Sound absorption in a He^3-He^4 solution near the λ point

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A complete set of hydrodynamic equations for He^3-He^4 solutions is obtained which takes into account the relaxation of the density ρ_s of the superfluid component near the λ point. The propagation of first, second, and fourth sound is investigated on the basis of the set of equations. The second-viscosity coefficients, which are connected with the relaxation of ρ_s near the λ point, are calculated. The temperature dependence of the sound absorption coefficients, which is related to the second viscosity and impurity diffusion, is found.

The propagation of sound near second-order phase transition points exhibits a number of interesting features.[1] Near the λ point, those effects become important which are connected with the relaxation of the density of the superfluid component of the liquid, ρ_s. Phenomena are observed here that are similar to what takes place in ordinary hydrodynamics in the presence of slow relaxation processes.[2] However, there are also important differences, connected with the possibility of propagation of two types of oscillations in the superfluid liquid. The presence of He^3 impurities, which do not take part in the superfluid motion, also complicates the picture. Whereas only the pressure oscillates in a wave of first sound in pure He^4, and only the temperature in a wave of second sound, oscillations of temperature, pressure and concentration take place in both waves in the solution.

The necessity of taking the relaxation of ρ_s into account leads to the result that, near the λ point, the ordinary equations of two-velocity hydrodynamics are inapplicable, and it is necessary to use a set of equations which takes slow relaxation processes into account. Such equations of two-velocity hydrodynamics were obtained for pure He^4 by Pitaevskiĭ,[3] and were made more exact in the work of Khalatnikov.[4] The equations obtained by the author in[5] for superfluid solutions of He^3—He^4, also require similar improvement.

After the necessary transformations, we obtain a set of equations for the He^3—He^4 solutions which differs from the equations for pure He^4[4] in the fact that it contains an additional equation which describes the law of conservation of density of the impurities, and the role of the chemical potential of He^4 in the solution is played by the quantity $\mu + \mu_s - Zc/\rho$, where

$$\mu=\left(\frac{\partial E_0}{\partial \rho}\right)_{\rho_s,p,c},\quad \mu_s=\left(\frac{\partial E_0}{\partial \rho_s}\right)_{\rho,s,c},\quad Z=\left(\frac{\partial E_0}{\partial c}\right)_{\rho_s,\rho,s}$$

(E_0 is the internal energy per unit volume of the liquid). In the equation for the increase in entropy, however, it is necessary to take the flow of impurities into account. For the study of sound propagation, we can limit ourselves to the case of small gradients of ρ_s. We then get the following simplified set of equations:

$$\frac{\partial \mathbf{v}_s}{\partial t}+\nabla\left(\frac{v_s^2}{2}+\mu+\mu_s-\frac{Z}{\rho}c\right)=0, \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho_s\mathbf{v}_s+\rho_n\mathbf{v}_n)=0, \tag{2}$$

$$\frac{\partial(\rho c)}{\partial t}+\operatorname{div}(\rho c\mathbf{v}_n+g)=0, \tag{3}$$

$$\begin{gathered}\frac{\partial}{\partial t}(\rho_n v_{ni}+\rho_s v_{si})+\frac{\partial}{\partial x_k}(\rho_n v_{ni}v_{nk}+\rho_s v_{si}v_{sk}+p\delta_{ik})\\ +\eta\frac{\partial}{\partial x_k}\left(\frac{\partial v_{ni}}{\partial x_k}+\frac{\partial v_{nk}}{\partial x_i}-\frac{2}{3}\operatorname{div}\mathbf{v}_n\right)=0,\end{gathered} \tag{4}$$

$$\frac{\partial S}{\partial t}+\operatorname{div}\left[S\mathbf{v}_n+\frac{1}{T}\left(\mathbf{q}-\frac{\mathbf{g}Z}{\rho}\right)\right]=\frac{R}{T}, \tag{5}$$

$$\frac{\partial \rho_s}{\partial t}+\operatorname{div}\rho_s\mathbf{v}_s=-\frac{2\Lambda m}{\hbar}\left[\mu_s+\frac{(\mathbf{v}_n-\mathbf{v}_s)^2}{2}\right]\rho_s. \tag{6}$$

The dissipative function of the liquid is

$$\begin{gathered}R=\frac{2\Lambda m}{\hbar}\left[\frac{1}{2}(\mathbf{v}_n-\mathbf{v}_s)^2+\mu_s\right]^2\rho_s+\frac{1}{2}\eta\left(\frac{\partial v_{ni}}{\partial x_k}+\frac{\partial v_{nk}}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_{ne}}{\partial x_e}\right)^2\\ -\mathbf{q}\frac{\nabla T}{T}-\mathbf{g}T\nabla\frac{Z}{\rho T}.\end{gathered} \tag{7}$$

The impurity flux g and the heat flux q are expressed in terms of the usual formulas.[6] The dimensionless kinetic coefficient Λ in the equation depends on the specific mechanism of relaxation of ρ_s.

The set of equations (1)—(6) is identical in form with the usual set of equations of the hydrodynamics of He^3—He^4 solutions, with the difference, however, that ρ_s is not a given function of p, T and c, but an independent quantity, the approach of which to its equilibrium value is described by the additional equation (6).

By linearizing the equations and assuming the departure of all the quantities from their equilibrium values to be proportional to $e^{-i\omega t}$, we can calculate the nonequilibrium increment to μ_s that is produced by propagation of a sound wave:

$$\begin{gathered}\mu_s'=\frac{1}{1-i\omega\tau}\frac{\hbar\rho}{2\Lambda m\rho_s}\left\{\left[\left(\frac{\partial \rho_s}{\partial \rho}\right)_{\sigma,\mu_s,c}-\frac{\rho_s}{\rho}\right]\operatorname{div}\mathbf{v}_n\right.\\ \left.+\frac{1}{\rho}\left[\left(\frac{\partial \rho_s}{\partial \rho}\right)_{\sigma,\mu_s,c}-\frac{\sigma}{\rho}\left(\frac{\partial \rho_s}{\partial \sigma}\right)_{\mu_s,\rho,c}-\frac{c}{\rho}\left(\frac{\partial \rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}-1\right]\operatorname{div}\rho_s(\mathbf{v}_s-\mathbf{v}_n)\right\},\end{gathered} \tag{8}$$

where we have introduced the system relaxation time

$$\tau=\frac{\hbar}{2\Lambda m\rho_s}\left(\frac{\partial \rho_s}{\partial \mu_s}\right)_{\sigma,\rho,c}. \tag{9}$$

Substituting the values of μ_s' in (1) and (6), we obtain

$$\begin{gathered}\frac{\partial \mathbf{v}_s}{\partial t}+\nabla\mu=\nabla\left\{\frac{1}{1-i\omega\tau}\frac{\hbar}{2\Lambda m\rho_s}\left(\frac{\partial p}{\partial \mu_s}\right)_{\sigma,\rho,c}\left[\left(\frac{\partial \mu}{\partial \mu_s}\right)_{\sigma,\rho,c}\right.\right.\\ \left.\left.+\frac{c}{\rho}\left(\frac{\partial \rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right]\operatorname{div}\mathbf{v}_n\right\}+\nabla\left\{\frac{1}{1-i\omega\tau}\frac{\hbar}{2\Lambda m\rho_s}\left[\left(\frac{\partial \mu}{\partial \mu_s}\right)_{\sigma,\rho,c}\right.\right.\\ \left.\left.+\frac{c}{\rho}\left(\frac{\partial \rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right]^2\operatorname{div}\rho_s(\mathbf{v}_s-\mathbf{v}_n)\right\},\end{gathered} \tag{10}$$

$$\begin{gathered}\frac{\partial j_i}{\partial t}+\nabla p=\eta\frac{\partial}{\partial x_k}\left(\frac{\partial v_{ni}}{\partial x_k}+\frac{\partial v_{nk}}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_{ne}}{\partial x_e}\right)\\ +\nabla\left\{\frac{1}{1-i\omega\tau}\frac{\hbar}{2\Lambda m\rho_s}\left(\frac{\partial p}{\partial \mu_s}\right)^2_{\sigma,\rho,c}\operatorname{div}\mathbf{v}_n\right\}+\nabla\left\{\frac{1}{1-i\omega\tau}\frac{\hbar}{2\Lambda m\rho_s}\left[\left(\frac{\partial \mu}{\partial \mu_s}\right)_{\sigma,\rho,c}\right.\right.\\ \left.\left.+\frac{c}{\rho}\left(\frac{\partial \rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right]\operatorname{div}\rho_s(\mathbf{v}_s-\mathbf{v}_n)\right\}.\end{gathered} \tag{11}$$

Comparing (10)—(11) with the usual set of hydrodynamic equations of He^3—He^4 solutions,[6] we find that the terms in front of div $\mathbf{v}_n$ and div $\rho_s(\mathbf{v}_s - \mathbf{v}_n)$ play the role of second-viscosity coefficients, which are connected with the slow processes of relaxation of ρ_s:

$$\zeta_1=\zeta_4=\frac{\tau}{1-i\omega\tau}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\rho,\sigma,c}\left(\frac{\partial p}{\partial\mu_s}\right)_{\rho,\sigma,c}\left[\left(\frac{\partial\mu}{\partial\mu_s}\right)_{\rho,\sigma,c}+\frac{c}{\rho}\left(\frac{\partial\rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right], \tag{12}$$

$$\zeta_2=\frac{\tau}{1-i\omega\tau}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\rho,\sigma,c}\left(\frac{\partial p}{\partial\mu_s}\right)^2_{\rho,\sigma,c}, \tag{13}$$

$$\zeta_3=\frac{\tau}{1-i\omega\tau}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\rho,\sigma,c}\left[\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\rho,\sigma,c}+\frac{c}{\rho}\left(\frac{\partial\rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right]^2. \tag{14}$$

In view of the fact that the second viscosity coefficients found here are singular near the λ point, they make a fundamental contribution to sound absorption and in further calculations, we need not take account of the usual (nonsingular) second-viscosity coefficients.

Solving the set of equations (1)—(6) in the linear approximation, without account of the diffusion or thermal conductivity, we obtain an expression which determines the dispersion of first sound:

$$\begin{gathered}\frac{1}{u_1^2}=\frac{1}{u_{10}^2}+\frac{i\omega\tau}{1-i\omega\tau}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\\ \times\frac{1}{\rho u_{10}^4}\left(\frac{\partial p}{\partial\mu_s}\right)^2_{\sigma,\rho,c}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\sigma,\rho,c},\qquad \beta=\frac{c}{\rho}\frac{\partial\rho}{\partial c},\end{gathered} \tag{15}$$

where u_{10} is the equilibrium velocity of first sound at small ω.

Using (15), it is easy to obtain an expression which determines the dispersion of the sound velocity near the λ point, and the absorption coefficient:

$$\begin{gathered}u_1^2=u_{10}^2\left[1+\frac{1}{2\rho u_{10}^2}\frac{\omega^2\tau^2}{1+\omega^2\tau^2}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\right.\\ \left.\times\left(\frac{\partial p}{\partial\mu_s}\right)^2_{\sigma,\rho,c}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\sigma,\rho,c}\right],\end{gathered} \tag{16}$$

$$\begin{gathered}\alpha_1=\frac{1}{2\rho u_{10}^3}\frac{\omega^2\tau}{1+\omega^2\tau^2}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\\ \times\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\sigma,\rho,c}\left(\frac{\partial p}{\partial\mu_s}\right)^2_{\sigma,\rho,c}.\end{gathered} \tag{17}$$

So far as the second sound is concerned, our treatment is valid only for the case $\omega\tau \ll 1$, since for $\omega\tau \sim 1$ second sound is completely absorbed. The coefficient of absorption of second sound for small frequencies is equal to

$$\begin{gathered}\alpha_2=\frac{\omega^2\tau}{2u_2^3}\frac{\rho_s}{\rho_n\rho}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\sigma,\rho,c}\left\{(1+\beta)\left(\frac{\partial p}{\partial\mu_s}\right)_{\sigma,\rho,c}\right.\\ \left.-\rho\left[\left(\frac{\partial\mu}{\partial\mu_s}\right)_{\sigma,\rho,c}+\frac{1}{\rho}\left(\frac{\partial\rho_s}{\partial c}\right)_{\mu_s,\rho,\sigma}+1\right]\right\}^2.\end{gathered} \tag{18}$$

Since the derivatives of thermodynamic quantities are usually measured in the variables p, T and c, it is convenient to write out the expressions for the sound absorption coefficients in these variables:

$$\begin{gathered}\alpha_1=\frac{\omega^2}{1+\omega^2\tau^2}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\frac{\hbar\rho}{4\Lambda m\rho_s u_{10}^3}\cdot\\ \times\left[\frac{1}{\rho}\left(\frac{\partial\rho_s}{\partial T}\right)_{p,c}\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho_s}{\partial p}\right)_{T,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\right]^2\cdot\\ \times\left[\left(\frac{\partial\rho}{\partial p}\right)_{T,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho}{\partial T}\right)_{p,c}\left(\frac{\partial\sigma}{\partial p}\right)_{T,c}\right]^{-2},\end{gathered} \tag{19}$$

$$\begin{gathered}\alpha_2=\frac{\omega^2}{4\Lambda m u_2^3}\left\{\frac{\bar\sigma}{\rho}\left[\left(\frac{\partial\rho_s}{\partial p}\right)_{T,c}\left(\frac{\partial\rho}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho}{\partial p}\right)_{T,c}\left(\frac{\partial\rho_s}{\partial T}\right)_{p,c}\right]\cdot\right.\\ \left.\times\left[\left(\frac{\partial\rho}{\partial p}\right)_{T,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho}{\partial T}\right)_{p,c}\left(\frac{\partial\sigma}{\partial p}\right)_{T,c}\right]^{-1}-\frac{c}{\rho}\left(\frac{\partial\rho_s}{\partial c}\right)_{p,T}-1\right\}^2.\end{gathered} \tag{20}$$

For clarification of the temperature dependence of the sound absorption coefficients near the λ point, it is convenient to isolate the factors singular in the temperature and to connect the derivatives entering into Eqs. (19) and (20) with the specific heat of the solution. We shall determine the singularity of the thermodynamic functions near the λ point in what follows on the basis of the similarity hypothesis.[7,8] For this purpose, it is appropriate to make use of the general thermodynamic relations which connect the derivatives of the thermodynamic quantities with the derivatives along the λ line and with the specific heat.[9]

The derivatives of any thermodynamic quantity W(p, T, c) that is a function of the three variables p, T and c, satisfy the following relations at constant p:

$$\left(\frac{\partial W}{\partial T}\right)_{p,t}=\left(\frac{\partial W}{\partial T}\right)_{p,c}+\left(\frac{\partial W}{\partial c}\right)_{p,T}\left(\frac{\partial c}{\partial T}\right)_{p,t}, \tag{21}$$

$$\left(\frac{\partial W}{\partial c}\right)_{p,t}=\left(\frac{\partial W}{\partial c}\right)_{p,T}+\left(\frac{\partial W}{\partial T}\right)_{p,c}\left(\frac{\partial T}{\partial c}\right)_{p,t}. \tag{22}$$

Here the derivative $(\ldots)_{p,t}$ is calculated along any curve in T—c space that passes through the point we need. In our case, the line of the λ transition in the solution is such a curve. Similar relations can be written down in p—c and T—p spaces, respectively, for constant values of T and c.

After simple calculations, we can obtain the following by using (21), (22) and similar relations for the first-sound absorption coefficient:

$$\begin{gathered}\alpha_1=\frac{\omega^2}{1+\omega^2\tau^2}\frac{\hbar\rho}{4\Lambda m\rho_s u_{10}^3}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\\ \times\left(\frac{\partial\rho_s}{\partial T}\right)^2_{p,c}\left(\frac{\partial\sigma}{\partial T}\right)^{-2}_{p,c}A_\lambda^2,\end{gathered} \tag{23}$$

$$\begin{gathered}A_\lambda=\left(\frac{\partial\sigma}{\partial p}\right)_{\lambda,c}\left[\left(\frac{\partial\rho}{\partial p}\right)_{\lambda,c}+\rho^2\left(\frac{\partial\sigma}{\partial p}\right)_{\lambda,c}\frac{\partial T_\lambda}{\partial p}\right.\\ \left.-\rho^2\left(\frac{\partial\sigma}{\partial p}\right)_{\lambda,c}\left(\frac{\partial\sigma}{\partial T}\right)^{-1}_{p,c}\right]^{-1}\end{gathered} \tag{24}$$

The symbol λ on the derivatives indicates that the derivative is taken along the λ transition curve.

In a similar way, we obtain the following for the second-sound absorption coefficient:

$$\alpha_2=\frac{\omega^2\hbar}{4\Lambda m u_2^3}\left[\left(\frac{\partial\rho_s}{\partial T}\right)_{p,c}\left(\frac{\partial\sigma}{\partial T}\right)^{-1}_{p,c}\left(\frac{\bar\sigma}{\rho}B_\lambda-\frac{c}{\rho}\frac{\partial T_\lambda}{\partial c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\right)+1\right]^2; \tag{25}$$

$$\begin{gathered}B_\lambda=A_\lambda\left(\frac{\partial\rho}{\partial p}\right)_{\lambda,c}\left(\frac{\partial\sigma}{\partial p}\right)^{-1}_{\lambda,c},\\ \bar\sigma=\sigma-c\left(\frac{\partial\sigma}{\partial c}\right)_{p,T}=\bar\sigma_0+c\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\frac{\partial T_\lambda}{\partial c},\\ \bar\sigma_0=\sigma-c\left(\frac{\partial\sigma}{\partial T}\right)_{p,\lambda}\frac{\partial T_\lambda}{\partial c}.\end{gathered} \tag{26}$$

Taking into account that the derivative $c^2\partial(Z/\rho)/\partial c$ tends to zero near the λ point,[10,11] we can write down the velocity of second sound in solutions, in the immediate vicinity of the λ point, in the following fashion:

$$u_2=u_{20}\left[1+\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\frac{c}{\bar\sigma_0}\frac{\partial T_\lambda}{\partial c}\right], \tag{27}$$

$$u_{20}=\left[\frac{\rho_s}{\rho}\bar\sigma_0^2\left(\frac{\partial\sigma}{\partial T}\right)^{-1}_{p,c}\right]^{1/2}. \tag{28}$$

In view of the fact that all the derivatives entering into the expressions for the absorption coefficients can be calculated from thermodynamic measurements, Eqs. (23) and (25) allow us to determine the temperature and concentration behavior of the basic kinetic characteristics of the given theory of Λ from measurements of the absorption of first or second sound near the λ point. On the other hand, we can determine the temperature and concentration dependence of Λ if we take the mechanism of sound dissipation near the λ point into account.[12]

Near the λ point, the system can be characterized by some correlation length ξ. Second-sound quanta with wavelength of the order of ξ will be completely dissipated and we can introduce a characteristic time $\tau = \xi/u_2$ which determines the rate of dissipation of second-sound energy. On the other hand, recognizing the connection (9) between the coefficient Λ and the relaxation time τ, we get

$$\Lambda \approx \frac{mu_2\xi}{\hbar} = \frac{m\xi}{\hbar} u_{20}\left[1+\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\frac{c}{\bar{\sigma}_0}\frac{\partial T_\lambda}{\partial c}\right]. \tag{29}$$

Energy dissipation of first sound takes place by decay of the waves of first sound into quanta of second sound and is characterized by the same relaxation time τ.

As we see from the formulas given above, the temperature dependence of τ, Λ, α_1 and α_2 is essentially determined by the behavior of ρ_S near the λ point. The temperature dependence associated with the specific heat in He^3—He^4 solutions (in contrast with pure He^4) is insignificant, since a renormalization of the critical indices takes place,[13,14] and the specific heat at constant pressure and concentration is finite, although it has a sharp peak near the λ point.[11,15]

If the change of ρ_S with temperature near the λ point takes place according to the law $\rho_S \sim \epsilon^{\zeta_\lambda}$ (ζ_λ is the critical index for change of ρ_S, $\epsilon = (T_\lambda - T)/T_\lambda$), we get

$$\tau\sim\varepsilon^{-3\zeta_\lambda/2}, \quad \Lambda\sim\varepsilon^{-\zeta_\lambda/2}, \tag{30}$$

and for the sound absorption coefficients we obtain

$$\alpha_1\sim\varepsilon^{-2+3\zeta_\lambda/2}, \quad \alpha_2\sim\varepsilon^{-2+\zeta_\lambda}. \tag{31}$$

For pure He^4, $\zeta_\lambda = (2 - \alpha)/3$ (α is the critical specific heat exponent). As experimental data show, the temperature dependence of ρ_S in solutions is satisfactorily described by the law $\zeta_\lambda = 2/3$ for the case $\epsilon \lesssim 2 \times 10^{-3}$.[16] In such a case, $\alpha_1 \sim \epsilon^{-1}$ and $\alpha_2 \sim \epsilon^{-4/3}$. Experiments[17] on the absorption of first sound in an He^3—He^4 solution indicate the validity of the law $\alpha_1 \sim \epsilon^{-1}$. To calculate the difference between the high-frequency sound velocity $u_{1\infty}$ and the equilibrium value from Eq. (16), we must also take into account the dependence of the derivatives, τ and Λ on the specific heat. For this difference we obtain

$$u_{1\infty}-u_{10}\sim\varepsilon^{-2+3\zeta_\lambda}/C_{p,c}^2. \tag{32}$$

In the case of pure He^4, it is found that $u_{1\infty} - u_{10} \sim C_p^{-1}$. However, experiments show[18,19] that both for He^4 and for He^3—He^4 solutions, this law gives only the first term of the dependence of the velocity difference, and it is necessary to use the exact formula (16), rewriting it in the variables p, T, c:

$$\begin{aligned} u_1-u_{10} &= \frac{1}{2u_{10}}\frac{\omega^2\tau}{1+\omega^2\tau^2}\frac{\hbar\rho}{2\Lambda m\rho_s}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1} \\ &\times\left[\frac{1}{\rho}\left(\frac{\partial\rho_s}{\partial T}\right)_{p,c}\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho}{\partial p}\right)_{T,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\right]^2 \\ &\times\left[\left(\frac{\partial\rho}{\partial p}\right)_{T,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}-\left(\frac{\partial\rho}{\partial T}\right)_{p,c}\left(\frac{\partial\sigma}{\partial p}\right)_{T,c}\right]^{-2}. \end{aligned} \tag{33}$$

The diffusion of the impurities also makes a contribution to the sound absorption, along with the second-viscosity coefficients. If we take into account that $\rho_S \to 0$ near the λ point, the expressions[20] for the first and second sound absorption coefficients simplify and take the following form:

$$\alpha_1^{(D)}=\frac{1}{2}\frac{\omega^2}{u_1^2}\beta^2 D\left\{c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\right\}^{-1},$$

$$\alpha_2^{(D)}=\frac{\omega^2}{2u_2^5}\frac{\rho_s}{\rho_n}D\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(c+\frac{k_T\bar{\sigma}}{C_{p,c}}\right)^2, \tag{34}$$

where D is the diffusion coefficient and $k_T D$ the thermal diffusion coefficient.

As discussions based on the theory of dynamic similarity have shown,[21] the temperature dependence of the diffusion coefficient is determined by the law $D \sim u_2\rho_S^{-1}$, which, without account of the specific heat, gives the temperature dependence $D \sim \epsilon^{-\zeta_\lambda/2}$ ($D \sim \epsilon^{-1/2}$ if $\zeta_\lambda = 2/3$). Consequently, $\alpha_1^{(D)} \sim \epsilon^{-\zeta_\lambda/2}$, $\alpha_2^{(D)} \sim \epsilon^{-2\zeta_\lambda}$ (the weak singularity associated with the factor $\partial(Z/\rho)/\partial c$ need not be taken into account). For $\zeta_\lambda = 2/3$, we get $\alpha_1^{(D)} \sim \epsilon^{-1/3}$, $\alpha_2^{(D)} \sim \epsilon^{-4/3}$.

Fourth sound can be propagated under specific conditions of the total stopping of the normal component. The set of equations describing the propagation of fourth sound can be obtained from (1)—(6) if we set $v_n = 0$ in them and eliminate the equation which describes the law of momentum conservation. Of the coefficients of second viscosity only ζ_3 enters into the sound absorption coefficient. Since only the limit $\omega\tau \ll 1$ is important for fourth sound, we have for the absorption coefficient

$$\alpha_4=\frac{\omega^2\tau}{2u_4^3}\left(\frac{\partial\mu_s}{\partial\rho_s}\right)_{\sigma,\rho,c}\left[\left(\frac{\partial\mu}{\partial\mu_s}\right)_{\rho,\sigma,c}+\frac{c}{\rho}\left(\frac{\partial\rho_s}{\partial c}\right)_{\rho,\sigma,\mu_s}+1\right]^2. \tag{35}$$

Separating the singular parts in the derivatives, we get

$$\begin{aligned} \alpha_4 &= \frac{\omega^2\hbar}{4\Lambda m u_4^3}\left[\left(\frac{\partial\rho_s}{\partial T}\right)_{p,c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}^{-1}\left(\frac{\bar{\sigma}}{\rho}B_\lambda\right.\right. \\ &\left.\left.+(1+\beta)A_\lambda-\frac{c}{\rho}\frac{\partial T_\lambda}{\partial c}\left(\frac{\partial\sigma}{\partial T}\right)_{p,c}\right)+1\right]^2. \end{aligned} \tag{36}$$

The mechanism of energy dissipation in fourth sound is decay of waves of fourth sound into quanta of second sound.[22] Taking into account the temperature dependence of Λ, we get

$$\alpha_4\sim\varepsilon^{-2+\zeta_\lambda}.$$

The absorption of fourth sound, which is connected with diffusion of the impurities, is determined by the formula

$$\alpha_4^{(D)}=\frac{\omega^2}{2u_4^5}\frac{\rho_s}{\rho}D\left[c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\right]^{-1}\left[\left(c+\frac{k_T\bar{\sigma}}{C_{p,c}}\right)c\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)+u_1^2\beta(1+\beta)\right]^2. \tag{37}$$

Taking into account the temperature dependence of u_4 and D, we get $\alpha_4^{(D)} \sim \epsilon^{-2\zeta_\lambda}$. Consequently, the temperature dependence of the absorption coefficient of fourth sound is the same as that for second sound.

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