Microwave breakdown and exciton condensation in germanium

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(Submitted December 20, 1973)
Zh. Eksp. Teor. Fiz. **66**, 2178–2190 (June 1974)

Exciton breakdown induced in germanium by a pulsed microwave field of frequency $f = 10$ GHz is investigated in detail at helium temperatures. The germanium is optically excited by a pulsed YAG laser ($\lambda = 1.06$ μm). The maximum density of the nonequlibrium carriers is $\sim 10^{15}$ cm^{-3}. The dependences of the threshold breakdown power on the duration and the delay time of the microwave pulse relative to the laser pulse and on the optical excitation level are studied at $T = 1.3$ °K. A theory of the effect is proposed, based on the mechanism of impact ionization of excitons at equilibrium with electron-hole drops. The breakdown effect is also discussed from the viewpoint of the biexciton model. The experimental dependences obtained are in satisfactory agreement with theoretical calculations based on the exciton-drop model.

1. INTRODUCTION

Investigation of excitons in germanium at low temperatures and relatively high concentrations is of considerable interest in connection with the problem of their collective properties. According to the existing theoretical concepts, excitons in germanium form, under certain conditions, bound states regarded by most workers as drops of an electron-hole liquid[1-9] and by others as exciton molecules or biexcitons[10-12]. Consequently, great importance attaches to new experimental approaches that yield additional information on the nature of exciton complexes. Work in this direction, connected with the study of properties of excitons at microwave frequencies, has been recently reported[8, 12-14]. Observation of exciton breakdown in germanium by a microwave field was reported in[15]. The observed characteristics of this breakdown were discussed on the basis of the models of the biexciton gas and of the electron-hole drops, but no reliable conclusion was drawn concerning the nature of the broken-down exciton objects.

We have undertaken a detailed study of the effect of microwave breakdown, for the purpose of obtaining data that would explain the nature of the exciton complexes produced in germanium. Assuming an analogy between the exciton-breakdown mechanism and the known features of laser and microwave breakdown of gases in solids[16-18], we investigated the microwave breakdown in germanium under pulsed conditions, varying the duration of the microwave pulses and their delays relative to the exciting laser pulse. The observed dependences of the threshold breakdown power on the duration and delay time of the microwave pulse, and also on the intensity of the optical excitation, have made it possible to analyze in detail the exciton breakdown process and to propose for this effect a theory based on the mechanism of impact ionization of excitons in the presence of electron-hole drops.

2. EXPERIMENTAL SETUP AND PROCEDURE

We investigated the microwave conductivity of germanium under conditions of pulsed laser excitation. The experiments were performed with a setup whose clock diagram is shown in Fig. 1. The investigated germanium samples 1 were placed in a rectangular microwave resonator 2, in which H_{102} oscillations were excited at f = 10 GHz. The resonator was immersed in a cryostat 3 with liquid helium. The loaded Q of the resonator with the sample was $\sim 10^3$ at T = 1.3°K. The microwave source

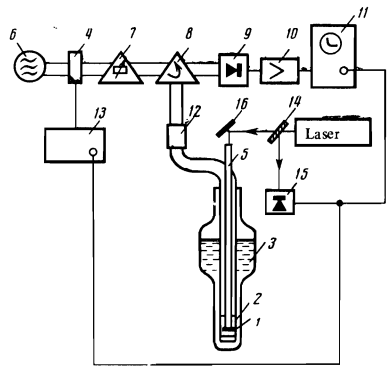

FIG. 1. Block diagram of setup: 1–sample, 2–resonator, 3–helium cryostat, 4–microwave modulator, 5–light pipe, 6–klystron, 7–attenuator, 8–Y circulator, 9–microwave detector, 10–amplifier, 11–oscilloscope, 12–matching section, 13–G5-7A generator, 14–beam splitter, 15–photodiode, 16–mirror.

was a continuously operating klystron with output power ~100 mW. A diode modulator 4 was connected in the waveguide line and made it possible to shape rectangular microwave pulses by applying a voltage from a G5-7A generator. The pulse duration could range from 0.2 to 100 μsec, with rise times ~0.2 μsec. The ratio of the pulse power to the continuous level of the power at the modulator output was ~20dB. The signal reflected from the resonator was observed at a continuous transmitted power level < 0.5 mW. The signal reflected from the resonator was detected with a video receiver and displayed on an oscilloscope screen. The time resolution of the recording apparatus was $\sim 10^{-7}$ sec. When a microwave pulse of ≥ 5 mW power was applied, the microwave conductivity of the sample increased abruptly, thus indicating the appearance of a large number of free carriers in the germanium as a result of the destruction (breakdown) of the excitons.

Optical excitation of the germanium was with a laser based on yttrium-aluminum garnet with neodymium ($\lambda = 1.06$ μ) producing giant pulses of 100 nsec duration, with repetition frequency 100 Hz. The maximum pulse energy was 10^{-4} J. The laser beam passed through a quartz light pipe 5, to the end face of which the sample was secured. The laser-excitation intensity was varied with calibrated neutral filters.

We investigated pure germanium samples with residual impurity density $N_A + N_D$ from 5×10^{12} to 10^{10} cm^{-3}, in the form of plates with typical dimensions 5 × 5 × 0.5 mm, etched in a boiling NaOH + H_2O_2 mixture. Some

samples had a smoothly polished surface. The density of the free carriers produced by optical excitation under the conditions of our experiments was estimated from the intensity of the light incident on the sample, and was assumed to be uniform in the entire volume of the sample. The last assumption is based on the following circumstance: Although carriers are generated by light only in a thin surface layer (the coefficient of optical absorption of germanium at $\lambda = 1.06\ \mu$ is $\sim 10^4\ cm^{-1}$), they fill a sample ~1 mm thick within a time $\sim 10^{-6}$ sec, because of their rapid diffusion[19]. The maximum initial carrier density in the samples of our experiments reached $n = 10^{15}\ cm^{-3}$, corresponding to an exciting laser pulse energy 10^{-6} J.

3. EXPERIMENTAL RESULTS

The effect of exciton breakdown in germanium was investigated in the temperature interval 2.5–1.3°K. The principal data presented below were obtained at T = 1.3°K.

At low microwave levels, the waveform of the microwave conductivity signal following the laser pulse is a sum of two exponentials with characteristic times $\tau_1 = 2\ \mu$sec and $\tau_2 = 50\ \mu$sec, which reflect the dynamics of the binding and recombination of the carriers and excitons in the sample after the pulsed laser excitation[1)]. Investigations of the signal waveform following application of a constant magnetic field up to 10 kG have shown that the amplitudes of the indicated exponentials vary in accordance with the cyclotron-resonance lines. We conclude therefore that the observed signal is due to the free carriers, which contribute to the real and imaginary parts of the dielectric constant of the sample. This contribution is determined by the known relations[2]

$$\varepsilon' = -\frac{4\pi n_e e^2}{m(\omega^2+\nu^2)}, \quad \varepsilon'' = \frac{4\pi n_e e^2}{m\omega}\frac{\nu}{(\omega^2+\nu^2)}, \qquad (1)$$

where n_e is the density of the free carriers, ω is the frequency of the electromagnetic field, ν is the effective collision frequency, and m is the effective carrier mass.

The observations have shown that the initial section of the signal (exponential with time τ_1) includes both the absorption of the microwave power and the detuning, in accord with (1) at relatively large carrier density (when $\nu \gtrsim \omega$ is possible). This initial section describes the binding of the carriers into excitons and into exciton complexes. The exponential with the time τ_2 is determined mainly by the detuning of the resonator, i.e., by the change of ϵ', in accord with relations (1) at low carriers densities and $\nu \ll \omega$. We assume that the long exponential is connected with residual free carriers that are in equilibrium with excitons and exciton complexes. Measurement of the sign of the frequency detuning of the resonator, corresponding to the signal with the long exponential (τ_2), has shown that the change $\Delta\epsilon'$ is negative. This confirms that the observed signal is due to the residual free carriers[2)].

Application of a microwave pulse that is delayed relative to the exciting laser pulse causes an abrupt burst of sample conductivity, when the pulse power P exceeds a certain threshold value P_d (Fig. 2). The duration of the breakdown spike is ~0.5 μsec. The breakdown has a strongly pronounced threshold character: the spike amplitude increases very rapidly even at a slight excess of power above the threshold $(P-P_d)/P_d \sim 0.01$.

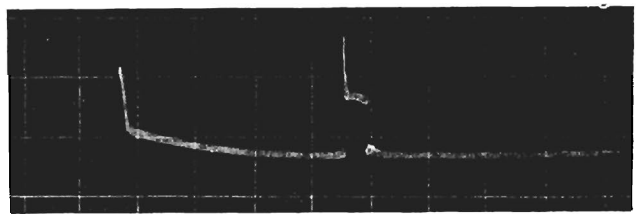

FIG. 2. Waveform of signal reflected from microwave resonator and observed following pulsed laser excitation of germanium. The microwave pulse, on the top of which is seen the breakdown spike, was delayed relatively to the laser pulse by ~80 μsec. The time sweep scale is 20 μsec/div.

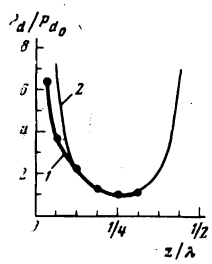

FIG. 3

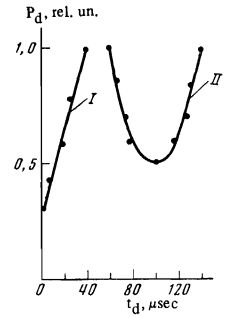

FIG. 4

FIG. 3. Dependence of the exciton breakdown threshold on the sample position along the resonator z axis (curve 1). For comparison, the figure shows the amplitude distribution of the electric microwave field component $(E_0/E_y)^2$ in the resonator (curve 2). P_{d_0} is the value of the breakdown threshold at the maximum of the electric field E_0.

FIG. 4. Dependence of the exciton breakdown threshold P_d on the microwave pulse delay time t_d relative to the exciting laser pulse. The microwave pulse duration is $\tau = 0.2$ sec. The initial density of the electron-hole pairs, is $\bar{n} = 10^{15}\ cm^{-3}$.

This makes it possible to determine very accurately the threshold of the breakdown. Since the threshold is one of the most important characteristics of the breakdown, and is connected with the nature of the broken-down objects, we have measured this quantity under various conditions. By breakdown threshold we mean here and below the minimum resonator microwave input power, at which breakdown is observed.

It was assumed that the breakdown threshold can depend on the relative values of the electric and magnetic components of the microwave field, on the dimensions of the exciton complexes, and on the initial densities of the carriers and excitons. It was therefore of interest to study the dependence of P_d on the position of the samples in the resonator, on the duration τ and on the delay time t_d of the microwave pulse relative to the laser pulse, and on the intensity of the optical excitation. We describe below the experimental results concerning these dependences.

a) Dependence of P_d on the position of the sample in the resonator. In a rectangular microwave resonator, the E and H components of the field are sufficiently well separated, so that the relatively thin samples used by us (≤ 1 mm) could be located either in an electric or in a magnetic microwave field, and we could determine which of these fields is responsible for the breakdown. Figure 3 shows the dependence of the breakdown threshold on the sample position along the z axis of the resonator at fixed durations and delay times of the microwave pulse and of the exciting laser pulse power. We see that the threshold is minimal when the sample is in an electric microwave field, and that there is no breakdown in

a magnetic microwave field. The shape of the $P_d(z)$ curve deviates somewhat from the relation $E_y^2(z) = E_0^2 \sin^2 k_z z$ for the y component of the electric field of the normal H_{102} mode in the empty resonator. This deviation may be due to the change in the spatial distribution of the field in the resonator due to the introduction of the sample and the subsequent retuning of the resonator.

The obtained dependence of the breakdown threshold on the position of the sample in the resonator indicates that the cause of the observed effect is heating of the free carriers by the electric microwave field. The breakdown of the metallic electron-hole drops is more readily expected in a microwave magnetic field.

b) Dependence of P_d on the microwave pulse delay time t_d. Figure 4 shows the dependence of the threshold breakdown power on the microwave-pulse delay time relative to the laser pulse at a fixed microwave pulse duration $\tau = 0.2$ μsec and at an initial density $\bar{n} = 10^{15}$ cm^{-3} of the nonequilibrium carriers in the sample. We see that the curve has two branches, the first in the region 3–40 μsec and the second at $t_d = 50$–150 μsec, the second branch having a minimum at $t_d = 100$ μsec.

The character of the breakdown is somewhat different on the indicated branches. This difference pertains both to the sharpness of the breakdown threshold and to the time of its development. In the first section of the region where this breakdown exists, the threshold is indistinct, and the amplitude of the breakdown spike increases relatively slowly with increasing microwave pulse power. The duration of the leading and trailing edges for this branch is $\tau' \approx \tau'' \approx 0.2$ μsec. For the second branch, the breakdown threshold is very strongly pronounced, and the rise and fall-off times of the breakdown spikes are $\tau' \approx 0.1$ μsec and $\tau'' \approx 0.2$ μsec.

We note that the first branch of the $P_d(t_d)$ curve exists only at sufficiently high laser-pump levels. At an excitation intensity corresponding to the initial carrier density $\bar{n} \leq 5 \times 10^{14}$ cm^{-3} in the sample, only the second branch with a minimum appears on the $P_d(t_d)$ curve.

It was found that the breakdown effect under discussion is observed in all the investigated samples with different contents of residual impurity, including ultrapure germanium ($N_A + N_D = 10^{10}$ cm^{-3}),[3)] and has the same characteristics (values of P_d, τ', and τ'') for all the samples.

c) Dependence of P_d on the microwave pulse duration τ. Figure 5 shows the dependence of the breakdown threshold power on the microwave pulse duration at an initial electron-hole pair density in the sample $\bar{n} = 10^{15}$ cm^{-3} and a delay time $t_d = 100$ μsec. The $P_d(\tau)$ curve is drooping and flattens into a horizontal straight line at $\tau = 0.5$ μsec.

Experiments were also performed on breakdown by two successive microwave pulses of equal duration and amplitude. It turns out that if the time interval between the microwave pulses is shorter than 10–15 μsec, then the breakdown is produced only by the first pulse, but if the second pulse is delayed relative to the first by more than 10–15 μsec, then the second pulse also produces breakdown.

In addition to the exciton breakdown, we observed also an optically-induced impurity breakdown. The characteristics of the impurity breakdown differed significantly from those of the exciton breakdown and varied from sample to sample. Thus, in samples with residual-impurity concentration 10^{11}–5×10^{12} cm^{-3} the impurity-breakdown threshold was independent of the delay time t_d and existed up to $t_d = 10$ msec. In ultrapure germanium, however, the impurity of breakdown was observed only at delay times up to 250–300 μsec following the laser pulse. In addition, the duration of the breakdown spike for the impurity was much larger than for the exciton breakdown, and amounted to ~2 μsec with a rise time ~1 μsec.

We investigated the dependence of the impurity-breakdown threshold on the microwave pulse duration (curve b on Fig. 5). The strong difference between the plots of $P_d(\tau)$ for the exciton and impurity breakdowns makes it possible to separate the two effects by varying τ. For example, a microwave pulse of duration less than 1 μsec (as seen from Fig. 5) produces only exciton breakdown. We note also that exciton breakdown is observed only at $T \leq 2.5$°K, whereas impurity breakdown is observed also at higher temperatures (up to 4.2°K).

d) Dependence of P_d on the optical-excitation intensity. Greatest interest attaches to the second branch of exciton breakdown. As will be shown below, the existence of a minimum of the breakdown threshold can be interpreted on the basis of the model with breakdown of an exciton gas in equilibrium with an electron-hole drop. We have therefore investigated in detail the dependence of the exciton breakdown on the initial concentration of the generated electron-hole pairs, which is determined by the intensity of the optical excitation. Figure 6 shows a family of $P_d(t_d)$ curves for different optical-excitation levels I. We see that $P_{d\,min}$ shifts towards shorter delays with decreasing light intensity. Figure 7 shows a plot of instant $t^*_d(I)$ when $P_{d\,min}$ occurs against the laser-pulse intensity.

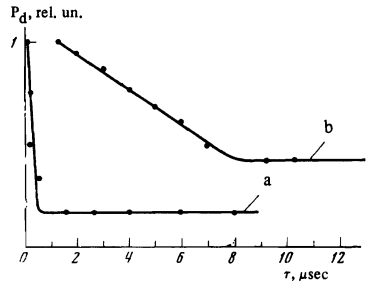

FIG. 5

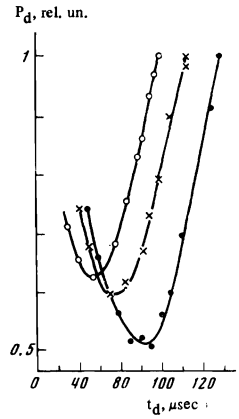

FIG. 6

FIG. 5. Dependence of the breakdown threshold P_d on the microwave pulse duration τ: a—exciton breakdown, b—impurity breakdown. The exciton-breakdown data were obtained at a microwave pulse delay time (relative to the exciting laser pulse) $t_d = 100$ μsec, and at initial electron-hole pair density $\bar{n} = 10^{15}$ cm^{-3}. The impurity density in the sample is 5×10^{12} cm^{-3}.

FIG. 6. Dependence of the exciton breakdown threshold P_d on the microwave-pulse delay time t_d for different optical-excitation levels (initial concentrations of the generated electron-hole pairs). ●—$\bar{n} = 10^{15}$ cm^{-3}, X—$\bar{n} = 5 \times 10^{14}$ cm^{-3}, O—$n = 2.5 \times 10^{14}$ cm^{-3}. Microwave pulse duration $\tau = 0.2$ μsec.

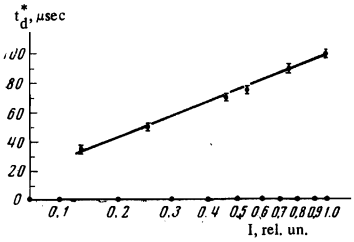

FIG. 7. Dependence of the time t_d^* of the minimum exciton breakdown threshold on the laser-excitation intensity I. The abscissa scale is logarithmic. The point I = 1 corresponds to an initial electron-hole pair density $\bar{n} = 10^{15}$ cm^{-3}.

4. THEORY OF MICROWAVE BREAKDOWN OF EXCITONS. DISCUSSION OF EXPERIMENTAL RESULTS

The most characteristic of the discussed qualitative factors is the considerable time delay of the breakdown relative to the perturbing pulse. The optimal conditions for exciton breakdown are produced in the sample within ~100 μsec after the appearance of the excitons in the sample (Fig. 6), a time that exceeds by more than one order of magnitude the lifetime of the free excitons[20]. On the other hand, it is known from numerous recent experimental investigations[9-15] that at low temperatures the excitons in Ge stick together to form certain complexes, electron-hole drops (EHD), or biexcitons. The lifetimes of these complexes are much larger than the lifetime of the free excitons. There is therefore practically no doubt that the singularities of the observed microwave breakdown of excitons are connected in some manner with the presence of these complexes. In other words, the observed breakdown developes in a system comprising electrons + excitons + EHD or in a system comprising electrons + excitons + biexcitons. We shall discuss the experimental data described above on the basis of both the EHD and biexciton models.

We start with the EHD model, since, as we shall show, it makes it possible to describe in a natural fashion most of the results. In this model we start from the fact that the bulk of the nonequilibrium carriers in the sample are bound into drops of average radius R and of particle density n_0. The drop concentration N is connected with the average density of the carriers introduced in the sample by the obvious relation

$$\bar{n} = {}^4/_3 \pi R^3 N n_0. \qquad (2)$$

In addition to the EHD, the sample contains also excitons with concentration n and free carriers with concentration n_e. We, however, did not take them into account in (2), inasmuch as $\bar{n} \gg n \gg n_e$ at the sufficiently low temperatures of interest to us. During the first several microseconds after the exciting pulse, a dynamic equilibrium is established between these three groups of bound and free carriers, namely, the free carriers are bound into excitons and stick to the drops, the excitons dissociate into free carriers or are absorbed by the drops, and the drops capture and evaporate the excitons and free carriers. All the concentrations, (n, n_e, and $\bar{n}$) decrease with time as the result of recombination, but the equilibrium between them is preserved for a long time.

The main cause of the breakdown is undoubtedly the heating of the free carriers by the microwave field. The magnitude of this heating can be easily estimated in the usual manner by equating the energy absorbed from the field to the phonon radiation losses:

$$\frac{e^2\mathscr{E}^2}{m\omega^2}\nu = \frac{\hbar\bar{\omega}\nu}{1+2N_{\bar{\omega}}}. \qquad (3)$$

Here $\mathscr{E}$ is the field intensity in the sample, ω is the field frequency, e and m are the charge and effective mass of the electron, ν is the frequency of the collisions with the phonons, $\bar{\omega}$ is the average frequency of the emitted and absorbed phonons, and is equal, by virtue of the energy and momentum conservation, to $\bar{\omega} = \sqrt{2\bar{\epsilon}ms^2}/\hbar$, where $\bar{\epsilon}$ is the average electron energy, s is the speed of sound, and $N_{\bar{\omega}} [e^{\hbar\bar{\omega}/kT} - 1]^{-1}$ is the equilibrium phonon distribution function. In the case of weak overheating we have $(\bar{\epsilon} \sim kT)\hbar\bar{\omega} \ll kT$ by virtue of $ms^2 \ll kT$, and it is then easy to obtain from (3)

$$\bar{\epsilon} = \frac{kT}{ms^2}\frac{e^2\mathscr{E}^2}{m\omega^2}, \qquad (4)$$

and in stronger fields, when $\hbar\bar{\omega} > kT$, we have

$$\bar{\epsilon} = \frac{1}{2ms^2}\left(\frac{e^2\mathscr{E}^2}{m\omega^2}\right)^2. \qquad (5)$$

We are interested mainly in the region (5), since $\bar{\epsilon} \sim \epsilon_0 \gg kT$ at breakdown (ϵ_0 is the exciton binding energy). In this region $\bar{\epsilon}$ increases very rapidly with increasing field ($\sim\mathscr{E}^4$). It is easy to estimate also the fields corresponding to the start of the overheating, putting for this purpose $\bar{\epsilon} \sim kT$ in (4). Then $e^2\mathscr{E}^2 \approx (ms\omega)^2$, which yields $\mathscr{E} \approx 2$ V/cm at $m \approx 10^{-28}$ g, $s = 5 \times 10^5$ cm/sec, and $\omega = 7 \times 10^{10}$ sec^{-1}. In fields 10 V/cm, the average electron energy becomes ~10°K, which should correspond to breakdown of the free excitons present in the sample.

The key factor in the explanation of the entire aggregate of the described experimental data is that the EHD, without being noticeably overheated in the considered field and without being destroyed by bombardment with free carriers, are at the same time effective trapping centers for electrons and holes. To verify this, let us estimate the time Γ_e^{-1} of trapping of an electron by one of the drops

$$\Gamma_e^{-1} = [4\pi N\bar{R}^2 v_\epsilon]^{-1} = \frac{n_0}{3\bar{n}}\frac{\bar{R}}{v_\epsilon}; \qquad (6)$$

Here $v_\epsilon = \sqrt{\bar{\epsilon}/m}$ is the average random electron velocity in a microwave field of intensity $\mathscr{E}$. Substituting in (6) $n_0 = 2 \times 10^{17}$ cm^{-3}, $\bar{n} \approx 10^{15}$ cm^{-3}, $\bar{R} \approx 10^{-3}$ cm and $v_\epsilon \approx 10^7$ cm/sec, we obtain $\Gamma_e^{-1} \sim 10^{-8}$ sec.

Thus, in a sufficiently pure sample or under conditions when the other capture centers (due to impurities and defects) are filled after the exciting pulse, the mechanism considered here turns out to be decisive for the free-carrier density. The electric breakdown of the excitons is determined in this case by the ratio of the rate of multiplication of the free carriers as the result of impact ionization $\alpha(\mathscr{E})nn_e$ ($\alpha = \langle\sigma_i v\rangle$ is the impact-ionization coefficient and σ_i is the cross section for the disintegration of the exciton by an electron) to the rate of capture by the drops, and also of the reciprocal process of the binding into excitons βn_e^2. The equations describing the change in the concentrations n and n_e thus take the form

$$\frac{dn_e}{dt} = \alpha n n_e - \beta n_e^2 - \left(\Gamma_e + \frac{1}{\tau_e}\right) n_e = \alpha n n_e - \beta n_e^2 - \tilde{\Gamma}_e n_e, \qquad (7a)$$

$$\frac{dn}{dt} = -\alpha n n_e + \beta n_e^2 - \Gamma(n - n_T) - \frac{n}{\tau_{ex}}. \qquad (7b)$$

In the right-hand side of (7) we have introduced, in addition to the already mentioned terms, also terms that take into account the recombination of the exciton

n/τ_{ex}, their capture by the drops Γ_n, and evaporation of the excitons from the drops Γ_{n_T}, and also other electron-capture mechanisms not connected with the drops, n_e/τ_e, so that $\tilde{\Gamma}_e = \Gamma_e + 1/\tau_e$. Obviously, $\Gamma = v_T\Gamma_e/v_\epsilon$, where v_T is the thermal velocity of the excitons. (We assume that both the excitons and the free carriers are trapped by the drop in each collision.) The quantity n_T, which characterizes the rate of evaporation from the EHD, is determined by the temperature of the carriers in the drop, and if the drops are not overheated relative to the crystal lattice, it is equal to the concentration of the excitons that are in thermodynamic equilibrium with the EHD at the specified temperature T. In (7) we did not take into account the thermal dissociation of the excitons, assuming it to have low probability and not to play a fundamental role in the breakdown. In addition, we did not introduce explicitly any source of free carriers, since the latter are apparently of nonequilibrium origin.

The coefficients α and β depend on the carrier energy distribution and consequently on the field. The coefficient α increases rapidly (exponentially) with increasing $\mathscr{E}$, while β decreases. On the other hand the coefficients $\tilde{\Gamma}_e$ and Γ depend explicitly on the time, for owing to the recombination of the carriers in the drop, with a time constant τ_0, the total surface area of all the drops decreases like $\exp(-2t/3\tau_0)$. In the absence of an electric field, the number of free carriers is quite small, and the solution of (7b), within a very short time after the exciting pulse, $\sim(\Gamma + 1/\tau_{ex})^{-1}$, assumes a quasistationary form

$$n(t) = \frac{\Gamma(t)\tau_{ex}}{1+\Gamma(t)\tau_{ex}} n_T. \tag{8}$$

When a field pulse is applied at a certain instant of time, the criterion for the breakdown is obviously the condition $dn_e/dt > 0$, i.e., by virtue of (7a) we have

$$\alpha n(t_0) - \beta n_e(t_0) - \tilde{\Gamma}_e(t_0) > 0. \tag{9}$$

Since the breakdown develops within a very short time, the additional evaporation of the excitons from the EHD within this time can be neglected, i.e., to describe the breakdown spike one can omit from (7b) the term $\Gamma(n_T - n)$ and put $\tilde{\Gamma}_e(t) \approx \tilde{\Gamma}_e(t_0) = \text{const}$. Then the system (7) can be integrated and reduced to the form

$$\frac{d\ln n_e}{dt} = \frac{\alpha\tilde{\Gamma}_e}{\alpha+\beta}\chi(n_e), \tag{10}$$

while the function $\chi(n_e)$ is determined by the transcendental equation

$$(n_e - n_{e\,max})\frac{(\alpha+\beta)^2}{\alpha\tilde{\Gamma}_e} = -\chi + \ln(1+\chi). \tag{11}$$

The integration constant $n_{e\,max}$ corresponds to the solution of (11) with $\chi = 0$, i.e., by virtue of (10), to the maximum value of n_e in the breakdown spike, and can be expressed in terms of the initial values $n(t_0) = \tilde{n}$ and $n_e(t_0) = \tilde{n}_e$ by comparing (7a) with (10)–(11) at $t = t_0$:

$$n_{e\,max} = \tilde{n}_e + \frac{\alpha\tilde{\Gamma}_e}{(\alpha+\beta)^2}\{\chi(\tilde{n}_e) - \ln(1+\chi(\tilde{n}_e))\} = \tilde{n}_e + \left\{\frac{1}{(\alpha+\beta)\tilde{n}_e}\frac{dn_e}{dt} - \frac{\alpha\tilde{\Gamma}_e}{(\alpha+\beta)^2}\ln\left(1 + \frac{\alpha+\beta}{\alpha\tilde{\Gamma}_e}\frac{1}{n_e}\frac{dn_e}{dt}\right)\right\}\Big|_{t=t_0} = \tilde{n}_e + \frac{\alpha\tilde{n} - \beta\tilde{n}_e - \tilde{\Gamma}_e}{\alpha+\beta} - \frac{\alpha\tilde{\Gamma}_e}{(\alpha+\beta)^2}\ln\left[-\frac{\beta}{\alpha} + \left(1+\frac{\beta}{\alpha}\right)\frac{\alpha\tilde{n} - \beta\tilde{n}_e}{\tilde{\Gamma}_e}\right]. \tag{12}$$

The time of cascade development can be obtained by direct integration of (10):

$$t_{max} - t_0 = \frac{\alpha+\beta}{\alpha\tilde{\Gamma}_e}\int_{\tilde{n}_e}^{n_{e\,max}}\frac{dn_e}{n_e\chi(n_e)}. \tag{13}$$

The subsequent decrease occurs with a time constant $\tilde{\Gamma}_e/(1+\beta/\alpha)$ since it is easy to verify from (10) and (11) that $\chi(n_e) \to -1$ at $dn_e/dt < 0$ and $(n_{e\,max} - n_e) \times (\alpha+\beta)^2/\alpha\tilde{\Gamma}_e \gg 1$.

To simplify the subsequent formulas and estimates, we shall assume that $\beta n_{e\,max} \ll \tilde{\Gamma}_e$, which is apparently correct in the case of interest to us, for if the process of binding into excitons, which is quadratic in n_e, were to play an essential role, the breakdown could not be abrupt. Assuming therefore $\beta/\alpha \ll 1$, we can rewrite (12) in the compact form

$$n_{e\,max} = \tilde{n}_e + \tilde{n}\left[1 - \frac{\tilde{\Gamma}_e}{\alpha\tilde{n}}\left(1 + \ln\frac{\alpha\tilde{n}}{\tilde{\Gamma}_e}\right)\right]. \tag{14}$$

Near the threshold, when $\alpha n_{e\,max} \ll \tilde{\Gamma}_e$, relation (11) reduces to

$$\chi^2 \approx \frac{2\alpha}{\tilde{\Gamma}_e}(n_{e\,max} - n_e), \tag{11a}$$

and Eq. (10)

$$\frac{d\ln n_e}{dt} = \sqrt{2\alpha\tilde{\Gamma}_e(n_{e\,max} - n_e)} \tag{10a}$$

has the solution

$$n_e = n_{e\,max}/\mathrm{ch}^2\left[\sqrt{2\alpha\tilde{\Gamma}_e n_{e\,max}}(t - t_{max})\right]. \tag{15}$$

The breakdown spike is in this case almost symmetrical, and its duration $(\alpha\tilde{\Gamma}_e n_{e\,max})^{-1/2}$ decreases with increasing $n_{e\,max}$, i.e., of the field. When the breakdown is fully developed and $\alpha\tilde{n} \gg \tilde{\Gamma}_l$, $n_{e\,max} \sim \tilde{n}$, and the expansion (11a) is no longer valid, the spike becomes strongly asymmetrical, namely, its trailing edge falls off, as already noted, with a time constant $\tilde{\Gamma}_e^{-1}$, and the leading front increases much more rapidly within a time $\sim(\alpha n)^{-1}$.

The foregoing solution is valid in a time interval $\ll \Gamma^{-1} \sim v_\epsilon\Gamma_e^{-1}/v_T$. At larger times it is no longer possible to neglect in (7b) the evaporation of the excitons from the drops. Accordingly, after the number of free carriers has decreased very strongly following the breakdown, the number of excitons again begins to increase slowly, and after a time $\sim\Gamma^{-1}$, which exceeds by one or two orders of magnitude the duration of the breakdown spike, the breakdown can repeat, in agreement with the experimental results. The considered picture makes it possible to describe in a natural manner also other observed regularities in the breakdown. The breakdown criterion (9) as $\beta \to 0$, with allowance made for (8) and for the explicit form of the dependence of Γ on Γ_e, on the time and the connection between Γ and Γ_e, can be easily reduced to the form

$$\alpha n_T\tau_{ex} = \frac{v_s}{v_T} + \frac{\tau_{ex}}{\tau_e} + 2\sqrt{\frac{v_s}{v_T}\frac{\tau_{ex}}{\tau_e}}\,\mathrm{ch}\left(\frac{2}{3}\frac{t-t_0}{\tau_0}\right), \tag{16}$$

where

$$t_0 = \frac{3}{2}\tau_0\ln\left(\sqrt{\frac{v_s}{v_T}}\tau_e\tau_{ex}\Gamma_0\right), \tag{17}$$

$\Gamma_0 = \Gamma(t=0)$ is the value at the initial instant, directly after the formation of the drops, which is obviously proportional $I^{2/3}$, where I is the intensity of the exciting pulse. Formulas (16) and (17) agree fully with the experimental relations (figs. 4, 6, 7). The time constant τ_0 determined by comparing (17) with Fig. 7 turns out

to be 35 μsec, in reasonable agreement with the lifetimes of EHD known from other studies.

The presence of the first breakdown region at relatively small delays, $t_d < 40$ μsec, is more readily due to the weak overheating of the sample after the exciting pulse, as a result of which the evaporation of the exciton from the drops increases, and consequently an increase takes place in the value of $\tilde{n}$ in the criterion (9). We note that this overheating can be very small ($< 1°K$), since the dependence of n_T on the temperature is exponential.

Let us discuss now the dependence of the breakdown threshold on the duration of the microwave pulse.

In all the preceding arguments we have assumed that the field does not change during the time of development of the breakdown spike. However, if $\Gamma_e \tau < 1$, i.e., the carriers do not manage to be captured by the drops within the time of action of the pulse, then it is clear that the breakdown criterion is equivalent to the condition $\alpha \tilde{n} \tau \sim 1$, i.e., to the requirement that each electron experience during the time of action of the pulse at least one ionization collision. Consequently, with further decrease of τ the impact ionization coefficient α corresponding to the type of the breakdown should increase, and with it also the threshold breakdown field. In experiment (Fig. 5) this growth begins at $\tau < 0.5 \times 10^{-6}$ sec, from which it follows that $\Gamma_e \gtrsim 2 \times 10^6$ sec^{-1}, in qualitative agreement with the estimates based on the trailing edge of the breakdown spike $\Gamma_e \approx 5 \times 10^6$ sec^{-1}. Assuming as an average estimate $\Gamma_e = 3\bar{n} v_\epsilon / n_0 R \approx 3 \cdot 10^6$ sec^{-1} and taking $v_e \approx 10^7$ cm/sec, $n_0 \approx 2 \times 10^{17}$ cm^{-3} and $\bar{n} \approx 2 \times 10^{13}$ cm^{-3} (the last figure was calculated under the assumption that each incident photon produces on electron-hole pair with allowance for the subsequent decay of the carriers within 100 μsec with a time constant $\tau_0 \approx 35$ μsec), we can estimate the average drop radius $R \approx 10^{-3}$ cm and the number of drops per unit volume $N \approx 5 \times 10^4$ cm^{-3}. Both estimates agree in order of magnitude with the results of the direct measurement of N and R by the light-scattering method[22, 23]. Thus, the model in which the electron-hole drops play the role of trapping centers for the free carriers explains practically all the experimentally observed regularities of exciton microwave breakdown in germanium at $T \lesssim 2°K$.

We are unable at present to suggest a satisfactory alternate description for all these facts, based on the assumption that the excitons are bound not into electron-hole drops but into biexcitons. The model proposed in[15] attributed the breakdown delay to the fact that after the exciting pulse the electron collision frequency ν, which enters in (1), is determined by electron-biexciton collisions and $\nu \gg \omega$. As the biexciton concentration decreases this frequency decreases and the energy absorbed by each electron increases, so that the large heating, at a given microwave power, occurs at the instant determined by the condition $\nu \approx \alpha$. However, under the conditions $\nu \gg \omega$ we have $\Delta\epsilon'' \gg \Delta\epsilon'$, which contradicts the experimental results, as does also the fact that $\Delta\epsilon' \sim n_e/\nu^2 \sim n_e/n_B^2$ should increase as n_B decreases with time. In addition, direct measurements by the cyclotron-resonance method[24] show that the inverse condition, $\nu \ll \omega$, is satisfied.

An attempt to explain the delay of the breakdown in analogy with the procedure used above for the EHD model, assuming that the biexcitons capture free carriers, also leads to difficulties, even if complexes of the negative-ion type were to exist. The point is that the biexciton ionization energy (the energy for its disintegration into an electron, hole, and an exciton) exceeds the exciton ionization energy by certainly less than a factor of 2, and the average electron energy $\bar{\epsilon}$ increases with increasing field, as we have seen, in proportion to $\mathscr{E}^4$. Therefore the breakdown field for the bielectrons therefore exceeds the breakdown field for excitons by not more than 20%. In fact, the breakdown of the biexcitons is more likely to set in earlier, inasmuch as under the experimental conditions their concentration should be higher by several orders of magnitude than the exciton concentration. The condition for biexciton breakdown is $\alpha_B n_B n_e - \gamma n_B n_e > 0$, where α_B is the coefficient of biexciton impact ionization, n_B is the biexciton concentration, and γ is a coefficient that determines the rate of capture of the electrons by the biexcitons. This criterion reduces simply to the obvious condition $\alpha_B > \gamma$ and does not depend on the concentration n_B, and consequently on the time, i.e., it cannot explain the delay of the breakdown. No less important a difficulty in the considered "gas" model of the breakdown is that within the framework of this model there is no visible explanation of the fact that the breakdown spike develops and terminates within a very short time (< 1 μsec), and then does not repeat after a rather long time (~ 10 μsec), in spite of the continuing action of the microwave pulse.

Thus, the theory of microwave breakdown of excitons in the presence of EHD explains the entire aggregate of the presented experimental data, including also the presence and position of the minimum on the plot of the breakdown threshold against the microwave-pulse delay time, the waveform of the breakdown spike, the dependence of the threshold power on the microwave pulse duration and on the optical-excitation power, and the possibility of a repeated breakdown only after a sufficiently long time.

Using the investigated dependence of the breakdown threshold on the microwave pulse duration, we were able to calculate the radius and the number of the EHD, which hitherto were known only from experiments on light scattering[22, 23]. A similar analysis of the breakdown effect assuming biexciton existence does not make it possible to explain the observed regularities of the breakdown.

The authors are grateful to A. M. Prokhorov, V. S. Vavilov, V. S. Bagaev, and T. N. Galkina for useful discussions, and to V. A. Sanina, A. S. Seferov, and S. P. Smolin for help with the experiments.

[1)] We note that approximately the same characteristic times were observed in the kinetics of recombination radiation of germanium [20].

[2)] We note that by virtue of (1) the contribution made to ϵ' by the free carriers is much lower at higher frequencies. This may be due to the fact that in [8] they observed $\Delta\epsilon' > 0$, i.e., the effect was not determined by the free carriers.

[3)] This sample was manufactured by the General Electric Company [21].

[1] L. V. Keldysh, Proc. 9-th Internat. Conf. on Semiconductor Physics, Mir (1968), p. 1387.
[2] L. V. Keldysh, in: Éksitony v poluprovodnikakh (Excitons in Semiconductors), Nauka (1971), p. 5.
[3] M. Combescot and P. Noziers, J. Phys. C. Sol. Stat. Phys., 5, 2369 (1972).

[4]W. Brinkman, T. Rice, P. Anderson and S. Chui, Phys. Rev. Lett., 28, 961 (1972).
[5]Ya. E. Pokrovskiĭ and K. I. Svistunova, ZhETF Pis. Red. 9, 435 (1969) [JETP Lett. 9, 261 (1969)].
[6]V. S. Bagaev, T. I. Galkina, and O. V. Gogolin, in: Éksitony v poluprovodnikakh (Excitons in Semiconductors), Nauka (1971), p. 19.
[7]V. S. Vavilov, V. A. Zayats, and V. N. Murzin, ZhETF Pis. Red. 10, 304 (1969) [JETP Lett. 10, 192 (1969)].
[8]I. C. Hensel and T. G. Phillips
[9]C. Benoit a là Guillaume and M. Voos, Phys. Rev. B7, 4, 1723 (1973).
[10]V. M. Asnin, B. V. Zubov, T. M. Murina, A. M. Prokhorov, A. A. Rogachev, and N. I. Savlina, Zh. Eksp. Teor. Fiz. 62, 737 (1972) [Sov. Phys.-JETP 35, 390 (1972)].
[11]V. M. Asnin, A. A. Rogachev, and N. I. Savlina, Tekh. Poluprovdn. 5, 802 (1971) [Sov. Phys.-Semicond. 5, 712 (1971)].
[12]P. S. Gladkov, B. G. Zhurkin, and N. A. Penin, Fiz. Tekh. Poluprovodn. 6, 1919 (1972) [Sov. Phys.-Semicond. 6, 1649 (1973)].
[13]B. M. Ashkinadze and F. K. Sultanov, ZhETF Pis. Red. 16, No. 5, 271 (1972 [JETP Lett. 16, 190 (1972)].
[14]T. Sanada, T. Ohyama and E. Otsuka, Sol. St. Comm., 12, 1201 (1972).
[15]A. A. Manenkov, V. A. Milyaev, G. N. Mikhaĭlova, and S. P. Smolin, ZhETF Pis. Red. 16, 454 (1972) [JETP Lett. 16, 322 (1972)].
[16]P. Suleebka and R. Snrau, J. Phys. D. Appl. Phys., 5, 97 (1972).
[17]Yu. K. Danileĭko, A. A Manenkov, A. M. Prokhorov, and V. A. Khaimov-Mal'kov, Zh. Eksp. Teor. Fiz. 58, 31 (1970) [Sov. Phys.-JETP 31, 18 (1970)].
[18]A. D. MacDonald, Microwave Breakdown in Gases, Wiley, 1966.
[19]B. V. Novikov, E. F. Gross, and M. A. Drygin, ZhETF Pis. Red. 8, 15 (1968) [JETP Lett. 8, 8 (1968)].
[20]B. V. Zubov, V. P. Kalinushkin, T. M. Murina, A. M. Prokhorov, and A. A. Rogachev, Fiz. Tekh. Poluprovdn. 7, 1614 (1973) [Sov. Phys.-Semicond. 7, 1077 (1974)].
[21]R. N. Hall and T. G. Solys, IEEE Trans., NS-18, 160 (1971).
[22]Ya. E. Pokrovskiĭ and K. I. Svistunova, ZhETF Pis. Red. 13, 297 (1971) [JETP Lett. 13, 212 (1971)].
[23]V. S. Bagaev, N. A. Penin, N. N. Sibel'din, and V. A. Tsvetkov, Fiz. Tverd. Tela 15, 177 (1973) [Sov. Phys.-Solid State, 15, 121 (1973)].
[24]P. S. Gladkov, Candidate's dissertation, Moscow, FIAN (1972).

Translated by J. G. Adashko.
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