Magnetic-field-induced interaction of collective excitations in a weak ferromagnet with single-ion anisotropy

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(Submitted June 15, 1973)
Zh. Eksp. Teor. Fiz. 65, 2334–2342 (December 1973)

The spectrum of the collective excitations of a weak ferromagnet with easy-plane anisotropy and with spin S = 1 at each site is investigated in the zero-order approximation of the Vaks-Larkin-Pikin method [1,2]. Exact allowance for the single-ion anisotropy produces in the spectrum a number of additional modes besides the usual spin waves. When the magnetic field is turned on, all the modes interact, and this is manifest, in particular, in the absence of a field intersection point for the AFMR frequencies. The field dependence of the frequencies at low temperatures is calculated numerically. It is also shown that the anisotropy constant cannot exceed the value of the exchange potential.

1. INTRODUCTION

The main sources of anisotropic interactions in antiferromagnetic crystals are the anisotropic exchange interaction and single-ion anisotropy. The role of these interactions can become manifest differently in the formation of the spin-wave spectrum. The anisotropic exchange interaction, since it takes place between unlike ions, is described sufficiently well by the self-consistent field approximation [1,2] and by the macroscopic theory described in detail in Turov's book [3]. At the same time, the fact that the energy spectrum of the isolated spin in nonequidistant character because of single-ion anisotropy calls for a consistent microscopic calculation of the spin-wave spectrum. Such an approach was attempted by us in [4], where we considered in detail the case of a compensated antiferromagnet in a zero magnetic field. It was shown that as a result of the exact allowance for the single-ion anisotropy a new group of oscillations appears in the high-frequency region of the spectrum of the collective excitations and is due to the nonconservation of the projection of the isolated spin on the molecular-field direction.

In this paper we consider a weak ferromagnet with easy-plane anisotropy under the assumption that the spin S of each magnetic line is equal to unity. It follows from the calculation that in the absence of a magnetic field the spectrum of the weak ferromagnet breaks up into two noninteracting groups of excitations, one of which, as in the case of the compensated antiferromagnet, is connected with the nonconservation of the spin projection on the direction of the molecular field. Turning on the magnetic field leads to interaction of these groups of oscillations. The calculation is carried out with the aid of the Vaks-Larkin-Pikin (VLP) self-consistent-field method [1,2], which is generalized to include the case when arbitrary single-ion anisotropy is taken into account [4].

2. THE HAMILTONIAN

If we neglect the interaction of the spins inside each of the sublattices of the weak ferromagnet, then the Hamiltonian, according to [5], is given by

$$\beta\mathcal{H}=\sum_{\mathbf{r}_f,\mathbf{r}_g}\{V^{\parallel}(\mathbf{r}_f-\mathbf{r}_g)S^x_{\mathbf{r}_f}S^x_{\mathbf{r}_g}+V^{\perp}(\mathbf{r}_f-\mathbf{r}_g)(S^x_{\mathbf{r}_g}S^x_{\mathbf{r}_g}+S^y_{\mathbf{r}_f}S^y_{\mathbf{r}_g}) -\varkappa(\mathbf{r}_f-\mathbf{r}_g)(S^x_{\mathbf{r}_f}S^y_{\mathbf{r}_g}-S^y_{\mathbf{r}_f}S^x_{\mathbf{r}_g})\}+\sum_{\mathbf{r}_f}[2\gamma(S^z_{\mathbf{r}_f})^2-hS^y_{\mathbf{r}_f}]+\sum_{\mathbf{r}_g}[2\gamma(S^z_{\mathbf{r}_g})^2-hS^y_{\mathbf{r}_g}], \tag{1}$$

where $V^{\parallel}(\mathbf{r}_f-\mathbf{r}_g)$ and $V^{\perp}(\mathbf{r}_f-\mathbf{r}_g)$ are the potentials of exchange interaction of a pair of ions with S = 1, located at the site $\mathbf{r}_f$ and $\mathbf{r}_g$ of the sublattices f and g, respectively; $\varkappa(\mathbf{r}_f-\mathbf{r}_g)$ is the Dzyaloshinskiĭ interaction potential; γ is the single-ion anisotropy constant; h is the reduced external magnetic field (the g-factor is assumed for simplicity to be isotropic). The single-ion and exchange anisotropy interactions are chosen in such a way that they ensure anisotropy of the easy xy plane type. All the quantities in (1) are given for convenience in dimensionless form ($\beta=T^{-1}$, where T is the temperature).

We separate from (1) the zero-order interaction Hamiltonians

$$\begin{aligned}\beta\mathcal{H}_{0f}&=\langle S\rangle(V_0^{\perp}\cos\psi+\varkappa_0\sin\psi)\sum_{\mathbf{r}_f}S^x_{\mathbf{r}_f}+\langle S\rangle\left(\frac{h}{\langle S\rangle}-V_0^{\perp}\sin\psi+\varkappa_0\cos\psi\right)\\&\quad\times\sum_{\mathbf{r}_f}S^y_{\mathbf{r}_f}-2\gamma\sum_{\mathbf{r}_f}(S^z_{\mathbf{r}_f})^2,\\-\beta\mathcal{H}_{0g}&=-\langle S\rangle(V_0^{\perp}\cos\psi+\varkappa_0\sin\psi)\sum_{\mathbf{r}_g}S^x_{\mathbf{r}_g}\\&\quad+\langle S\rangle\left(\frac{h}{\langle S\rangle}-V_0^{\perp}\sin\psi+\varkappa_0\cos\psi\right)\sum_{\mathbf{r}_g}S^y_{\mathbf{r}_g}-2\gamma\sum_{\mathbf{r}_g}(S^z_{\mathbf{r}_g})^2,\end{aligned} \tag{2}$$

which describe the behavior of the spins in the corresponding molecular fields with allowance for the single-ion anisotropy. In (2) $\langle S\rangle$ stands for the average spin at the site, and ψ is the angle between the average spin of each sublattice and the x axis.

The self-consistency condition (equality of the angle ψ to the angle between the x axis and the molecular field) leads to a transcendental equation for the equilibrium value of the angle:

$$\frac{\sin 2\eta}{\cos(\eta+\psi_0)}=\frac{h}{\langle S(\eta)\rangle V_0}, \tag{3}$$

$$\eta=\psi-\psi_0,\ \psi_0=\psi(h=0),\ \operatorname{tg}2\psi_0=\varkappa_0/V_0^{\perp},\ V_0=((V_0^{\perp})^2+\varkappa_0^2)^{1/2}. \tag{4}$$

The explicit form of $\langle S\rangle$ will be obtained later on.

As seen from (3), there is no critical magnetic collapse field in this system. This corresponds to the effect of induction of antiferromagnetic order above the Neel point by a magnetic field [6].

When (3) and (4) are taken into account, expressions (2) for the zero-order Hamiltonians become

$$\begin{aligned}-\beta\mathcal{H}_{0f}&=\sum_{\mathbf{r}_f}\left[\frac{y}{\sqrt{2}}(e^{-i\psi}S^+_{\mathbf{r}_f}+e^{i\psi}S^-_{\mathbf{r}_f})-2\gamma(S^z_{\mathbf{r}_f})^2\right],\\-\beta\mathcal{H}_{0g}&=-\sum_{\mathbf{r}_g}\left[\frac{y}{\sqrt{2}}(e^{i\psi}S^+_{\mathbf{r}_g}+e^{-i\psi}S^-_{\mathbf{r}_g})+2\gamma(S^z_{\mathbf{r}_g})^2\right],\end{aligned} \tag{5}$$

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where the square of the modulus of the molecular field is

$$y_i^2=\langle S\rangle^2V_0^2+h^2\sin\psi_0/\sin 2\eta. \tag{6}$$

3. DISPERSION EQUATION

The spectrum of the collective excitations of the spin system is determined by the poles of the analytic continuation $i\omega_n\to\omega$ of the matrix of the correlation functions $\hat{K}(\mathbf{q}, i\omega_n)$ with the matrix elements

$$K_{l\,l'}^{m\,m'}(\mathbf{q}, i\omega_n)=\frac{1}{2\beta}\int_{-\beta}^{\beta}e^{i\omega_n\tau}\sum_{\mathbf{r}_m,\mathbf{r}_{m'}}\exp[i\mathbf{q}(\mathbf{r}_{l'}-\mathbf{r}_{m'})]$$
$$\times\langle\hat{T}\{(S_{\mathbf{r}_l^l}(\tau)-\langle S_{\mathbf{r}_l^l}\rangle)(S_{\mathbf{r}_{m'}}-\langle S_{\mathbf{r}_{m'}}^{m}\rangle)\}\rangle d\tau. \tag{7}$$

The indices l, m, l', and m′ in (7) label the spin-operator components and the sublattice to which the operator pertains.

Introduction of the irreducible matrix part $\hat{\Sigma}(\mathbf{q}, i\omega_n)$ in the VLP method [7] makes it possible to represent $\hat{K}(\mathbf{q}, i\omega_n)$ in the form

$$\hat{K}(\mathbf{q}, i\omega_n)=(1-\hat{\Sigma}(\mathbf{q}, i\omega_n)\hat{V}_q)^{-1}\hat{\Sigma}(\mathbf{q}, i\omega_n). \tag{8}$$

The dispersion equation then becomes

$$\det[1-\hat{\Sigma}(\mathbf{q}, i\omega_n)\hat{V}_q]=0. \tag{9}$$

In the zeroth approximation, $\hat{\Sigma}(\mathbf{q}, i\omega_n)$ constitutes a matrix made up of blocks $\Gamma_{l'm'}^{lm}$, which are obtained from formula (7) for $K_{l'm'}^{lm}(\mathbf{q}, i\omega_n)$ with replacement of $\mathscr{H}$ by the single-particle Hamiltonians (5). The calculation of the blocks is made difficult by the fact that the zeroth Hamiltonians (5) are not diagonal in the representation of the operator S^z. Proceeding in analogy with [4], we obtain the unitary transformations $U_{f,g}$ that diagonalize $\mathscr{H}_{f,g}$:

$$U_{f,g}\mathscr{H}_{0f,g}U_{f,g}^{+}=\bar{\mathscr{H}}_{0f,g},\quad U_{f,g}=\prod_{\mathbf{r}_f,g}u_{\mathbf{r}_f,g}. \tag{10}$$

In this case any $\hat{T}$-ordered mean value $\langle\hat{T}S_{\mathbf{r}_{l_1}}^{l_1}(\tau_{l_1})\ldots S_{\mathbf{r}_{l_n}}^{l_n}(\tau_{l_n})\rangle_0$ ($\hat{T}$ is the chronological-ordering operator) is expressed only in terms of the transformed operators:

$$\langle\hat{T}S_{\mathbf{r}_{l_1}}^{l_1}(\tau_{l_1})\ldots S_{\mathbf{r}_{l_n}}^{l_n}(\tau_{l_n})\rangle_0=\langle\hat{T}\bar{S}_{\mathbf{r}_{l_1}}^{l_1}(\tau_{l_1})\ldots\bar{S}_{\mathbf{r}_{l_n}}^{l_n}(\tau_{l_n})\rangle_0, \tag{11}$$

and the problem reduces to the case of a diagonal zero-order Hamiltonian [8].

For the f-sublattice, for example, we obtain

$$u_{\mathbf{r}_f}=\frac{1}{2\sqrt{2}}(\sqrt{A}-\sqrt{B})S_{\mathbf{r}_f}^{z}+\frac{1}{2\sqrt{2}}(\sqrt{A}+\sqrt{B})(S_{\mathbf{r}_f}^{z})^2$$
$$+\frac{1}{2}e^{-i\psi}\left(\sqrt{B}-\frac{1}{\sqrt{2}}\right)S_{\mathbf{r}_f}^{+}+\frac{1}{2}e^{-i\psi}\left(\sqrt{B}+\frac{1}{\sqrt{2}}\right)$$
$$\times\{S_{\mathbf{r}_f}^{+},S_{\mathbf{r}_f}^{z}\}_{+}+\frac{1}{2}e^{i\psi}\left(\frac{1}{\sqrt{2}}-\sqrt{A}\right)S_{\mathbf{r}_f}^{-}$$
$$+\frac{1}{2}e^{i\psi}\left(\frac{1}{\sqrt{2}}+\sqrt{A}\right)\{S_{\mathbf{r}_f}^{-},S_{\mathbf{r}_f}^{z}\}_{+}+\frac{1}{\sqrt{2}}e^{-i2\psi}\sqrt{A}(S_{\mathbf{r}_f}^{+})^2$$
$$+\frac{1}{\sqrt{2}}e^{i2\psi}\sqrt{B}(S_{\mathbf{r}_f}^{-})^2, \tag{12}$$
$$-\beta\bar{\mathscr{H}}_{0f}=\sum_{\mathbf{r}_f}[\delta S_{\mathbf{r}_f}^{z}+\gamma(S_{\mathbf{r}_f}^{z})^2],$$
$$\bar{S}_{\mathbf{r}_f}^{z}=\frac{1}{2}(\sqrt{A}+\sqrt{B})(e^{-i\psi}S_{\mathbf{r}_f}^{+}+e^{i\psi}S_{\mathbf{r}_f}^{-})$$
$$+\frac{1}{2}(\sqrt{A}-\sqrt{B})\{(e^{-i\psi}S_{\mathbf{r}_f}^{+}+e^{i\psi}S_{\mathbf{r}_f}^{-}),S_{\mathbf{r}_f}^{z}\}_{+},$$
$$\bar{S}_{\mathbf{r}_f}^{+}=\frac{e^{i\psi}}{\sqrt{2}}\left\{\frac{y}{\delta}S_{\mathbf{r}_f}^{z}+\frac{\gamma}{\delta}[e^{-i2\psi}(S_{\mathbf{r}_f}^{+})^2+e^{i2\psi}(S_{\mathbf{r}_f}^{-})^2]-\frac{1}{2}(\sqrt{A}+\sqrt{B})\right.$$
$$\left.\times(e^{-i\psi}S_{\mathbf{r}_f}^{+}-e^{i\psi}S_{\mathbf{r}_f}^{-})-\frac{1}{2}(\sqrt{B}-\sqrt{A})\{(e^{-i\psi}S_{\mathbf{r}_f}^{+}-e^{i\psi}S_{\mathbf{r}_f}^{-}),S_{\mathbf{r}_f}^{z}\}_{+}\right\}.$$

From this we easily obtain the average value $\langle S\rangle$ of the spin at the site:

$$\langle S\rangle=yb/\delta,\ b=2e^{\gamma}\operatorname{sh}\delta/(1+2e^{\gamma}\operatorname{ch}\delta). \tag{13}$$

In (12) and (13) we used the notation

$$\delta=(y^2+\gamma^2)^{1/2},\ A=(\delta-\gamma)/2\delta,\ B=(\delta+\gamma)/2\delta, \tag{14}$$

and $\{\ldots,\ldots\}_+$ denotes the anticommutation operation.

The blocks needed to determine the spectrum and obtained with the aid of (13) are given in the Appendix. After substituting them in (9) and calculating the sixth-order determinant, the dispersion equation becomes

$$1-V_q^2[\alpha^2\Gamma_1^2-2\alpha\cos 2\eta\Gamma_4^2+\cos^2 2\eta(\Gamma_2^2+\Gamma_3^2)$$
$$+2\sin^2 2\eta\Gamma_2\Gamma_3]+V_q^4\{\alpha^2(\Gamma_1\Gamma_3-\Gamma_4^2)[\cos^2 2\eta(\Gamma_1\Gamma_3-\Gamma_4^2)] \tag{15}$$

$$+2\sin^2 2\eta\Gamma_1\Gamma_2]+\Gamma_2^2[\Gamma_3^2-2\alpha\cos 2\eta\Gamma_4^2$$
$$+\alpha^2\cos^2 2\eta\Gamma_1^2]\}-V_q^6\alpha^2\Gamma_2^2(\Gamma_1\Gamma_3-\Gamma_4^2)^2=0, \tag{16}$$
$$V_q^2=(V_q^{\perp})^2+\varkappa_q^2,\ \alpha=V_q^{\parallel}/V_q.$$

4. SPECTRUM IN ZERO MAGNETIC FIELD

For $h=0$ ($\eta=0$), the dispersion equation (15) breaks up into three. The first two of them

$$1\pm V_q(\Gamma_3-\alpha\Gamma_1)-\alpha V_q^2(\Gamma_1\Gamma_3-\Gamma_4^2)=0 \tag{17}$$

determine the two branches corresponding to the ordinary spin-wave theory, and the two branches connected with the non-equivalence of the local transitions of the spin from the first to the second excited energy sublevel and from the second to the third, just as in the case of an easy-axis compensated antiferromagnet [4,9]. The frequencies of all four branches are given by the following expressions:

$$(\beta\omega_{1,2,3,4})^2=\delta^2+\gamma^2(\pm)\frac{b}{\delta}V_q[\delta\gamma R+\gamma^2\cos^2\rho+\delta^2\sin^2\rho]$$
$$-\frac{1}{2}\cos 2\rho\left(\frac{y}{\delta}bV_q\right)^2\pm\left\{\left(\frac{1}{2}\cos 2\rho\frac{y^2}{\delta^2}b^2V_q^2\right)^2\right.$$
$$-\left(\frac{y}{\delta}bV_q\right)^2\left[\delta^2+\gamma^2\cos 2\rho-2\gamma\delta R\sin^2\rho\right. \tag{18}$$
$$\left.-y^2\sin^4\rho(\pm)\frac{b}{\delta}V_q\cos 2\rho(\delta\gamma R+\gamma^2\cos^2\rho+\delta^2\sin^2\rho)-\frac{\delta^2}{y^2}(\delta-\gamma R\cos 2\rho)^2\right]$$
$$\left.+4\gamma\delta\left(\delta(\pm)\frac{\gamma}{\delta}bV_qR\cos^2\rho\right)\left[\gamma(\pm)\frac{b}{\delta}V_q(\gamma+\delta R\sin^2\rho)\right]\right\}^{1/2},$$

where the signs in the parentheses are in mutual correspondence, $\cos 2\rho=\alpha$, and

$$R=(\operatorname{ch}\delta-e^{-\gamma})/\operatorname{sh}\delta. \tag{19}$$

It is easy to verify that the gap of the lowest-frequency mode $\beta\omega_1$ vanishes, as it should. At $\rho=0$ (compensated antiferromagnet), the frequencies (18) coincide with the analogous ones in [4].

If $\gamma=0$, then only the ordinary spin-wave modes remain in the spectrum (18), and $\beta\omega_{3,4}=\delta$ corresponds to local transitions of the spin with a change of energy by δ. In a magnetic field we easily obtain for $\beta\omega_{1,2}$

$$(\beta\omega_{1,2})^2=b^2[V_0^2-V_q^2\cos 2\eta\cos 2\rho]$$
$$+h^2\sin\psi_0/\sin 2\eta\pm bV_qy(\cos 2\rho-\cos 2\eta). \tag{20}$$

It follows from (20) that intersection of the spin-wave branches takes place at $\eta=\rho$.

At low temperatures ($b \approx R \approx 1$), the modes $\beta\omega_{3,4} = \delta - \gamma$ in (18) correspond to local transitions from the excited energy sublevel, and therefore are excluded from the collective-excitation spectrum. The frequencies $\beta\omega_{1,2}$ thus take the form

$$(\beta\omega_{1,2})^2 = (\delta+\gamma)^2 \mp 2\frac{V_q}{\delta}[\delta\gamma+\gamma^2\cos^2\rho+\delta^2\sin^2\rho] - \cos 2\rho\left(\frac{y}{\delta}V_q\right)^2. \quad (21)$$

In the model considered by us, the projection of the spin operator on the direction of its molecular field is not conserved even in the zeroth approximation, and this leads to a mixing of the states of the isolated spin with different magnetic quantum numbers m. Connected with this fact is the last dispersion equation

$$1 - V_q^2\Gamma_2^2 = 0, \quad (22)$$

which is separated from (15). It determines the two modes of the spectrum

$$(\beta\omega_{5,6})^2 = 4\delta\left(\delta \pm \frac{\gamma^2}{\delta^2}bV_q\right), \quad (23)$$

the frequencies of which coincide at $\gamma = 0$ with the energies of the local spin transitions with $\Delta m = 2$.

5. INTERACTION OF COLLECTIVE EXCITATIONS IN AN EXTERNAL MAGNETIC FIELD

If the external magnetic field $h \neq 0$ ($\eta \neq 0$), then Eq. (15) no longer breaks up into three separate equations, and the calculation becomes cumbersome. We therefore confine ourselves here to low temperatures, when the modes $\beta\omega_{3,4}$ are excluded from the collective-excitation spectrum. The solutions of the fourth-degree dispersion equation (15) can be represented in this case in the form

$$(\beta\omega_{1,2})^2 = 4\delta^2 + 2x - 2\sqrt{z} \mp 2\{P_1 - z + [(2z-P_1)^2 - P_3]^{1/2}\}^{1/2},$$
$$(\beta\omega_{5,6})^2 = 4\delta^2 + 2x + 2\sqrt{z} \pm 2\{P_1 - z - [(2z-P_1)^2 - P_3]^{1/2}\}^{1/2}, \quad (24)$$

$$P_1 = x^2 + 2V_q^2\left[\delta^2B^2g^2 + 2B^2\gamma^2\sin^2 2\eta + \frac{\gamma^4}{\delta^2}\cos^2 2\eta\right],$$

$$P_2 = V_q^2\left(\delta Bg + \frac{\gamma^2}{\delta}\cos 2\eta\right)\left\{x\left(\delta Bg - \frac{\gamma^2}{\delta}\cos 2\eta\right) + 2V_q^2\frac{\gamma^2}{\delta}\alpha AB\sin^2 2\eta\right\},$$

$$P_3 = [x^2 - (2V_q\delta Bg)^2]\left[x^2 - \left(2V_q\frac{\gamma^2}{\delta}\cos 2\eta\right)^2\right]$$

$$-8V_q^2\gamma^2\sin^2 2\eta\left\{B^2x(2\alpha AgV_q^2 - x) + 2V_q^2\frac{\gamma^2}{\delta^2}[\delta^2B(\alpha A + B^2g)\cos 2\eta\right.$$

$$\left. - B^2(\delta^2B^2 - \alpha^2A^2V_q^2)]\right\},$$

$$x = \delta^2(B^2-1) - \alpha ABV_q^2\cos 2\eta,\ g = \alpha A - B\cos 2\eta,$$

and z is the root of the equation

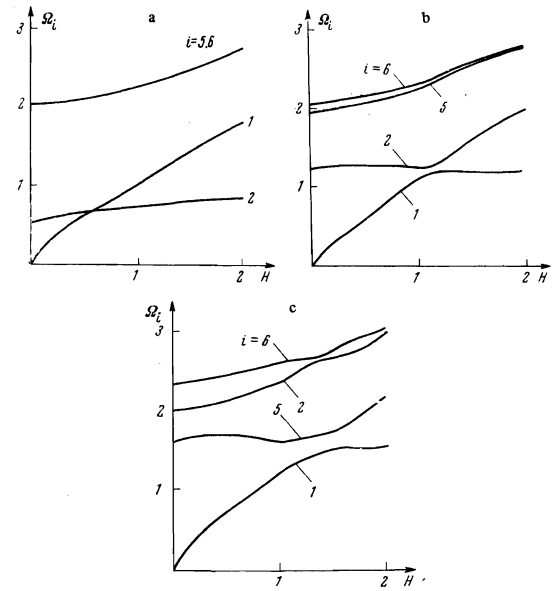
$$z^3 - z^2P_1 + \frac{1}{4}z(P_1^2 - P_3) - P_2^2 = 0 \quad (25)$$

with the initial condition $z(h=0) = x^2$.

The figure shows the results of a numerical calculation of the frequencies (24) in the homogeneous case ($q = 0$) at $\kappa_0 = 0.5\ V_0^{\perp}(\psi_0 \approx 13°)$ and $\alpha = 0.86$. We have used here the notation

$$\{\Omega_i, \Gamma, H, \Delta, X\} = V_0^{-1}\{\beta\omega_i, \gamma, h, \delta, x\}.$$

For $\Gamma = (0)$ (Fig. a) we have $\Omega_{5,6} = 2\Delta$, and the antiferromagnetic resonance (AFMR) frequencies $\Omega_{1,2}$ behave in the manner predicted by the phenomenological theory [10,11]. The intersection point is determined, as noted above, by the condition $\eta = \rho$ or $g = 0$. At a finite value of the anisotropy (Fig. b), the modes $\Omega_{5,6}$ are



Dependence of the frequencies Ω_i on the magnetic field for different values of the anisotropy constant: a) $\Gamma = 0$, b) $\Gamma = 0.25$, c) $\Gamma = 0.6$ (Ω_i and H are given in relative units).

split, and the AFMR modes no longer have an intersection point, as a result of the interaction with these modes. In the case of a weak anisotropy, the frequencies Ω_i at $g = 0$ are given by

$$\Omega_{1,2}^2 \approx 4\left\{\Delta^2 + X \pm \frac{\Gamma^2\cos 2\eta}{\Delta X}[\Delta^2(B^2-1) - B^2]\right\},$$
$$\Omega_{5,6}^2 \approx 4\left[\Delta^2 \pm \frac{\Gamma^2B^2\cos 2\eta\sin^2 2\eta}{\Delta X}\right]. \quad (26)$$

For $\Gamma > \Gamma_0$ (Fig. c), where

$$\Gamma_0 = [\Delta(1+\alpha)]/(3+\alpha), \quad (27)$$

the high-frequency AFMR mode lies above the lower component of the doublet $\Omega_{5,6}$. It should be noted here that the anisotropy constant Γ cannot be larger than unity. From (6) and (13) we easily obtain at $h = 0$

$$\Delta = [\Gamma^2 + (y/V_0)^2]^{1/2} = 1. \quad (28)$$

It is impossible to satisfy (28) at $\Gamma > 1$. This limitation on the value of Γ is due to the fact that the exchange interaction via the molecular field tends to "unmix" the states of the isolated spin with definite magnetic quantum numbers, whereas single-anisotropy tends to "mix" them. Thus, when the anisotropy increases the antiferromagnetic ordering of the easy plane axis becomes impossible starting with a certain value of Γ.

6. CONCLUSION

From the results obtained in this paper we can draw the following conclusions.

1. To ascertain the role of single-ion anisotropy in the formation of the spectrum of the collective excitations in antiferromagnets, it is necessary to use essentially a microscopic approach. For example, in the case of a spin $S = 1$ there appear in the spectrum, in addition to ordinary spin waves, a number of additional branches connected with the nonequidistant character of the system of energy sublevels of the isolated spin in the molecular field, and with the nonconservation of the spin projection on the corresponding direction of the molecular field. Knowledge of the frequencies of all the collective

excitations may be useful in the interpretation of the spectrum of the magnetic-dipole absorption in the infrared region.

2. Application of an external magnetic field leads to an interaction between all the collective excitations, which is manifest, in particular, in the absence of an intersection point in the field of the AFMR frequencies. At sufficiently large anisotropy, the energy of the high-frequency AFMR branch may turn out to be larger than the energy of one of the modes connected with the nonconservation of the spin projection on the molecular field.

3. The anisotropy constant γ cannot exceed V_0. Otherwise, the strong "mixing" of isolated-spin states with definite magnetic quantum numbers makes antiferromagnetic ordering of the easy-plane type impossible.

A suitable crystal with which to check the proposed theory may be nickel carbonate, whose low-frequency AFMR branch was investigated by Prozorova [12]. The nickel ion in this compound has a spin $S=1$, and the Dzyaloshinskiĭ interaction amounts to ≈ 0.5 of the exchange interaction [12,13]. One should therefore expect also an appreciable single-ion anisotropy constant.

The authors thank Academician A. S. Borovik-Romanov, E. A. Turov, and V. E. Naĭsh for a discussion of the results and for useful remarks.

APPENDIX

The Fourier components of the simplest blocks are determined by the following expressions:

$$\begin{gathered}\Gamma_f^{zz}=\Gamma_1,\ \Gamma_f^{++}={}^1/_2 e^{i2\psi}[\Gamma_2-\Gamma_3],\ \Gamma_f^{--}={}^1/_2 e^{-i2\psi}[\Gamma_2-\Gamma_3],\\ \Gamma_f^{+-}=\Gamma_f^{-+}={}^1/_2(\Gamma_2+\Gamma_3),\ \Gamma_f^{z+}=e^{i\psi}\Gamma_4/\sqrt{2},\\ \Gamma_f^{+z}=-e^{i\psi}\Gamma_4/\sqrt{2},\ \Gamma_f^{z-}=-e^{-i\psi}\Gamma_4/\sqrt{2},\\ \Gamma_f^{-z}=e^{-i\psi}\Gamma_4/\sqrt{2},\end{gathered} \tag{A.1}$$

where

$$\begin{gathered}\Gamma_1=A\varphi_1(G_n^1+G_{-n}^1)+B\varphi_2(G_n^2+G_{-n}^2),\\ \Gamma_2=\frac{y^2}{\delta^2}b'\delta_n+\frac{\gamma^2}{\delta^2}b(G_n^{1+2}+G_{-n}^{1+2}),\\ \Gamma_3=B\varphi_1(G_n^1+G_{-n}^1)+A\varphi_2(G_n^2+G_{-n}^2),\\ \Gamma_4=\frac{y}{2\delta}[\varphi_1(G_n^1-G_{-n}^1)+\varphi_2(G_n^2-G_{-n}^2)].\end{gathered} \tag{A.2}$$

Here $G_{\pm n}^i=(y_i\mp i\omega_n\beta)^{-1}$, $y_1=\delta+\gamma$, $y_2=\delta-\gamma$,

$$\varphi_1=\frac{e^{\delta+\gamma}-1}{1+2e^{\gamma}\operatorname{ch}\delta},\quad \varphi_2=\frac{1-e^{\gamma-\delta}}{1+2e^{\gamma}\operatorname{ch}\delta}. \tag{A.3}$$

The blocks with the sublattice index g are obtained from (A.1) by replacing ψ with $\pi-\psi$.

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Translated by J. G. Adashko
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