Mixing of degenerate states of an excited hydrogen atom in long-range collisions with a charged particle

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We calculate the differential cross section for elastic scattering for the collision of a charged particle and an excited hydrogen atom, a collision accompanied by transition between degenerate states of the atoms having the same principal quantum number n. The Schrödinger equation is solved in a basis of n^2 degenerate states of the atom; the dipole approximation is used for the interaction with the incident particle. The problem is solved in the quasiclassical approximation in the motion of the colliding particles; this approximation is constructed here in a somewhat modified form. This gives rise to dynamic wave functions of the quasimolecule, analogous to those obtained earlier in the nonstationary problem [1] and making it possible in this case to describe in a self-consistent manner the state of the atom and the motion of the colliding particles. Since the effective interaction depends strongly on the dynamic term, it follows that scattering through a given angle corresponds to several different quasiclassical trajectories, and this leads to interference oscillations in the cross sections. The mixing of $2s$ and $2p$ states is considered in detail.

1. INTRODUCTION

Demkov and the authors [1] considered scattering of a heavy charged particle by a hydrogen-like ion in an excited state with principal quantum number n. The potential of the interaction between the ion and the particle was taken into account in the dipole approximation[1)]

$$V(\mathbf{r}, \mathbf{R}) = \frac{(Z_A - 1)Z_B}{R} - Z_B \frac{\mathbf{rR}}{R^3}, \qquad (1)$$

where Z_A and Z_B are respectively the charges of the nucleus of the ion A and of the incident particle B, $\mathbf{r}$ is the radius vector of the electron relative to A, and $\mathbf{R}$ is the radius vector of B relative to A. Since the ratio of the electron mass to the masses M_A and M_B of the heavy particles (nuclei) is small, the Born-Oppenheimer approximation can be used and the motion of the heavy particles and of the electron can be separated. Then the problem of calculating the differential scattering cross sections breaks up into two stages.

We first solved the nonstationary quantum-mechanical problem for the atomic electron at a certain given nuclear trajectory $\mathbf{R}(t)$. Neglecting transitions between states with different n, it becomes possible to construct the dynamic terms of the quasimolecule, and also the corresponding dynamic wave functions $\psi_{nn'n''}(\mathbf{r}, t)$ between which no transitions take place during the course of the collision.

In the second stage of the solution of the problem we considered the motion of the nuclei. In [1] we analyzed scattering by a hydrogen-like ion ($Z_A \neq 1$), so that the principal interaction determining the motion of the nuclei was the Coulomb interaction (the first term in formula (1)). It is essential that the indicated interaction is independent of the state of the atomic electron. In this case, the motion of the nuclei is specified, and the differential scattering cross sections are easy to calculate [1].

This approach cannot be used for the important and more complicated case considered in the present paper, that of the scattering of a charged particle by a hydrogen atom ($Z_A = 1$). Since there is no Coulomb interaction here, the motion of the nuclei is determined by the dipole potential connected with the linear Stark effect on the excited states of the hydrogen atom. It is important that this dipole potential depends in a decisive manner on the state of the atom and can, in particular, correspond to attraction as well as repulsion between the nuclei. Thus, to find the differential scattering cross sections it is necessary to obtain for the electron problem a solution compatible with the motion of the nuclei. The trajectory of the nuclei then splits, as it were, in accordance with the different states of the atom, and interference effects must be taken into account for different trajectories corresponding to scattering through one and the same angle.

Similar problems always arise in the theory of atomic collisions in the description of nonrectilinear motion of nuclei with allowance for several electronic states. The problem is usually solved by using adiabatic terms for the electrons and a quasiclassical approximation for the nuclei. The simplest case is resonant charge exchange, where two adiabatic electronic terms are taken into account (see, e.g., [2]).

From the general point of view, the process considered in the present paper has two interesting distinguishing features. First, several (n^2) electronic states take part. More important is the use of dynamic rather than adiabatic electronic terms. The dynamic terms are constructed in such a way that there are no transitions between them even at finite velocities of the nuclei, whereas for the adiabatic terms these transitions are absent only in the adiabatic limit, i.e., when the relative velocity of the nuclei tends to zero. In this problem we have succeeded (for the first time, insofar as we know) in obtaining dynamic terms that depend not only on the relative distance between the heavy particles, but also on their angular momentum. Inasmuch as the motions of the electron and of the nuclei are not fully separated in the wave functions $\Psi_{nn'n''}(\mathbf{r}, \mathbf{R})$ corresponding to the dynamic terms, the quasiclassical approximation is constructed in a somewhat different form than is customary. This case is unique in that after simple and natural approximations it becomes possible to solve a multichannel nonadiabatic problem in a quasiclassical approximation.

2. QUASICLASSICAL APPROXIMATION

To calculate the differential scattering cross sections, we first diagonalize the Hamiltonian operator on the hydrogen functions of the given n-layer for the incident charged particle interacting in dipole fashion with the hydrogen atom. The motion of the incident particle is assumed here to be quasiclassical. The obtained eigenfunctions will then be used to construct the scattering amplitude.

The Hamiltonian operator for the considered system, after separating the motion of the mass center, takes the form

$$\mathcal{H}=\mathcal{H}_0-\frac{1}{2M}\nabla_R^2-Z_B\frac{\mathbf{rR}}{R^3},\quad \mathcal{H}_0=-\frac{1}{2}\nabla_r^2-\frac{1}{r}, \tag{2}$$

where $M = M_A M_B/(M_A + M_B)$. In the wave-function space of the hydrogen atom with given n, the Schrödinger equation for the wave function of the entire system $\Psi(\mathbf{r}, \mathbf{R})$ takes the form

$$\left[-\frac{1}{2M}\nabla_R^2-Z_B\frac{\mathbf{rR}}{R^3}\right]\Psi(\mathbf{r},\mathbf{R})=E\Psi(\mathbf{r},\mathbf{R}), \tag{3}$$

where E is the heavy-particle collision energy in the c.m.s.

In addition to the usual integrals of motion (the square of the total angular momentum, L^2, and its projection on the selected axis), the Schrödinger equation (3) has in this problem a specific integral of motion Λ [2)]

$$\Lambda\Psi(\mathbf{r},\mathbf{R})=\left[-\mathcal{L}^2+2MZ_B\frac{\mathbf{rR}}{R}\right]\Psi(\mathbf{r},\mathbf{R})=\lambda\Psi(\mathbf{r},\mathbf{R}), \tag{4}$$

Here $\vec{\mathcal{L}}$ is the angular momentum-operator of the relative motion of the heavy particles. Owing to the presence of the additional integral of motion Λ, it is possible to separate the angular and radial dependences in the wave function and to seek a solution of (3) in the form

$$\Psi(\mathbf{r},\mathbf{R})=f(R)\sum_i c_i(\mathbf{N})\varphi_i(\mathbf{r},\mathbf{N}), \tag{5}$$

where $\mathbf{N} = \mathbf{R}/R$ is a unit vector. The radial function f(R) satisfies the equation

$$-\frac{1}{2M}\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{d}{dR}f(R)\right)+\frac{\lambda}{2MR^2}f(R)=Ef(R). \tag{6}$$

The functions $\varphi_i(\mathbf{r}, \mathbf{N})$ are certain hydrogen wave functions with given n, taken in a coordinate system with x′ axis directed along **N** and with the plane x′y′ coinciding with the plane of the vector **N** and the z axis. It follows from this definition of the functions φ_i that

$$L_{z'}\varphi_i=(\mathcal{L}_{z'}+l_{z'})\varphi_i=0, \tag{7}$$

$$L_z\varphi_i=(\mathcal{L}_z+l_z)\varphi_i=0, \tag{8}$$

where **I** is the angular momentum of the atomic electron. This leaves a leeway, which will be eliminated later on, in the choice of the functions φ_i.

We shall consider henceforth scattering of particles moving along the z axis with large angular momentum. Since the angular momentum of the electron is $l \lesssim n \sim 1$, the total angular momentum is $L \gg 1$. For our purposes it suffices to diagonalize the operator L^2 accurate to terms of the order of unity. Consequently, we can omit the fourth, fifth, and sixth terms from the equation

$$(\vec{\mathcal{L}}+\mathbf{l})^2\sum_i c_i(\mathbf{N})\varphi_i(\mathbf{r},\mathbf{N})=\sum_i\left[(\vec{\mathcal{L}}^2c_i)\varphi_i+2(\vec{\mathcal{L}}c_i)(\vec{\mathcal{L}}\varphi_i)\right. \tag{9}$$

$$\left.+2(\vec{\mathcal{L}}c_i)(\mathbf{l}\varphi_i)+2c_i(\vec{\mathcal{L}}\mathbf{l}\varphi_i)+c_i(\mathbf{l}^2\varphi_i)+c_i(\vec{\mathcal{L}}^2\varphi_i)\right]=L(L+1)\sum_i c_i(\mathbf{N})\varphi_i(\mathbf{r},\mathbf{N}).$$

Choosing c_i to depend only on the polar angle θ (this agrees with the motion of the particle along the z axis) and using relation (7), we can easily verify that the second and third terms of (9) cancel each other, and we obtain for c_i the equation

$$\mathcal{L}^2c_i(\theta)=L(L+1)c_i(\theta). \tag{10}$$

The solutions of (10) are Legendre polynomials that depend on L and do not depend on the choice of the functions φ_i in the expansion (5). In the quasiclassical approximation, the Legendre polynomials

$$P_L(\cos\theta)\approx-\frac{i}{\sqrt{2\pi L\sin\theta}}\left\{\exp\left(i\left[\left(L+\frac{1}{2}\right)\theta+\frac{\pi}{4}\right]\right)\right.$$
$$\left.-\exp\left(-i\left[\left(L+\frac{1}{2}\right)\theta+\frac{\pi}{4}\right]\right)\right\} \tag{11}$$

contain two exponentials that describe particles that are scattered into one and the same solid angle (θ, Φ), but move in different sides of the scatterer in the plane of the azimuthal angle Φ (this corresponds to attraction and repulsion potentials, and in our problem both cases are realized, depending on the state of the atom). As will be shown later on, this leads in the electron problem to the appearance of effective magnetic fields perpendicular to the scattering plane and having opposite directions (cf. [1]). To fix the direction of the magnetic field it is therefore necessary to choose only one exponential

$$c_i(\theta)\equiv c^{\pm}(\theta,L)=-\frac{i}{\sqrt{2\pi L\sin\theta}}\exp\left\{\pm i\left[\left(L+\frac{1}{2}\right)\theta+\frac{\pi}{4}\right]\right\}. \tag{12}$$

To determine the functions φ_i and the eigenvalues λ, we add equations (4) and (9) term by term and, neglecting terms of order of unity, obtain

$$\sum_i\left[2(\vec{\mathcal{L}}c^{\pm})(\mathbf{l}\varphi_i)+2MZ_B\mathbf{rN}c^{\pm}\varphi_i\right]=\left[L(L+1)+\lambda\right]\sum_i c^{\pm}\varphi_i. \tag{13}$$

When (12) is taken into account, Eq. (13) coincides with Eq. (14) of [1]. To diagonalize (13) it suffices therefore to choose for φ_i the functions $\psi_{nn'n''}$ introduced in the cited paper. The physical meaning of the functions $\psi_{nn'n''}$ is connected with the effective magnetic field **H** that results from changing over to a rotating coordinate system, and is directed along the angular momentum of the heavy particles, whereas in our case for the functions c^+ the field **H** should be regarded as directed along the vector $\mathbf{e}_z \times \mathbf{N}$ ($\mathbf{e}_z$ is a unit vector along the z axis), with the direction of H reversed for the functions c^-. Thus, Eq. (13) is diagonalized by the functions

$$\psi^{\pm}_{nn'n''}=\sum_{i_1,i_2=-j}^{j}D^j_{n'i_1}(0,\beta,0)D^j_{n''i_2}(0,-\beta,0)\psi^{\pm}_{ni_1i_2}, \tag{14}$$
$$n', n''=-j, -j+1, \ldots, j;\quad j=(n-1)/2,$$

where $D^j_{mm'}(\alpha, \beta, \gamma)$ is the Wigner function with $\pi/2 < \beta < \pi$ and $\tan\beta = -3nZ_BM/2L$. The basis functions $\psi^{+}_{ni_1i_2}$ are specified here in terms of the rotated coordinated systems introduced above, with the z′ axis directed along the corresponding magnetic field, i.e., for the functions $\psi^{+}_{ni_1i_2}$ along the vector $\mathbf{e}_z \times \mathbf{N}$, and for $\psi^{-}_{ni_1i_2}$ in the opposite direction, with the quantum numbers i_1 and i_2 connected in simple manner with the parabolic quantum numbers n_1, n_2, and m:

$$i_1=\tfrac{1}{2}(m+n_2-n_1),\quad i_2=\tfrac{1}{2}(m-n_2+n_1). \tag{15}$$

The values of the parameter λ corresponding to $\psi^{\pm}_{nn'n''}$ do not depend on the index ±:

$$\lambda_{nn'n''}=-L(L+1)+\left[4L^2+(3nZ_BM)^2\right]^{1/2}(n'+n''). \tag{16}$$

Regular solutions for Eq. (6) are spherical Bessel functions

$$f_{nn'n''}(R)=[\pi/2kR]^{1/2}J_{[-\lambda_{nn'n''}+1/4]^{1/2}}(kR), \tag{17}$$

where $k = [2ME]^{1/2}$. The question of the choice of the solution is discussed in detail in Sec. 3.

Thus, under the assumed approximations, the operators $\mathscr{H}$, L^2, L_z, and Λ are simultaneously diagonalized on the functions

$$\Psi^{\pm}_{nn'n''}(\mathbf{r},\mathbf{R})=f_{nn'n''}(R)\psi^{\pm}_{nn'n''}(\mathbf{r},\mathbf{N})c^{\pm}(\theta,L). \tag{18}$$

Taking (14) into account, we construct from the functions $\Psi^{+}_{nn'n''}(\mathbf{r},\mathbf{R})$ a solution of the Schrödinger equation (3) containing a plane wave incident along the z axis on a hydrogen atom in a certain initial state[3)]. Separating the diverging wave in this solution, we obtain the scattering amplitude. For the scattering amplitude in the basis of spherical hydrogen functions we obtain

$$\begin{aligned}F_{lml'm'}(\theta)=&-\sum_{L,n',n''}\frac{(2L+1)}{2ik\sqrt{2\pi L\sin\theta}}\Big\{A_{lmn'n''}(\pi)A^{*}_{l'm'n'n''}(\theta)\cdot\\ &\times\exp\Big(i\Big[\Big(L+\frac{1}{2}\Big)\theta+\frac{\pi}{4}+2\delta_{nn'n''}\Big]\Big)-A^{*}_{lmn'n''}(\pi)A_{l'm'n'n''}(\theta)\cdot\\ &\times\exp\Big(-i\Big[\Big(L+\frac{1}{2}\Big)\theta+\frac{\pi}{4}-2\delta_{nn'n''}\Big]\Big)\Big\},\\ A_{lmn'n''}(\theta)=&\sum_{i_1i_2}D^{l}_{m\mu}\Big(\frac{3\pi}{2},\frac{3\pi}{2},0\Big)(ji_1ji_2|l\mu)e^{i(i_1+i_2)\theta}\cdot\\ &\cdot D^{j}_{n'i_1}(0,\beta,0)D^{j}_{n''i_2}(0,-\beta,0),\end{aligned} \tag{19}$$

where $(ji_1ji_2|lm)$ are Clebsch-Gordan coefficients. We have used here the connection of the functions $\psi_{ni_1i_2}$ with the spherical functions φ_{nlm} [5]

$$\psi_{ni_1i_2}=\sum_{l=|m|}^{2j}(ji_1ji_2|lm)\varphi_{nlm}. \tag{20}$$

The Wigner functions $D^{l}_{mm'}(3\pi/2,\ 3\pi/2,\ 0)$ appear on going over from the primed coordinate system to the unprimed one. The phase shift $\delta_{nn'n''}$ is determined in accordance with the form of the radial function $f_{nn'n''}$ (17):

$$\frac{2}{\pi}\delta_{nn'n''}=L+\frac{1}{2}-\Big(-\lambda_{nn'n''}+\frac{1}{4}\Big)^{1/2}. \tag{21}$$

Replacing the summation over L by integration, as is customary in the quasiclassical approximation [6], and calculating the integral by the stationary-phase method, we obtain for the scattering amplitude

$$\begin{aligned}F_{lml'm'}(\theta)=&-\frac{i}{k}\Big[\sum_{n'+n''>0}\Big(\frac{L_{nn'n''}(\theta)}{\sin\theta}\Big|\frac{dL_{nn'n''}(\theta)}{d\theta}\Big|\Big)^{1/2}\\ &\times A_{lmn'n''}(\pi)A^{*}_{l'm'n'n''}(\theta)\cdot\exp\Big(i\Big[\Big(L_{nn'n''}+\frac{1}{2}\Big)\theta+2\delta_{nn'n''}\Big]\Big)\\ &+\sum_{n'+n''<0}\Big(\frac{L_{nn'n''}(\theta)}{\sin\theta}\Big|\frac{dL_{nn'n''}(\theta)}{d\theta}\Big|\Big)^{1/2}A^{*}_{lmn'n''}(\pi)A_{l'm'n'n''}(\theta)\\ &\times\exp\Big(-i\Big[\Big(L_{nn'n''}+\frac{1}{2}\Big)\theta-2\delta_{nn'n''}\Big]\Big)\Big]\end{aligned} \tag{22}$$

where $L_{nn'n''}$ is a function of θ defined by the equation

$$2d\delta_{nn'n''}/dL=\mp\theta, \tag{23}$$

The upper and lower signs being taken for $n' + n'' > 0$ and $n' + n'' < 0$, respectively.

3. CROSS SECTION FOR THE TRANSITIONS 2s → 2p AND THE REGION WHERE THE THEORY IS APPLICABLE

As will be shown later on, the quasiclassical approximation is applicable for not too large angular momenta

$$L\ll{}^{3}/_{2}n(n-1)Z_BM \tag{24}$$

(in the present section, the difference between L and $\mathscr{L}$ is immaterial). In this case Eq. (23) takes the form

$$1\mp\frac{\theta}{\pi}=\Big[1+\frac{3n(n'+n'')Z_BM}{L^2_{nn'n''}}\Big]^{-1/2} \tag{25}$$

and the $L_{nn'n''}(\theta)$ dependence thus turns out to be the same as in the quasiclassical problem of scattering by a potential

$$V_{nn'n''}(R)=-{}^{3}/_{2}n(n'+n'')Z_B/R^2. \tag{26}$$

In classical mechanics, for an attraction potential inversely proportional to the square of the distance, orbiting can take place, wherein the particles trace several orbits around the common mass center in the course of the collision. In this case the quasiclassical expression (22) for the scattering amplitude must be altered. Collisions with orbiting, however, are connected with relatively small L, and therefore make a relatively small contribution to the scattering through angles that are small in comparison with π:

$$\theta/\pi\ll1, \tag{27}$$

and this corresponds to the inequality

$$L^2\gg{}^{3}/_{2}n(n-1)Z_BM. \tag{28}$$

It is assumed in this paper that the conditions (27) and (28) are satisfied, and collisions with orbiting are disregarded.

Limitations on the side of small scattering angles are due to the known condition for the applicability of the quasiclassical approximation to scattering problems

$$L\theta\gg1, \tag{29}$$

which can be reduced to the form (24) if (25) is taken into account, or can be represented in the form

$$\theta/\pi\gg[{}^{3}/_{2}n(n-1)Z_BM]^{-1}. \tag{30}$$

In the region of small R ($R \lesssim 3n^2/2$), the dipole approximation for the interaction of a charged particle and an atom is not applicable [1]. However, if the angular momentum of the incident particle is large, then the behavior of the wave function in this region is determined by the centrifugal barrier and depends little on the form of the interaction potential. Therefore, if the condition (28) is satisfied, it is necessary to choose a regular solution of Eq. (6), as was indeed done in Sec. 2.

The region of applicability of the other employed approximations was considered in detail in [1]. The inequalities (23), (28), and (30)—(33) obtained in that reference remain in force also in our case. For the reactions

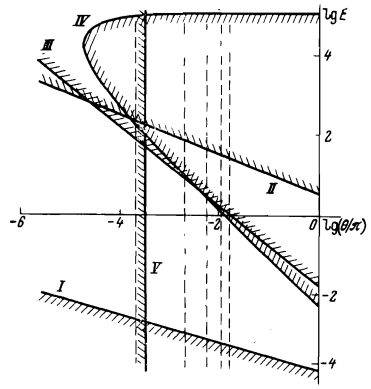
$$H(2s)+H^{+}\to H(2p)+H^{+}, \tag{31}$$

$$H(2s)+H^{+}\to H(2s)+H^{+} \tag{32}$$

the region of applicability is illustrated in the figure. It can be noted that neglect of the charge-exchange process leads in this symmetrical case to more significant limitations than in [1].

Among the potentials $V_{nn'n''}$ (26) there are some that differ only in sign, for example $V_{nn'n''} = -V_{n-n'-n''}$. In the approximation (28), such potentials lead to scattering through one and the same angle; the corresponding trajectories lie in the scattering plane on different sides of the interaction center. In particular, at n = 2 we have

$$V_{2-1/2\,1/2}=V_{21/2-1/2}=0,\quad V_{21/2\,1/2}=-V_{2-1/2-1/2}=-3Z_B/R^2,$$



Limits of the validity of the approximations made in the calculations for the reactions (31) and (32). I–Neglect of relativistic level splitting–condition (28) of [1]; II–neglect of transitions with change of n–condition (33) of [1]; III–neglect of the deviation from a dipole potential–condition (30) of [1], and also neglect of mixing of terms with different n in the effective electric field–condition (32) of [1]; IV–neglect of resonant charge exchange–condition (31) of [1]; V–limit of applicability of the quasiclassical approximation–condition (30) of the present paper. The dashed lines show the positions of the first five maxima of the cross section for the $2s \to 2p$ transition.

so that there exists only pair of the indicated trajectories, and the cross sections are particularly easy to calculate. With the aid of (22) and (23), and taking (25) and (27) into account, we obtain the differential scattering cross section for the reactions (31) and (32):

$$\sigma_{2s\to 2p}(\theta)=\frac{3\pi Z_B}{4E\theta^2\sin\theta}\sin^2\beta(1+\cos^2\beta)\sin^2\sqrt{12\pi Z_B M\theta}, \qquad (33)$$

$$\sigma_{2s\to 2s}(\theta)=\frac{3\pi Z_B}{4E\theta^2\sin\theta}\sin^4\beta\cos^2\sqrt{12\pi Z_B M\theta}, \qquad (34)$$

$$\operatorname{tg}\beta=-(3Z_B M\theta/\pi)^{1/2},$$

where $E = Mv^2/2$ is the collision energy in the c.m.s. and θ is the scattering angle in the same system. In the region where the theory is valid (see the inequalities (27) and (30)), the cross section oscillates as a function of the angle. The quantity $E\sigma(\theta)$ does not depend on the energy E. The latter follows directly from the fact that the dipole-interaction potential and the Laplace operator are transformed in the same manner under a scale transformation (compare with the form of the differential cross section in the simple $1/R^2$ potential [7]).

The positions of the first five maxima of the cross section are shown in the figure. We note also that, as indicated in [1], we can expect the very existence of interference oscillations to continue even outside of the limits of applicability of certain approximations assumed in this paper.

With the aid of the obtained formulas we can also calculate the cross sections of transitions between fine-structure sublevels (this is discussed in greater detail in [1]). The cross sections of the transitions $2s_{1/2} \rightleftarrows 2p_{1/2}$ and $2s_{1/2} \rightleftarrows 2p_{3/2}$ differ from the cross section (33) only by statistical factors, and for the transition $2p_{1/2} \rightleftarrows 2p_{3/2}$ we obtain

$$\sigma_{2p_{1/2}\to 2p_{3/2}}(\theta)=2\sigma_{2p_{3/2}\to 2p_{1/2}}(\theta)=\frac{3\pi Z_B}{18E\theta^2\sin\theta}(1+\cos^4\beta)\cos^2\sqrt{12\pi Z_B M\theta}. \qquad (35)$$

In this paper we do not calculate the total scattering cross sections. Inasmuch as the trajectories of the heavy particles differ little here from straight lines, it is difficult to expect the total cross section to differ appreciably from that obtained by Chibisov [8].

4. CONCLUSION

As was indicated in the Introduction, a question common to problems of atomic collisions is that of determining the potential in which the nuclei move if several states are considered in the quantum-mechanical electronic part of the problem and the motion of the nuclei is described by classical mechanics. The problem considered in the present paper differs in that we were able to determine explicitly the dynamic wave functions $\Psi_{nn'n''}(\mathbf{r}, \mathbf{R})$, between which no transitions take place, with the corresponding potential containing, in addition to the dipole interaction, also an effective magnetic field, and having consequently a dependence on the momentum of the heavy particles. The dependence of the dynamic terms on L is qualitatively due to the fact that the dipole moment of the hydrogen atom is not directed exactly at the incident particle, but lags it somewhat. However, our investigation has shown that for scattering through angles that are not too small [Eq. (30)] this lag can be neglected, which corresponds to the use of the adiabatic approximation. The magnetic field can then be disregarded ($\beta = \pi/2$), and the particle moves in the potential $V_{nn'n''}$ (26) which can be obtained by averaging the initial interaction potential $\mathbf{r}\cdot\mathbf{R}/R^3$ over the electronic states $\psi_{nn'n''}$ and taking the inequality (24) into account. The phase difference acquired in scattering through one and the same angle θ in the potentials $V_{nn'n''}$ leads to interference oscillations in the scattering cross sections.

Comparing our results with those of [1], where the trajectories of the heavy particles were governed by their Coulomb interaction, we indicate first that, in contrast to [1], in our case the states $\psi_{nn'n''}$ with $n' + n'' = 0$ make no contribution whatever to the differential scattering cross section, inasmuch as there is no particle interaction for these states in the employed approximation. In the adiabatic case with $n = 2$ the oscillations in the scattering cross section are due to the interference of two states that are split from the initial level of the hydrogen atom as a result of the linear Stark effect. The phase differences that determine the interference pattern are different because in [1] the $L(\theta)$ dependence was determined by scattering from a Coulomb potential, whereas in our case it is determined by scattering from a potential $1/R^2$. When the adiabatic approximation ceases to be valid, the interference pattern becomes more complicated, corresponding now to the mixing of three states (see Fig. 1 of [1]). In our problem, however, the condition for applicability of the quasiclassical approximation and the adiabaticity condition coincide, and all the results admit of a simple interpretation.

We note that the quasiclassical approximation constructed in the present paper (Sec. 2) can be generalized directly to include the case of scattering of a charged particle by a hydrogenlike ion, i.e., for the general potential (1). All that changes is the equation for the radial function (6), the solution of which is now a Whittaker function, and this leads to another expression for the phase (21). Thus, it becomes possible to refine the results of [1] by taking into account the influence of the dipole interaction on the motion of the charged particle.

An important role was played in the problems considered in the present paper and in [1] by the presence of the large parameter M, the reduced mass of the nuclei. It made it possible, in particular, to use a classical or quasiclassical description of the motion of the incident particle. For the scattering of an electron by hydrogen-

like systems (M = 1), the region where the theory is valid is strongly limited. It would be desirable in this case to consider the quantum problem, while retaining the remaining approximations.

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[1)]The atomic system of units is used.

[2)]The existence of an additional integral of motion in n-subspace was used in a somewhat implicit form by Seaton [3] and by Gaĭlitis and Damburg [4], where the operator Λ was diagonalized for the case n = 2 in the representation of the total angular momentum without using the quasiclassical approximation.

[3)]We note that the solution is obtained in this case in the form of an infinite sum, each term of which has formally a singularity at $\theta = 0$ and at $\theta = \pi$. It must be borne in mind, however, that the solution has been constructed in the quasiclassical approximation and does not hold near the indicated values of θ (see (30)).

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