P_{1/2}$ of the potassium doublet.

Theoretical analysis of the foregoing resonant effects is usually based on a two-level model of the atom. Under conditions when $\omega_{01} > \omega > \omega_{02}$, however (as, for example, in the experiment of[7]), the use of this model is insufficient, since it is necessary to take into account the contribution made to the refractive index by both of the indicated transitions, i.e., it is necessary in fact to consider a system of three-level atoms.

In this paper we consider, by a quasiclassical method, a three-level model of an atom in a strong field with allowance for degeneracy of the atomic levels. This degeneracy, as will be shown, leads to specific polarization effects that arise in the self-action of light beams. In particular, it will be shown that the degree of polarization of the initial radiation strongly influences the dynamics of the self-focusing (self-defocusing) process.

We consider a three-level atom having in the ground state an energy E_1 and a total angular momentum $j_1 = 1/2$, and having in the two excited states energies E_2 and E_3 and angular momenta $j_2 = 3/2$ and $j_3 = 1/2$. For the isolated atom, the states are degenerate with respect to the projections of the total angular momentum. This level system is typical of alkali-metal atoms.

To investigate the resonant interaction of strong radiation with such a system, we use the well-known procedure of simultaneously solving the classical Maxwell equations and the Schrödinger equation in an external field, in analogy with the procedure used for the two-level system[8].

Retaining only the terms quadratic in the field, we write down expressions for the refractive indices n_+ and n_- of the circular components of the field ($E^{\pm} = E_x \pm iE_y$), interacting with the three-level system of the atoms:

$$n_+=1+24\gamma\frac{\hbar^2\Delta^3}{\varepsilon}\left(2+\frac{\varepsilon}{\varepsilon-\Delta}\right)-\gamma A|E^+|^2-\gamma B|E^-|^2, \tag{1}$$

$$n_-=1+24\gamma\frac{\hbar^2\Delta^3}{\varepsilon}\left(2+\frac{\varepsilon}{\varepsilon-\Delta}\right)-\gamma B|E^+|^2-\gamma A|E^-|^2. \tag{2}$$

Here

$$\begin{gathered} A=\frac{\Delta^3}{\varepsilon^3}\left[5+\frac{\varepsilon}{\varepsilon-\Delta}+\frac{\varepsilon^2}{(\varepsilon-\Delta)^2}+\frac{2\varepsilon^3}{(\varepsilon-\Delta)^3}\right], \\ B=\frac{3\Delta^3}{\varepsilon^3}\left[1+\frac{\varepsilon}{\varepsilon-\Delta}+\frac{\varepsilon^2}{(\varepsilon-\Delta)^2}\right], \\ \gamma=\pi N|d|^4/72\hbar^3\Delta^3, \end{gathered} \tag{3}$$

where d is the reduced dipole-moment matrix element for the transition between the ground ($j_1 = 1/2$) and excited ($j_3 = 1/2$) states. Allowance is made here for the fact that the reduced matrix element for the transition between the states $j_1 = 1/2$ and $j_2 = 3/2$ is double the matrix element of the $1/2 - 1/2$ transition for alkali metals. N is the density of the atoms of the medium and $\Delta = \omega_{01} - \omega_{02}$. The nonlinear increment to the refractive index in expressions (1) and (2), which depends on the field intensity, leads to self-focusing or self-defocusing of the light beam, depending on the signs of A and B (or, in final analysis, depending on the sign of the detuning $\epsilon = \omega_{01} - \omega$).

As seen, for example, from (1), the refractive index n_+ depends on the intensity of not only the E^+ component of the field, but also on the E^- component. As a result, in the general case of elliptically polarized light, the propagations of the E^+ and E^- components of the field are interrelated.

An analysis of expressions (1) and (2) shows that if the radiation frequency ω exceeds the transition frequencies ω_{01} and ω_{02}, then the nonlinear increments for both polarizations are larger than zero, and self-focusing takes place. If, however, $\omega < \omega_{02}$, the nonlinear increments are negative, and self-defocusing takes place. The most interesting frequency region lies between the levels $P_{3/2}$ and $P_{1/2}$. Both self-focusing and self-defocusing can take place here, depending on the values of the detuning and on the parameter $\alpha = |E^-|^2/|E^+|^2$, which characterizes the degree of polarization. At incident-radiation frequencies satisfying the relations

$$A+\alpha B=0 \quad \text{for } n_+,$$
$$A\alpha+B=0 \quad \text{for } n_-, \tag{4}$$

the nonlinear increments vanish. Self-defocusing takes place if the incident-radiation frequency is close to the $P_{3/2}$ level, and self-focusing if it is close to $P_{1/2}$. The contributions of the levels $P_{3/2}$ and $P_{1/2}$ cancel at the frequencies determined by relations (4). We note that this end-point frequency depends on the parameter α. For example, in the case of linearly polarized light ($E^+ = E^-$) the nonlinearity vanishes for the frequency $\omega = \omega_{01} - 2\Delta/3$, in good agreement with the experimental results[7].

For a more detailed analysis of both the self-action dynamics and of the polarization effects it is necessary to solve the wave equations for the E^+ and E^- components of the field with refractive indices (1) and (2).

We shall solve the wave equations on the basis of the known procedure of the scalar theory [9]. We introduce dimensionless beam radii $f_\pm$ for the field components $E^\pm$, in accordance with the formulas

$$|E^\pm|^2=(|E_0^\pm|^2/f_\pm^2)\exp\{-2r^2/a_0 f_\pm^2\}, \tag{5}$$

where $E_0^\pm$ are the field components at the entry to the medium, and a_0 is the initial radius of the beam. Under the same assumptions as in[9], we obtain the following system of equations for $f_\pm$:

$$f_+''/f_+=A/f_+^4+\alpha_0 B/f_-^4, \tag{6}$$
$$f_-''/f_-=\alpha_0 A/f_-^4+B/f_+^4. \tag{7}$$

Here the primes denote differentiation with respect to the dimensionless distance $\tilde{z} = z/R$, where

$$R=a_0(\gamma|E_0^+|^2)^{-1/2}, \quad \alpha_0=|E_0^-|^2/|E_0^+|^2.$$

We have solved the system (6)—(7) numerically in three incident-radiation frequency ranges ($\omega < \omega_{02}$, $\omega > \omega_{01}$ and $\omega_{02} < \omega < \omega_{01}$).

In the first region ($A > 0$ and $B > 0$), self-defocusing of the beam takes place. Figure 1 shows the behavior of the dimensionless radii f_+ and f_- for two different circular components of the polarization. As seen from the figure, these components are defocused differently (the defocusing is larger for the circle with the larger intensity). This causes the intensities of the different field components on the beam axis, determined by the expressions $|E^\pm|^2/f_\pm^2$, to become equalized, so that the polarization ellipse turns into a line. For an ellipse with semiaxis ratio 10 : 1, this occurs at a length $\tilde{z} \approx 0.9$.

In the second region ($A < 0$ and $B < 0$), the beam

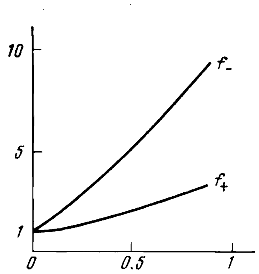

FIG. 1

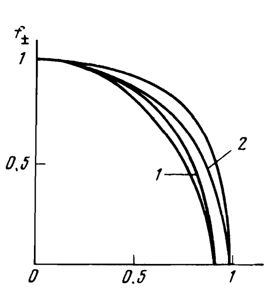

FIG. 2

FIG. 1. Self-defocusing of elliptically polarized light: $\alpha = 10, \beta = 14.5$.

FIG. 2. Self-focusing of elliptically polarized light. Curves: 1) $\alpha = 10$, $\beta = 0.6$; 2) $\alpha = 2, \beta = 0.6$.

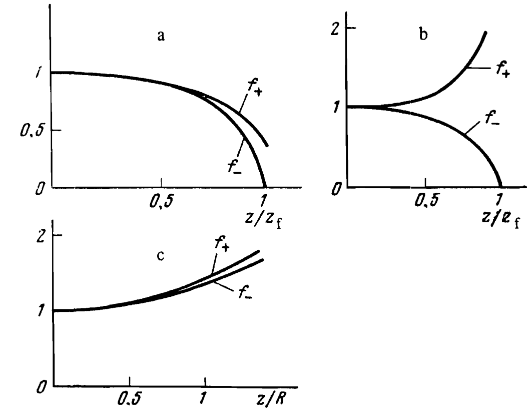

FIG. 3. Behavior of the dimensionless radii of a light beam with almost linear polarization and with semiaxis ratio $\alpha = 1.1$. a) $\beta = -150$, b) $\beta = -2$, c) $\beta = -1.41$.

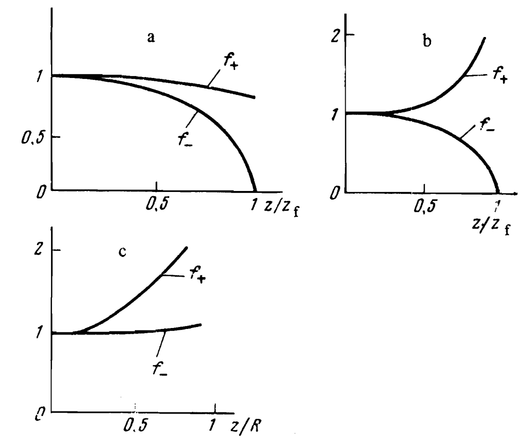

FIG. 4. Behavior of dimensionless radii of light beam with alliptic polarization and with semiaxis ratio $\alpha = 10$. a) $\beta = -150$, b) $\beta = -2$, c) $\beta = -1.41$.

becomes self-focused and the different polarization components are focused at one and the same point, albeit in different manners. This effect is most strongly manifested near the transition $S_{1/2} - P_{3/2}$ ($A/B = 5/3$), when the influence of the $P_{1/2}$ level can be neglected, i.e., one can in fact consider a two-level system. Figure 2 shows plots of the beam radii $f_\pm$ against distance. In both pairs, the upper curve pertains to f_+ and the lower to f_-. Inasmuch as the intensity of the component E^+ is smaller at the entry to the medium, its focusing would be slower than that of the component E^-, but the interaction between them causes them to be focused at the same point.

As already noted, the most interesting frequency region lies between the levels ω_{01} and ω_{02}, where the dynamics of the resonant self-focusing depends strongly on the presence of both levels. We have investigated the behavior of the dimensionless beam width in a wide range of variation of the parameters α and $\beta = \epsilon/(\epsilon - \Delta)$ ($0 < \epsilon < \Delta$), which characterize the degree of polarization of the radiation and the deviation of its frequency from the level $P_{3/2}$. We demonstate here only a few typical cases. Figure 3 shows plots of f_+ and f_- for $\alpha_0 = 1.1$ (small deviation from linearity of the polarization) and for different values of β. We see that at a certain value of the parameter β one of the circles focused and the other defocused, so that the radiation is polarized quite near the focus, i.e., spatial separation of one of the circles takes place at resonant self-focusing.

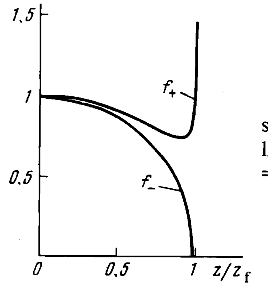

FIG. 5. Change in the character of the self-action during the propagation of a light beam in a resonant medium; $\alpha_0 = 1.1, \beta = -2.9$.

Figure 4 shows plots of f_+ and f_- for $\alpha_0 = 10$ (the intense component is E^-, with a small admixture of E^+). In this case, for a definite interval of values of β, a "purification" of the circular component of higher intensity takes place over a distance equal to the focal length. An analysis of expressions (4) and a reduction of the numerical data make it possible to determine the frequency region where the different components behave differently (even at the entry to the nonlinear medium, one circle becomes focused and the other defocused). This region is $-2.5 < \beta < -1.99$ for $\alpha_0 = 1.1$ and $-14.5 < \beta < -1.93$ for $\alpha_0 = 10$. The existence of this region is due to the fact that the points between the levels ω_{01} and ω_{02}, where the nonlinearity vanishes, are different for the two different field components, as follows from the conditions (4). For linearly polarized light ($\alpha_0 = 1$), there is no such region, since the points where the components E^+ and E^- are compensated coincide at $\beta = -2$.

We note that these regions are determined for the values of α_0 at the entrance to the medium. We have seen that as the beam propagates in the nonlinear medium, the intensity ratio of the two circular components of the field changes, and this leads to a shift in the compensation points, which in turn changes the character of the self-action. Thus, in the plot shown in Fig. 5, both circles are initially self-focused, and then at $z > 0.93z_f$, when α increases and becomes $\geq A/B$, the circle of lower intensity starts to be defocused.

Interest also attaches to a case when the frequency of the incident radiation lies between the levels $P_{1/2}$ and $P_{3/2}$ and is closer to $P_{1/2}$. Then $B = 0$, as follows from the condition that the matrix element of the transition $S_{1/2} - P_{3/2}$ vanish. Equations (6) and (7) are separated, and the different circular components $E^\pm$ of the field propagate completely independently and are focused at different points, the positions of which are determined by the initial intensity of each of the components. The focal distances are connected by the relation

$$z_f^+/z_f^- = \sqrt{\alpha_0}. \qquad (8)$$

The peculiarities of the resonant self-focusing, that were indicated above are connected with the asymmetry of the dependence of the nonlinear response of the medium on the intensity of the two circular components of the field, so that relatively simple polarization measurements make it possible to investigate the dynamics of the processes of self-focusing and self-defocusing. In addition, a nonlinear medium possessing self-action can serve as a polarizer.

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