Magnetic dipole forces and critical dynamics of ferromagnets above the Curie point

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The influence of magnetic dipole forces on the critical dynamics of cubic ferromagnets above the Curie point is considered. It is necessary to take such forces into account, since the exchange interaction conserves the total spin of the system and is therefore insufficient to describe the relaxation of the uniform mangetization and the absorption of long-wave electromagnetic oscillations. It is shown that, in the range of temperature for which $4\pi\chi \ll 1$ (χ is the static susceptibility), the dipole forces can be taken into account by perturbation theory, and with decrease of $\tau=(T-T_c)T_c^{-1}$ the critical damping increases approximately as τ^{-1}. In the region $4\pi\chi \gg 1$ and for momenta less than $q_0=a^{-1}(\omega_0/T_c)^{1/2}$, where ω_0 is the characteristic energy of the dipole forces, because of the long-range character of these forces the spin Green function becomes anisotropic and perturbation theory becomes inapplicable. In this temperature range and for momenta greater than q_0 the dynamic-scaling theory holds, with a critical index $z_e \approx 5/2$ that is the same as when the dipole forces are not taken into account. In the region of momenta less tha q_0 part of the momentum dependence is "frozen in" at momenta of the order of q_0 and dynamic scaling arises with index $z_d=z_e-\nu^{-1}\approx 1$, where ν is the correlation-length index. In this case the critical damping decreases as $\tau^{2/3}$. Thus, this quantity has a maximum in the region of temperatures for which $4\pi\chi \sim 1$. At the end of the paper a method for studying the critical dynamics by means of polarized neutrons is discussed.

1. INTRODUCTION

In the study of critical phenomena in ferromagnets, usually only the exchange interaction is taken into account (cf., e.g., the paper[1] by Halperin and Hohenberg). However, this turns out to be insufficient for the description of a number of phenomena. The point is that the exchange interaction is isotropic; it therefore conserves the total spin of the system. But there exist phenomena which occur only in the absence of this conservation law. These are, primarily, the relaxation of the uniform magnetization and the associated absorption of long-wave electromagnetic oscillations in ferrodielectrics. In cubic ferromagnets the magnetic anisotropy is small and the magnetic dipole forces are the principal interaction violating the conservation of the total spin. The present article is devoted to an analysis of the effect of these forces on the dynamics of the critical fluctuations above the Curie point. The main difficulty here is that, because of their long range, the dipole forces, despite their small magnitude, cannot, generally speaking, be taken into account by perturbation theory. We now consider this question in more detail.

The magnitude of the dipole forces is characterized by the parameter

$$\omega_0=4\pi(g\mu)^2v_0^{-1}\equiv 4\pi g\mu M_0, \tag{1}$$

where M_0 is the saturation magnetization at T = 0; usually, $\omega_0 \sim (10^{-2}-10^{-3})T_C$. We define the spin Green function by the equality

$$G_{\alpha\beta}(\mathbf{k},\omega)=i\int_0^\infty dt\, e^{i\omega t}\langle[\mathbf{S}_{\mathbf{k}}^{\alpha}(t),\mathbf{S}_{-\mathbf{k}}^{\beta}(0)]\rangle,$$
$$\mathbf{S}_{\mathbf{k}}=N^{-1/2}\sum_{\mathbf{R}}e^{i\mathbf{k}\mathbf{R}}\mathbf{S}_{\mathbf{R}}. \tag{2}$$

This Green function is related to the magnetic susceptibility of the solid, which determines the response to an external field, by the formula

$$\tilde{\chi}_{\alpha\beta}(\mathbf{k},\omega)=\frac{\omega_0}{4\pi}G_{\alpha\beta}(\mathbf{k},\omega). \tag{3}$$

Above the Curie point in zero magnetic field, the tensor $G_{\alpha\beta}$ decomposes into two parts, parallel and perpendicular to the momentum $\mathbf{k}$:

$$G_{\alpha\beta}=G_\perp(\delta_{\alpha\beta}-n_\alpha n_\beta)+G_\parallel n_\alpha n_\beta, \quad \mathbf{n}=\mathbf{k}k^{-1}. \tag{4}$$

The long-range character of the dipole forces leads to the result that in the limit of small k, as Krivoglaz[2] has shown, the following formulas hold (cf. also Appendix I):

$$G=\frac{G_\perp}{1+\omega_0 G_\perp}, \quad \tilde{\chi}=\frac{\tilde{\chi}}{1+4\pi\tilde{\chi}_\perp}. \tag{5}$$

We note also that $\tilde{\chi}_\perp \equiv \chi$ is the ordinary susceptibility, i.e., it determines the response to the internal field.

As T_C is approached, the susceptibility χ grows and the anisotropy of the Green function increases. In the temperature region in which $4\pi\chi \ll 1$ we shall have $G_\parallel \approx G_\perp$, i.e., the anisotropy is small and the dipole forces can be taken into account by perturbation theory; below we shall call this the exchange region of temperature. But if $4\pi\chi \gg 1$, then $G_\parallel \approx \omega_0^{-1} \ll G_\perp$, i.e., the critical fluctuations are extremely anisotropic; as T_C is approached they increase without limit only in the plane perpendicular to the momentum. Below we shall call this the dipole region of temperature. In this region perturbation theory in ω_0 is inapplicable, but in most cases we can neglect the longitudinal fluctuations, i.e., we can neglect the quantity $G_\parallel$ in comparison with $G_\perp$. Formulas (5) are valid for small momenta.

It is shown below that the anisotropy in the dipole region disappears when the momentum k becomes greater than the dipole momentum:

$$q_0=a^{-1}(\omega_0/T_c)^{1/2}\gg\varkappa, \tag{6}$$

where a is a quantity of the order of the lattice constant and $\varkappa = a^{-1}\tau^\nu$ is the momentum equal to the inverse range of the critical fluctuations. In accordance with the static-scaling law,

$$\chi=\omega_0 Z/4\pi T_c\tau^{\nu(2-\eta)}, \tag{7}$$

where $Z \sim 1$, $\tau = (T - T_c)T_c^{-1}$, $\nu \approx 2/3$ is the correlation-length index and $\eta \sim 10^{-2}$ is the Fisher parameter. It follows from this formula that the dipole region is very narrow: $\tau \lesssim 10^{-4}$—10^{-5}. But, at the present time, there are already experiments on critical absorption of electromagnetic waves in ferrites for $\tau \sim 10^{-4}$ (cf. the papers of Drabkin et al.[3], Luzyanin et al.[4], and Hashimoto et al.[5]), so that a theoretical treatment of the dynamics in this region is timely.

Recently, Fisher and Aharony[6] have studied the static properties of ferromagnets in the dipole region by means of the renormalization-group technique and continuation in the dimensionality of space. They showed that the static-scaling theory holds, with critical indices that differ insignificantly from the indices of the exchange region. In fact, this change in the indices is associated with the limitation of the growth of the longitudinal fluctuations and with the resulting lowering of the effective number of fluctuating components of the magnetization. In[6] a new index φ characterizing the temperature region in which it is necessary to take the dipole forces into account was also calculated. It coincided exactly with the index of the susceptibility in the exchange region, as indeed it should, by virtue of (5). Below we shall not specially distinguish the values of the indices in the two regions, since it will always be clear which temperature region is being discussed.

2. UNIFORM RELAXATION IN THE EXCHANGE REGION

The uniform relaxation has been calculated earlier by Guber[7] by means of perturbation theory and the random-phase approximation. He obtained the following result:

$$\Gamma_0 = C\frac{\omega_0^2}{T_c\tau^{2\nu(1-\eta)/2}} \approx \frac{C\omega_0^2}{T_c\tau}, \quad C \sim 1. \tag{8}$$

We shall now obtain this same formula somewhat differently. In passing, several formulas will be written out and a number of features that are important for the subsequent analysis will be discussed.

In the exchange region, the anisotropy can be neglected: $G_{\alpha\beta} = G\delta_{\alpha\beta}$, and, in accordance with the general theory (cf., e.g., the paper[8] by Schwabl and Michel and the Appendix of the author's paper[9]), for small k and ω we have

$$\begin{aligned} G(\mathbf{k},\omega) &= \frac{G(\mathbf{k},0)\,\Gamma_{\mathbf{k}}}{-i\omega+\Gamma_{\mathbf{k}}}, \\ \Gamma_{\mathbf{k}} &= G^{-1}(\mathbf{k},0)\lim_{\omega\to 0}(i\omega)^{-1}[\Phi(\mathbf{k},\omega)-\Phi(\mathbf{k},0)], \\ \Phi(\mathbf{k},\omega) &= \frac{i}{3}\int_0^\infty dt\, e^{i\omega t}\langle[\dot S_{\mathbf{k}}^{\alpha}(t), \dot S_{-\mathbf{k}}^{\alpha}(0)]\rangle, \end{aligned} \tag{9}$$

where the dot denotes time differentiation[1].

Furthermore, $\dot S_{\mathbf{k}}^{\alpha}$ consists of two (exchange and dipole) parts:

$$\begin{aligned} \dot S_{\mathbf{k}}^{\alpha} &= (\dot S_{\mathbf{k}}^{\alpha})_e + (\dot S_{\mathbf{k}}^{\alpha})_d, \\ (\dot S_{\mathbf{k}}^{\alpha})_e &= N^{-1/2}\sum_{\mathbf{k}_1}(V_{\mathbf{k}_1}-V_{\mathbf{k}-\mathbf{k}_1})\varepsilon_{\beta\alpha\gamma}S_{\mathbf{k}_1}^{\beta}S_{\mathbf{k}-\mathbf{k}_1}^{\gamma}, \\ (\dot S_{\mathbf{k}}^{\alpha})_d &= -\omega_0 N^{-1/2}\sum_{\mathbf{k}_1} n_1^{\beta}\varepsilon_{\beta\alpha\gamma}S_{\mathbf{k}-\mathbf{k}_1}^{\gamma}(\mathbf{n}_1, S_{\mathbf{k}_1}); \end{aligned} \tag{10}$$

here $V_{\mathbf{k}}$ is the Fourier transform of the exchange integral and $\mathbf{n}_1 = \mathbf{k}_1 k_1^{-1}$. In the limit $\mathbf{k} = 0$ the quantity $(\dot S_{\mathbf{k}}^{\alpha})_e$ vanishes and $(\dot S_{\mathbf{k}}^{\alpha})_d$ is finite.

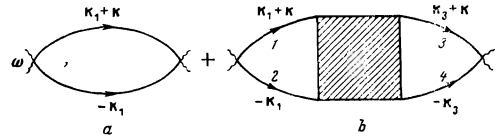

FIG. 1

The long-wave dynamics associated with the exchange part of $\dot S_{\mathbf{k}}^{\alpha}$ was analyzed earlier[1,9] and will be assumed to be given. The function $\Phi(\mathbf{k}, \omega)$ is the Fourier transform of the retarded commutator and can therefore be regarded as the analytic continuation of the corresponding function of discrete frequencies. For the latter, the diagram technique of Vaks, Larkin and Pikin[10] can be used. In this, the diagrams corresponding to the dipole part of $\Gamma_{\mathbf{k}}$ differ from the corresponding exchange diagrams only in the form of the extreme right and extreme left bare vertices. The complete set of them is depicted in Fig. 1, where the shaded rectangle is the exact four-point function, taking only the exchange forces into account, and the extreme right and left vertices are the dipole vertices[2].

We first consider diagram 1a. After summation over the discrete frequencies, the standard analytic continuation and replacement of $n(x) = (e^{x/T} - 1)^{-1}$ by T_c/x, the corresponding contribution to Γ_0 has the form

$$\Gamma_0^{(a)} = \frac{2}{3}\omega_0^2 G^{-1}(0,0)\frac{v_0}{(2\pi)^3}\int d\mathbf{k}\frac{T_c}{\pi}\int\frac{dx}{x^2}(\operatorname{Im} G(\mathbf{k},x))^2. \tag{11}$$

This expression is analogous to the corresponding formula in[9]; in deriving it, we took into account that ImG is an odd function of x. In accordance with the dynamic-scaling law[1],

$$G(\mathbf{k},\omega) = G(\mathbf{k},0)f\left(\frac{\mathbf{k}}{\varkappa}, \frac{\omega}{k^z}\right), \quad G(\mathbf{k},0) = T_c^{-1}k^{-2+\eta}g\left(\frac{k}{\varkappa}\right), \tag{12}$$

with $z = (5 - \eta)/2$. Substitution of this expression into (11) leads to Guber's formula (8).

We now show that the entire set of more complicated diagrams 1b does not alter this result; their contribution can be written in the following way:

$$\Gamma_0^{(b)} = \omega_0^2 G^{-1}(0,0)\frac{v_0^2}{(2\pi)^6}\int d\mathbf{k}_1\, d\mathbf{k}_3\, G^2(\mathbf{k}_1,0)G^2(\mathbf{k}_3,0)F(\mathbf{k}_1,\mathbf{k}_3), \tag{13}$$

where $F(\mathbf{k}_1, \mathbf{k}_3)$ is the integral over the energies that is obtained as a result of analytic continuation of the shaded block (cf. Appendix II). The concrete form of this expression is now unimportant to us. Only one thing is important—the convergence of the integrals over $\mathbf{k}_1$ and $\mathbf{k}_3$. If these integrals converge, then for $F(\mathbf{k}_1, \mathbf{k}_3)$ we can make use of the dimensional estimate $F \sim \varkappa^{1-2\eta-z}$, which follows from the properties of the vertex parts in scaling theory (cf. the papers of A. A. Migdal[11] and Polyakov[12,13]). As a result, it turns out that $\Gamma_0^{(b)}$ also has the form (8). But if even one of the integrals diverges in the region $k_{1,3} \gg \varkappa$, then formula (8) is obviously not valid.

From the principle of coalescence of correlations[12], we obtain the following estimate for F for large $k_{1,3}$:

$$F(\mathbf{k}_1,\mathbf{k}_3) \sim \varkappa^{-z}k_1^{1/\nu-1-\eta}k_3^{1/\nu-1-\eta}\varkappa^{3-2/\nu} \tag{14}$$

and the integral (13) does indeed converge. (Estimates of this type are considered in more detail below, in the analysis of the dipole region.)

It should be noted that in the treatment of the diffusion of the magnetization a divergence does indeed ap-

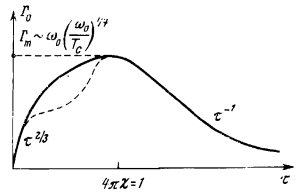

FIG. 2

pear, and the simple dimensional estimate leads to an incorrect expression for the diffusion coefficient D: $D \sim \kappa(1+\eta)/2$ instead of $D \sim \kappa(1-\eta)/2$. This divergence is associated with the vector character of the vertices, by virtue of which it is necessary to take into account the dependence of F on the angle between $\mathbf{k}_1$ and $\mathbf{k}_3$. In[9], therefore, a procedure different from that used here was used to estimate D.

3. THE DIPOLE REGION

Thus, we have shown that in the exchange region the critical damping Γ_0 grows with decrease of τ, and the uniform relaxation time $T_0 = \Gamma_0^{-1}$ decreases. But the uniform magnetization is the order parameter characterizing the phase transition in ferromagnets. This means that as $\tau \to 0$ critical slowing-down of the relaxation should occur, i.e., T_0 should become infinite. This argument once again shows the inapplicability of perturbation theory in the region $4\pi\chi \gtrsim 1$. Incidentally, in all the calculations of the preceding section we neglected Γ_0 compared with the characteristic exchange-scaling energy $T_c(\kappa a)^z$; using (8), it is easily verified that this can be done in precisely the region $4\pi\chi \ll 1$. Thus, we should expect that in the dipole region the quantity Γ_0 decreases as $\tau \to 0$ and, consequently, the dependence of Γ_0 on τ has the form of a curve with a maximum in the region $4\pi\chi \sim 1$, as depicted in Fig. 2.

We now consider the relation between $G_{\parallel}$ and $G_{\perp}$ in more detail. Substituting into (5) the Ornstein-Zernike formula for $G_{\perp}$:

$$G_{\perp} = \frac{Z}{T_c(k^2+\varkappa^2)a^2}, \tag{15}$$

where $Z \sim 1$, we obtain

$$G_{\parallel} = \frac{Z}{T_c[(k^2+\varkappa^2)a^2 + Z\omega_0 T_c^{-1}]} \tag{16}$$

whence the expression (6) for q_0 follows immediately[3)]. Assuming the scaling formula (12) for $G_{\perp}$ and taking into account that, generally speaking, $f(k/\kappa, \omega/k^z) \sim 1$ for $k \ll q_0$, from (5) we obtain

$$G_{\parallel} \approx \frac{1}{\omega_0}, \quad \operatorname{Im} G_{\parallel} \approx \frac{\operatorname{Im} G_{\perp}}{\omega_0^2 |G_{\perp}|^2} \sim \frac{1}{4\pi\chi\omega_0} \ll \omega_0^{-1}. \tag{17}$$

Thus, the imaginary part of $G_{\parallel}$ is small compared with its real part. It should be noted that formula (5) is correct only in the limit $k \to 0$. Therefore, to justify formulas (6) and (17) further investigation is necessary; this is carried out in Appendix I.

Thus, in the dipole region for momenta less than q_0, $G_{\parallel}$ can be neglected in comparison with $G_{\perp}$. Thus, if the scaling laws hold, for momenta less than q_0 they are due to the properties of $G_{\perp}$. Thus, if we estimate the vertex parts from the "unitarity condition"[11-13], we must substitute the imaginary parts of $G_{\perp}$ into the intermediate state; allowance for the imaginary parts of $G_{\parallel}$ leads to corrections that do not possess the scaling property (corrections of order $(4\pi\chi)^{-1}$ in the static theory and $(4\pi\chi)^{-2}$ in the dynamic theory).

Moreover, if all the external momenta are large compared with q_0, it is necessary to use the exact Green function in the unitarity estimates. Therefore, in the dipole region there are two regions of momenta in which there can be scaling properties—small and large momenta compared with q_0; the region of momenta of order q_0 is intermediate and in it there are no scaling properties. Clearly, the values of the critical indices in these two regions should be different. As regards the dynamic index z, this is shown below. The static indices ν and η in the first region have been calculated in the paper[6] by Fisher and Aharony.

Since $q_0 \gg \kappa$, there is only one static index η in the second region. Clearly, its value coincides with the value of η in the exchange region; this is connected with the fact that the values of the static indices are determined by the number of components of the field. Our primary purpose is to determine the dynamic-scaling index z_d in the region of momenta that are small compared with q_0. Since for small momenta the dipole contribution to $\dot{S}_{\mathbf{k}}^{\alpha}$ is greater than the exchange contribution, we shall start from the assumption that the dipole dynamics is determined by the dipole part of $\dot{S}_{\mathbf{k}}^{\alpha}$, and then verify the consistency of the picture thus obtained.

In place of (9), we now have:

$$\Gamma_{\mathbf{k}} = G_{\perp}^{-1}(\mathbf{k}, 0) \lim_{\omega \to 0} (i\omega)^{-1} [\Phi(\mathbf{k}, \omega) - \Phi(\mathbf{k}, 0)],$$
$$\Phi(\mathbf{k}, \omega) = \frac{i}{2} \int_0^{\infty} dt\, e^{i\omega t} \langle [S_{\mathbf{k}}^{\perp}(t), S_{-\mathbf{k}}^{\perp}(0)] \rangle, \tag{18}$$

where $\mathbf{S}_{\mathbf{k}}^{\perp}$ is the part of $\mathbf{S}_{\mathbf{k}}$ perpendicular to $\mathbf{k}$.

It is not difficult to convince oneself that in the limit $k = 0$ the function $\Phi(\mathbf{k}, \omega)$ does not depend on the way in which k tends to zero. For Φ, as before, we have the diagrams of Fig. 1. Now, however, it is necessary to take into account that, by virtue of (10), of the two lines emerging from the dipole vertices, one corresponds to $G_{\perp}$ (the upper lines in Fig. 1) and the other to $G_{\parallel}$ (the lower lines). This leads to the result that, in the integrals determining $\Gamma_{\mathbf{k}}$, the important momenta are those of order q_0, and not of order κ as in the exchange region.

We shall consider the diagram in Fig. 1a. In place of (11), we have

$$\Gamma_{\mathbf{k}}^{(a)} = \frac{2}{3}\omega_0^2 G_{\perp}^{-1}(\mathbf{k}, 0) \frac{T_c v_0}{(2\pi)^3} \int d\mathbf{k}_1 \frac{1}{i\pi^2} \times \int \frac{dx_1\, dx_2\, \operatorname{Im} G_{\perp}(\mathbf{k}_1+\mathbf{k}, x_1) \operatorname{Im} G_{\parallel}(\mathbf{k}_1, x_2)}{x_1 x_2 (x_1 + x_2 - i\delta)}. \tag{19}$$

By virtue of (17), for $k_1 < q_0$ the integrand in (19) is of the order of $\omega_0^{-2}\Gamma_{k_1}^{-1}$, and for $k_1 > q_0$ it decreases rapidly, so that the integral over $\mathbf{k}_1$ converges. This means that for $k \ll q_0$ we have the following order-of-magnitude estimate:

$$\Gamma_{\mathbf{k}}^{(a)} \sim G_{\perp}^{-1}(\mathbf{k}, 0) T_c \frac{v_0}{(2\pi)^3} \int_{k_1 < q_0} \frac{d\mathbf{k}_1}{\Gamma_{\mathbf{k}_1}}. \tag{20}$$

It is shown below that this contribution to $\Gamma_{\mathbf{k}}$ can be neglected.

In order to write down the complete expression for the contribution to $\Gamma_{\mathbf{k}}$ from the diagram 1b, it is necessary, by making use of the complete expression for the

shaded block that takes account of its analytic properties in the energies (cf. the paper [14] by the author), to sum over the frequencies ω_1 and ω_3, analytically continue in ω, and perform the subtraction appearing in the definition of $\Gamma_{\mathbf{k}}$. This is done in Appendix II. In addition, in the formulas given there, it is taken into account that $G_{\parallel}$ is constant for $k_{1,3} < q_0$ and we can neglect Im $G_{\parallel}$ by virtue of (17). This has been done because, as above, for the estimates we shall limit the range of integration over $\mathbf{k}_{1,3}$ by the momentum q_0. The expression thus obtained for $\Gamma_{\mathbf{k}}^{(b)}$ consists of several terms, each of which contains, in addition to the integrations over the momenta $\mathbf{k}_1$ and $\mathbf{k}_3$, integrations over two or three energies and a certain number of energy denominators. In the expression there is one term in which the number of energy denominators depending on integration variables is equal to the number of integrations (two), and in all the other terms the number of such denominators is greater than the number of integrations by unity. All the energy denominators are associated with variables emerging (outwards) from the shaded block, and the integration over the outward momenta $\mathbf{k}_{1,3}$ is performed over the large region $k_{1,3} \lesssim q_0$. Therefore, for a given number of integrations, each additional energy denominator should give a definite small quantity and the principal contribution to $\Gamma_{\mathbf{k}}^{(b)}$ is made by the above-mentioned term with two denominators. Below we shall confirm this qualitative argument by calculations.

Thus, taking (A.II.5) into account for $k \ll q_0$, we have the estimate

$$\Gamma_{\mathbf{k}}^{(b)} \sim G_{\perp}^{-1}(\mathbf{k},0)\frac{T_c^2 v_0^2}{(2\pi)^6}\int\limits_{k_{1,3}\leqslant q_0} d\mathbf{k}_1\, d\mathbf{k}_3 \frac{1}{\pi^2 i}\int \frac{dx_1\, dx_2}{x_1 x_2},$$
$$\Delta_1\Delta_3 G_{\perp}(\mathbf{k}_1+\mathbf{k}, x_1) G_{\perp}(\mathbf{k}_1, x_3)\frac{\partial}{\partial\omega} Z(x_1, 0, x_3, 0, \omega, \mathbf{k}_1, \mathbf{k}_3, \mathbf{k})|_{\omega=0}. \tag{21}$$

Here Z is the complete expression for the shaded vertex in Fig. 1b, x_1 and x_3 are the energies of the lines 1 and 3, and $\Delta_{1,3}$ are the discontinuities in these energies (cf. (A.II.3)). We have omitted a factor of order unity multiplying the whole expression; there is no point in taking it into account in our estimates.

The diagrams determining Z are organized in such a way that there is no choice of $G_{\parallel}$ or $G_{\perp}$ in the internal lines, and this means that it is necessary to associate $G_{\perp}$ with all the internal lines. In (20), momenta $k_{1,3} \sim q_0 \gg \kappa$ play the principle role in the integrals over $\mathbf{k}_{1,3}$. Also, because of the presence of the derivative with respect to the variable ω, which passes through the entire diagram from left to right (in (21) the energies of the lines appearing in Z do not depend on ω!), the momenta $\mathbf{k}_1$ and $\mathbf{k}_3$ are not coupled. In other words, there are necessarily internal lines, independent of $\mathbf{k}_1$ and $\mathbf{k}_3$, linking two blocks, of which the left depends on $\mathbf{k}_1$ and the right on $\mathbf{k}_3$ (cf. Fig. 3).

Below we shall see that in the integrals over these internal lines momenta of order κ or $k \ll q_0$ are important. In these conditions, to analyze Z we can make use of the principle of coalescence of correlations [12]. According to this principle, if there is an n-particle vertex and one of the momenta $\mathbf{k}$ is large compared with the others and with κ, the points of entry and exit of this momentum coalesce into one and the following formulas hold[4]:

$$\Gamma_n(\mathbf{k}, \ldots) = k^{1/\nu-\eta-1}\Gamma_{n-1}(\ldots),$$
$$\Gamma_3(\mathbf{k}, \mathbf{q}) = k^{1/\nu-\eta-1} q^{3-2/\nu} c(q/\kappa). \tag{22}$$

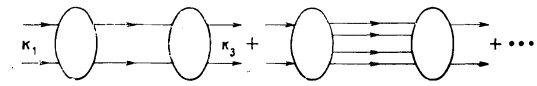

FIG. 3

Next, we assume dynamic scaling; this means that, along with (12) for $G_{\perp}$, we have the following expression for the vertices [13]:

$$\Gamma_n(\mathbf{k}, x_i) = \kappa^{3-n(1+\eta)/2}\gamma_n(\mathbf{k}/\kappa, x_i/\kappa^z). \tag{23}$$

Because of the formulas (22) and (23), we can separate the dependence on $\mathbf{k}_{1,3}$ from the dependence on $\mathbf{k}$ and ω, and as a result the estimate for $\Gamma_{\mathbf{k}}^{(b)}$ has the form

$$\Gamma_{\mathbf{k}}^{(b)} \sim G_{\perp}^{-1}(\mathbf{k},0)\frac{T_c^2 v_0^2}{(2\pi)^6}\int d\mathbf{k}_1\, d\mathbf{k}_3\, G_{\perp}(\mathbf{k}_1,0) G_{\perp}(\mathbf{k}_3,0)$$
$$\times [(k_1 a)(k_3 a)]^{1/\nu-\eta-1} H(\mathbf{k},\kappa) \sim G_{\perp}^{-1}(\mathbf{k},0)(q_0 a)^{2/\nu} H(\mathbf{k},\kappa), \tag{24}$$

where $H(\mathbf{k}, \kappa)$ is the quantity obtained by separating out the dependence on $\mathbf{k}_1$ and $\mathbf{k}_3$ in the diagrams of Fig. 3.

For estimates of the function $H(\mathbf{k}, \kappa)$ we can make use of the static vertices. As a result, we obtain the correct dependence on κ for $k \ll \kappa$ and on k for $k \gg \kappa$; this is connected with the assumption of scaling and with the convergence of the integrals obtained in this estimate (there is a different analysis of the problem of such estimates in [9]).

First, we consider the first diagram of Fig. 3. Taking (22) into account, we obtain

$$H(\mathbf{k},\kappa) \sim T_c \frac{v_0}{(2\pi)^3}\int d\mathbf{p}\, \Gamma_3^2(\mathbf{p},\mathbf{k})\frac{1}{\pi^2 i}\int \frac{dx_1\, dx_2\, \mathrm{Im}\, G_{\perp}(x_1, \mathbf{p}+\mathbf{k})\, \mathrm{Im}\, G_{\perp}(x_2, p)}{x_1 x_2 (x_1+x_2-i\delta)}$$
$$\sim T_c \frac{v_0}{(2\pi)^3}\int \frac{d\mathbf{p}\, G_{\perp}(\mathbf{p}+\mathbf{k},0) G_{\perp}(\mathbf{p},0)}{\Gamma_{\mathbf{p}+\mathbf{k}}+\Gamma_{\mathbf{p}}}\Gamma_3^2(\mathbf{p},\mathbf{k}). \tag{25}$$

Here we have isolated the sum $\Gamma_{\mathbf{p}+\mathbf{k}} + \Gamma_{\mathbf{p}}$ in the denominator, as if the energy dependence of $G_{\perp}$ had the form of a simple pole (9); for order-of-magnitude estimates, this is sufficient. For $p \gg k, \kappa$, the integrand is proportional to $p^{-3}(p^{2/\nu-3})\Gamma_{\mathbf{p}}^{-1}$ by virtue of (23). But $2/\nu - 3 = -\alpha/\nu$, where α, the specific-heat index, is small and negative [6]. Furthermore, in accordance with the assumption of dynamic scaling, $\Gamma_{\mathbf{p}} = T_c \varphi_1 p^{z_d}$ for $p \gg \kappa$. If $z_d \gg -\alpha/\nu$, the integral in (25) converges, and

$$H(\mathbf{k},\kappa) \sim \begin{cases} \varphi_1^{-1}(ka)^{-z_d+3-2/\nu} = \varphi_1^{-1}(ka)^{-z_d+\alpha/\nu}, & k \gg \kappa, \\ \varphi_2^{-1}(\kappa a)^{-z_d+3-2/\nu} = \varphi_2^{-1}(\kappa a)^{-z_d+\alpha/\nu}, & k \ll \kappa. \end{cases} \tag{26}$$

Substituting this formula into (24) and neglecting $\Gamma_{\mathbf{k}}^{(a)}$ in comparison with $\Gamma_{\mathbf{k}}^{(b)}$, it is easy to find z_d and $\varphi_{1,2}$:

$$z_d = 1/2(2-\eta+\alpha/\nu) = z_e - 1/\nu \approx 1,$$
$$\varphi_1 \sim \varphi_2 \sim (q_0 a)^{1/\nu} \approx (q_0 a)^{3/2}, \tag{27}$$

where $z_e = (5-\eta)/2$ is the dynamic exchange-scaling index. Next, using (20), (24) and (26), we obtain the estimate

$$\Gamma_{\mathbf{k}}^{(a)} \sim [\max(\kappa, k) q_0^{-1}]^{z_d-\alpha/\nu}\Gamma_{\mathbf{k}}^{(b)}.$$

It is also not difficult to verify that taking the more complicated diagrams of Fig. 3 into account does not change the formulas (26) and (27). Thus, we have obtained the following formula for the characteristic dipole-scaling energy:

$$\Gamma_{\mathbf{k}} = T_c(\kappa a)^{z_d}\varphi\left(\frac{k}{\kappa}\right) = T_c \begin{cases} A_1(q_0 a)^{1/\nu}(ka)^{z_e-1/\nu} \approx A_1(q_0 a)^{1/2} ka, & q_0 \gg k \gg \kappa, \\ A_2(q_0 a)^{1/\nu}(\kappa a)^{z_e-1/\nu} \approx A_2(q_0 a)^{1/2}\kappa a, & k \ll \kappa, \end{cases}$$
$$A_1 \sim A_2 \sim 1. \tag{28}$$

This is the principal result of this paper. As we have

already stated above, for momenta $k \gg q_0$ exchange scaling with the critical index z_e should occur. Thus, when the momentum k decreases through the region $k \sim q_0$, part of the momentum dependence of the quantity Γ_k is, as it were, frozen in at the value q_0. We note also that, from (28), for the uniform relaxation time we have

$$\Gamma_0 = \Gamma_0^{-1} \approx (A_2\omega_0)^{-1}(T_c/\omega_0)^{-1/2}\tau^{-3/2}. \quad (29)$$

Thus, the quantity T_0 becomes infinite as $\tau \to 0$, as followed from the qualitative arguments at the beginning of this Section.

It now remains to estimate the discarded terms. Primarily, these are γ_1 and γ_2 in (A.II.5). By virtue of dynamic scaling their contribution to Γ_k is of the order of $\Gamma_k\Gamma_{q_0}^{-1} \sim \max(k, \kappa)q_0^{-1}$, i.e., for $k, \kappa \ll q_0$ they are small. Furthermore, in the expression for Γ_k we neglected the contribution of the exchange part of S_k^{α} and the interference of the exchange and dipole parts. We shall consider the purely exchange contribution to Γ_k. The usual combination of functions $G_\perp$ appears in its vertices. With allowance for the renormalization of these vertices by means of the Ward identity[9], in the region $q_0 > k > \kappa$ we obtain

$$D(k)k^2 \sim k^{2-\eta}k^2 \int_k^{q_0} \frac{dk_1 k_1^2 k_1^{-2\eta}}{(k_1^{2-\eta})^2\Gamma_{k_1}} \sim k^{4-\eta}q_0^{-1/\nu}\max(k^{-z_e+1/\nu+1},\ q_0^{-z_e+1/\nu+1}). \quad (30)$$

This expression is small compared with Γ_k for $k \ll q_0$ and, as we should expect, is comparable with Γ_k for $k \sim q_0$. An analogous estimate is also valid for the interference contribution. It too is small for $k \ll q_0$ and comparable with Γ_k for $k \sim q_0$. We remark also that we started from the assumption that, for $k \ll q_0$, the dipole contribution to the energy is greater than the exchange contribution. By means of estimates analogous to the preceding ones, it is not difficult to verify that the opposite assumption is incorrect.

To conclude this section, we shall make one remark. The results obtained above are based on the possibility of neglecting $\Gamma_k^{(a)}$ in comparison with $\Gamma_k^{(b)}$. This can be done in the entire dipole region ($4\pi\chi \gg 1$) and for all momenta $k \ll q_0$ only if the block in Fig. 1b has no additional small quantity. But if there is such a quantity (we shall denote it by λ), there exists a temperature region $\kappa > q_0\lambda^{1/2}$, $4\pi\chi < \lambda^{-1}$ in which the main role is played by the first diagram in Fig. 1. In this region, the characteristic energy has the form

$$\Gamma_k = T_c(q_0a)^{1/2}(ka)^2\psi\left(\frac{k}{\kappa}\right) = T_c(q_0a)^{1/2}\begin{cases}\psi_1(ka)^2; & k \gg \kappa,\\ \psi_2(\kappa a)^2; & k \ll \kappa.\end{cases} \quad (31)$$

But in the temperature region $\kappa < q_0\lambda^{1/2}$, $4\pi\chi > \lambda^{-1}$, the formula (28) holds for momenta $k < q_0\lambda^{1/2}$, and for $q_0\lambda^{1/2} < k < q_0$ we shall have $\Gamma_k \sim T_c q_0^{1/2}k^2$. In the case under consideration, Γ_0 behaves as depicted by the dashed line in Fig. 2.

Whence can the small quantity arise? As A. A. Migdal has shown[15], the Fisher parameter η can be small only if the static vertices are small; Ginzburg[16] has confirmed this statement by calculating Γ_4 directly in the $(4-\epsilon)$-theory. A crude estimate gives $\Gamma_4 \sim \eta^{1/2} \sim 0.1$. If Z has a small parameter, then it is indeed necessary to consider two temperature regions. On the other hand, intermediate states with an odd number of particles, which are absent in the static theory[9], also make a contribution to the dynamics. But for odd vertices the estimates of[15,16] do not hold. Moreover, the even dynamic vertices are evidently not small. The point is that, as follows from the work of Halperin et al.[17], for $\omega \gg k^z$

$$G(\mathbf{k}, \omega) \sim \omega^{-(2-\eta)/z}$$

and it follows from the unitarity estimates for ImG[13] that the squares of the dynamic vertices are of the order of $\sin[\pi(2-\eta)/z]$. This means that for the exchange region $\lambda = (\sin 4\pi/5)^{1/2} \approx 0.85$, and for the dipole region $(\sin 3\pi|\alpha|)^{1/2} \approx 0.53$ (we have substituted $|\alpha| = 1/34$ from[6]). It is clear that we do not have a small parameter in either case; all that is possible is some additional numerical small factor, the existence of which can be established at the present time only from experiment.

4. CONCLUSION

We now discuss the question of the frequency dependence of the Green function $G_\perp$. It was shown in[9] that this dependence can be regarded as the dispersion of the damping coefficient $\Gamma_k(\omega)$. In complete analogy with[9], we can say that $\Gamma_k(\omega)$ has branch points $\omega_n(k) = -i\Gamma_k/n(0)$, where n are integers. From these points in the ω plane there emerge cuts, downward along the imaginary axis. Since $\Gamma_0(0) \neq 0$ for $k = 0$, the branch points move further away from the real axis with increasing n. Thus, it is necessary to take into account the dispersion of the uniform damping $\Gamma_0(\omega)$ for $\omega \sim \Gamma_0$; if $\omega \ll \Gamma_0$, the expressions for $\Gamma_0(\omega)$ and $G_\perp(0, \omega)$ have the form

$$\Gamma_0(\omega) = \Gamma_0 + i\alpha\omega - \beta\omega^2 - i\gamma\omega^3,$$
$$G_\perp(0,\omega) = G_\perp(0,0)\left(1 + \frac{i\omega}{\Gamma_0}\left\{1 + \frac{\omega^2}{\Gamma_0^2}[\Gamma_0\beta - (\alpha-1)^2]\right\} + \frac{\omega^2}{\Gamma_0^2}(\alpha-1)\right), \quad (32)$$

where, by virtue of dynamic scaling, $\alpha \sim 1$, $\beta \sim \Gamma_0^{-1}$ in the dipole region, and

$$\alpha \sim (\Gamma_0/T_c(\varkappa a)^{z_e})^{1/2} \sim (4\pi\chi)^{1/2} \ll 1,\ \beta \sim \Gamma_0^{-1}(4\pi\chi)^{1/2} \ll \Gamma_0^{-1}.$$

in the exchange region. To calculate the absorption of long-wave electromagnetic radiation by means of (32), it is necessary to take into account the shape of the sample, as explained in the paper[18] by Vugal'ter and the author.

The absorption of long-wave radiation near T_c has been studied experimentally[3-5], and a decrease of Γ_0 was found as the Curie point was approached. However, in these experiments the measurements were performed in the temperature region in which $4\pi\chi \sim 1$, and, therefore, there are as yet no data for comparing the quantity Γ_0 calculated above with experiment. The most interesting result of this paper is the change in the dynamic index on passing through the region of momenta of order q_0. Evidently, this prediction can be checked only by means of inelastic scattering of slow neutrons. However, since the characteristic energy $\Gamma_0 \sim \omega_0(\omega_0/T_c)^{1/4} \sim 0.1°K$ of interest is very small, its measurement lies at the limit of present-day capabilities. We shall now discuss, therefore, an experiment based on the use of polarized neutrons.

The point is that, as shown in the papers of Drabkin et al.[19] and the author[20], near T_c the change in the polarization vector of the neutrons in small-angle scattering is very sensitive to the magnitude of the transferred energy. If the incident neutrons are polarized parallel to the momentum, the polarization after scattering through a small angle ϑ is given by the formula[20]

$$-\frac{P}{P_0} = \frac{\langle\omega^2(k\vartheta)\rangle}{(2E\vartheta)^2}. \quad (33)$$

Here P_0 is the initial polarization, P is the polarization after the scattering, k is the momentum of the neutron, E is its energy, and $\langle\omega^2(k\theta)\rangle$ is the average energy of fluctuations with momentum $k\theta$; this formula is correct if $(-P)P_0^{-1} \ll 1$.

Substituting the quantity $\Gamma^2_{k\vartheta}$ into (33) in place of $\langle\omega^2\rangle$, we obtain the estimate

$$-\frac{P}{P_0}\sim\left(\frac{T_c}{2E}\right)^2\begin{cases}(ka)^5\vartheta^3, & k\vartheta>q_0,\\ (ka)^2(\omega_0/T_c)^{1/2}, & k\vartheta\leqslant q_0.\end{cases} \tag{34}$$

Thus, for angles $\vartheta < \vartheta_0 = (ka)^{-1}(\omega_0/T_c)^{1/2}$ this ratio ceases to depend on the angle. If we assume that a = 1 Å, then, for neutrons with long wavelength λ = 6 Å for T_c = 1000°K and $\omega_0 = 1^\circ$, $\vartheta_0 \approx 2^\circ$ and the ratio $-P/P_0$ is of the order of several percent, which is perfectly measurable. With increasing wavelength of the neutrons, the effect being discussed is magnified.

In conclusion, the author expresses his gratitude to G. M. Drabkin, Ya. A. Kasman, I. D. Luzyanin, and E. F. Shender for enabling the author to become acquainted with their papers[3,4] before publication, to S. L. Ginzburg for the same opportunity with regard to his paper[16], and to all of them for a large number of interesting discussions.

APPENDIX I

We shall now give a simple microscopic derivation of the formulas for $G_\perp$ and $G_\parallel$ and discuss the question of the momentum q_0 in more detail than in the main text. In[10], a diagram technique for spins was developed. Lying at the basis of this technique is the idea of single-cell blocks joined by interaction lines. In our case, the interaction has a tensor character:

$$V_{\alpha\beta}(\mathbf{k})=(V_{\mathbf{k}}+{}^1/_3\omega_0)\delta_{\alpha\beta}-\omega_0 n_\alpha n_\beta, \tag{A.I.1}$$

where the last term depends on the way in which the vector **k** tends to zero; in particular, for $\mathbf{k} \equiv 0$ the tensor $n_\alpha n_\beta$ must be replaced by the tensor of the demagnetizing coefficients $N_{\alpha\beta}$ ($N_{\alpha\alpha} = 1$). Introducing, as in[10], the set $\Sigma_{\alpha\beta}(\mathbf{k}, \omega)$ of irreducible diagrams, we obtain the equation

$$G_{\alpha\beta}(\mathbf{k},\ \omega)=\Sigma_{\alpha\beta}(\mathbf{k},\ \omega)+\Sigma_{\alpha\mu}(\mathbf{k},\ \omega)V_{\mu\nu}(\mathbf{k})G_{\nu\beta}(\mathbf{k},\ \omega). \tag{A.I.2}$$

In the presence of spontaneous magnetization this equation has been studied by Korenblit and Klochikhin[21].

In cubic crystals we have, in the most general form,

$$\Sigma_{\alpha\beta}=\Sigma_1\delta_{\alpha\beta}+\Sigma_2 k_\alpha k_\beta+\Sigma_3 h_\alpha h_\beta+\Sigma_4\varepsilon_{\alpha\beta\gamma}h_\gamma, \tag{A.I.3}$$

where **h** is the unit vector in the direction of the mean magnetic moment. Above T_c in zero magnetic field, $\Sigma_3 = \Sigma_4 = 0$.

For $k \neq 0$, Eq. (A.I.2) is easily solved:

$$\begin{gathered}G_{\alpha\beta}=G_{1\alpha\beta}-\frac{\omega_0 G_{1\alpha\nu}n_\nu n_\mu G_{1\mu\beta}}{1+\omega_0 n_\rho n_\varphi G_{1\rho\varphi}},\\ G_1=(1-\Sigma V_1)^{-1}\Sigma,\qquad V_1=V_{\mathbf{k}}+\omega_0/3.\end{gathered} \tag{A.I.4}$$

Above T_c in the absence of a magnetic field, it follows from (A.I.3) and (A.I.4) that

$$\begin{gathered}G_\perp=\frac{\Sigma_1}{1-V_1\Sigma_1},\\ G_\parallel=\frac{G_\perp(1+k^2\Sigma_2\Sigma_1^{-1})}{1+G_\perp[\omega_0-(V_1-\omega_0)k^2\Sigma_2\Sigma_1^{-1}]}.\end{gathered} \tag{A.I.5}$$

It is not difficult to convince oneself that the diagrams for $\Sigma_{\alpha\beta}$ have a well-defined limit, independent of the way in which **k** tends to zero. Therefore, for $\mathbf{k} \to 0$, the quantity $\Sigma_2 k^2$ vanishes and the formulas (4) and (5) of Krivoglaz are obtained from (A.I.5). It follows from the second formula of (A.I.5) that the expression (6) for q_0 given in the main text of the article will be unchanged if, for $k < q_0$,

$$V_1 k^2\Sigma_2\Sigma_1^{-1}<\omega_0.$$

The critical phenomena near T_c are associated with smallness of the denominator in the expression for $G_\perp$. This means that $\Sigma_1 \approx V_1^{-1} \sim T_c$, and we need to elucidate the values of k^2 for which the quantity $V_1^2 k^2\Sigma_2$ is comparable with ω_0. For this we remark first of all that anisotropy of the Green function is sufficient for the appearance of the second term in (A.I.3), and, therefore, in the dipole region there is no additional small term associated with the small quantity $\omega_0 T_c^{-1}$. Furthermore, in the scaling region, the k-dependent part of Σ must be estimated from the "unitarity condition"[11,13]. In this procedure, since the term $k^2\Sigma_2$ is determined from the same diagrams as the k^2-dependent part of Σ_1, these quantities are of the same order $(ka)^{2-\eta}T_c^{-1}$. This is sufficient for the validity of (6) and (17).

APPENDIX II

We now perform the analytic continuation in ω of the diagram in Fig. 1b. The initial expression under the integrals over $\mathbf{k}_1$ and $\mathbf{k}_3$ has the form

$$\begin{aligned}\Lambda(i\omega)=T^4\sum_{\omega_i}G_\perp(i\omega_1)G_\parallel(i\omega_2)\frac{1}{T}\delta_{\omega_1+\omega_2,\omega}\\ \times Z(i\omega_1,i\omega_2,i\omega_3,i\omega_4)\frac{1}{T}\delta_{\omega_3+\omega_4,\omega}G_\perp(i\omega_3)G_\parallel(i\omega_4).\end{aligned} \tag{A.II.1}$$

Here we have omitted the dependence on $\mathbf{k}_1$ and $\mathbf{k}_3$, which is unimportant for the following discussion. The letter Z denotes the complete expression for the shaded block. In[14] it was shown that the following formula holds:

$$\begin{aligned}Z(i\omega_1,\ i\omega_2,\ i\omega_3,\ i\omega_4)=Z_{12}(i\omega_1,\ i\omega_2,\ i\omega_3,\ i\omega_4,\ i\omega_1+i\omega_2)\\ +Z_{13}(i\omega_1,\ i\omega_2,\ i\omega_3,\ i\omega_4,\ i\omega_1-i\omega_3)+Z_{14}(i\omega_1,\ i\omega_2,\ i\omega_3,\ i\omega_4,\ i\omega_1-i\omega_4).\end{aligned} \tag{A.II.2}$$

The terms in this sum are analytic functions of each of the arguments written out, with cuts along the real axis. This property makes it possible to carry out the summation over the internal frequencies and the analytic continuation in ω, by expressing the answer in the form of an integral of the discontinuities of the function Z in its arguments; we recall that the discontinuity of a function across a cut is the name given to the quantity

$$\Delta_x f(x)=\frac{1}{2i}[f(x+i\delta)-f(x-i\delta)]. \tag{A.II.3}$$

We shall not carry out all the calculations in detail, since they are analogous to those available in[14]. We shall confine ourselves to indicating their consistency. By summing over ω_1 we obtain

$$\Lambda(i\omega)=\frac{T}{\pi^2}\int\frac{dx_1dx_2(x_1+x_2)}{x_1x_2(x_1+x_2-i\omega)}\Delta_1\Delta_2G_\perp(x_1)G_\parallel(x_2)K(x_1,x_2,i\omega). \tag{A.II.4}$$

Here K is the triangular diagram standing to the right of the lines 1 and 2 in Fig. 1b; it is an analytic function of each of its three arguments. The discontinuities taken in this formula are of the whole expression standing to the right of $\Delta_{1,2}$. In the derivation of (A.II.4) all the Planck functions n(x) are replaced by T/x. There is a formula of this type in the Appendix in[14].

If, using the analytic properties of Z, we now perform the summation over $\omega_{3,4}$, analytically continue in ω, carry out the subtraction procedure (18) that appears in

the definition of $\Gamma_{\mathbf{k}}$ and replace $G_{\|}$ by ω_0^{-1}, we obtain the following expressions:

$$\gamma = \lim_{\omega\to 0}(i\omega)^{-1}[\Lambda(\omega)-\Lambda(0)] = \gamma_0+\gamma_1+\gamma_2,$$

$$\gamma_0 = \frac{T^2}{i\omega_0^2\pi^2}\int \frac{dx_1dx_2}{x_1x_2}\Delta_1\Delta_3 G_\perp(x_1)G_\perp(x_3)\frac{\partial}{\partial\omega}Z(x_1,0,x_3,0,\omega)\Big|_{\omega=0},$$

$$\gamma_1 = \frac{T^2}{i\omega_0^2}\Big\{\frac{1}{\pi^3}\int \frac{dx_1\,dx_2\,dx_3}{x_1x_2x_3(x_1+x_2)}\Delta_1\Delta_3 G_\perp(x_1)G_\perp(x_3)Z(x_1,x_2,x_3,0,0) + \frac{1}{\pi^3}\int \frac{dx_1\,dx_3\,dx_4}{x_1x_3x_4(x_3+x_4)}\Delta_1\Delta_3 G_\perp(x_1)G_\perp(x_3)Z(x_1,0,x_3,x_4,0)\Big\}, \quad \text{(A.II.5)}$$

$$\gamma_2 = \frac{T^2}{i\omega_0^2}\Big\{\frac{1}{\pi^2}\int \frac{dx_1\,dx_2}{x_1x_2(x_1+x_2)}\Delta_1\Delta_2 G_\perp(x_1)\Big[\frac{1}{\pi}\int\frac{dy}{x_1-y}G_{3\perp}(x_1-y)\cdot \times\Delta_y Z_{13}(x_1,x_2,x_1-y,0,y)+\frac{1}{\pi^2}\int\frac{dx_3\,dy}{x_3(x_1-y)}\Delta_y Z_{14}(x_1,x_2,x_3,x_1-y,y)\Big] + \frac{1}{\pi}\int\frac{dx_1}{x_1}\Delta_1 G_\perp(x_1)\Big[\frac{1}{\pi^2}\int\frac{dx_4\,dy G_{3\perp}(x_1-y)}{x_4(x_1-y)(x_1+x_4-y)}\Delta_y Z_{13}(x_1,0,x_1-y,x_4,y) + \frac{1}{\pi^2}\int\frac{dx_3\,dy}{x_3(x_1-y)(x_1+x_3-y)}\Delta_3 G_\perp(x_3)\Delta_y Z_{14}(x_1,0,x_3,x_1-y,y)\Big]\Big\}.$$

In these formulas certain arguments have been found to be equal to zero, since after the replacement of $G_{\|}$ by ω_0^{-1} the integrals

$$\frac{1}{\pi}\int\frac{dx_i}{x_i}\Delta_{x_i}Z(\ldots,x_i,\ldots)=Z(\ldots,0,\ldots). \quad \text{(A.II.6)}$$

arose. This equality is a consequence of the analytic properties of Z and of the fact that the function Z should fall off for large values of the arguments. In the expression for γ_2 the operations of taking the discontinuities must be applied only to those arguments which do not depend on the combination $x_1 - y$. We note also that γ_2 is not symmetric with respect to the pairs $x_{1,2}$ and $x_{3,4}$. This is connected with the non-symmetric procedure of the calculations. It would also be possible to write γ_2 in a symmetrized form.

[1)] It was shown in [9] that the complete formula for $\Gamma_{\mathbf{k}}$, unlike (9), contains a further factor, which is usually close to unity. Starting from symmetry considerations it can be shown that it equals unity in the exchange region. In the dipole region there is a small correction to unity, vanishing in the limit $q_0 = 0$; we do not take this into account below.

[2)] The complete set of diagrams for $\Gamma_{\mathbf{k}}$ contains diagrams with a single-particle intermediate state. Starting from the symmetry under time reversal, it was shown in [9] that in the limit $\omega = 0$ such diagrams give zero contribution to $\Gamma_{\mathbf{k}}$.

[3)] For $k \gg \kappa$ we have $G_\perp \sim k^{-2+\eta}$ and therefore $q_0 = a^{-1}(\omega_0/T_c)^{1/(2-\eta)}$; but η is small and it is therefore necessary to neglect the deviation of the power exponent from 1/2.

[4)] In the static theory above T_c there are no odd vertices; therefore, the odd vertices in (22) must be understood as quantities obtained from even vertices by differentiation with respect to τ, so that for $q = 0$ the second of the formulas (22) is the Ward identity.

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187