Effect of laser radiation on superconductivity

R. A. Vardanyan and B. I. Ivlev

L. D. Landau Institute of Theoretical Physics,
USSR Academy of Sciences
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We consider the properties of the nonequilibrium state produced in a superconducting sample by laser radiation. We obtain the dependence of the change of the superconductor-spectrum gap on the temperature, radiation power, and the coordinates. The results agree with the existing experimental data. We investigate the behavior of the superconductor in an alternating field of frequency close to the value of the gap.

A number of experimental results have recently been published concerning the influence of laser radiation on the properties of superconducting films [1,2]. In these experiments, thin superconducting films evaporated on substrates where they irradiated from one side by laser light, either continuously or with sufficiently long pulses. Testardi [1] observed in this case the presence of resistivity below the transition temperature, while Parker and Williams [2], using an He-Ne laser, observed a unique dependence of the gap on the temperature under the influence of the radiation.

An alternating electromagnetic field, after penetrating into a sample, leads first of all to a simple heating of the latter via release of Joule heat. The electrons and the lattice are then in thermal equilibrium with each other. In addition, however, a change takes place in the electronic states because the electrons absorb quanta and go over into the region of higher energies. This process, generally speaking, cannot be described by introducing an effective electron temperature, since their distribution function becomes strongly nonequilibrium and can differ greatly from the Fermi function. If goot heat-removal conditions are created, then the heating of the sample as a whole is negligible, and the action of the electromagnetic field reduces only to a change in the electronic states. This is precisely the case realized in [1,2].

Nonequilibrium electronic excitations were produced in the field-penetration volume by the laser source and penetrated, by a diffusion mechanism, into the interior of the superconducting film. Owing to the quasiparticle energy relaxation, the picture remained stationary in time. The nonequilibrium distribution function of the quasiparticles leads, as is well known, to a change in the gap of the energy spectrum of the superconductor. The new value of the gap is determined from a relation analogous to the BCS relation, but with the nonequilibrium distribution function

$$1=\lambda\int_{\Delta}^{\omega_D}\frac{1-2n(\varepsilon)}{(\varepsilon^2-\Delta^2)^{1/2}}d\varepsilon. \qquad (1)$$

Parker and Williams [2] measured the gap as a function of the temperature with the field turned on. The results agreed with the phenomenological theory of Rothwarf and Taylor [3]. In this paper we obtain an expression that is exact within the framework of the considered model and connects the change of the gap with the radiation intensity and coincides in structure with the relation given in [3]. In addition, unlike the earlier study [1-3], where the distribution of the nonequilibrium excitations was assumed to be homogeneous over the film thickness, we consider the case of a coordinate dependence of all the quantities, which occurs in sufficiently thick samples.

The formulation of the problem reduces to the following. Assume that laser radiation is incident on one side of the superconducting film (or half-space). The value of the magnetic vector at the practically permissible radiation powers is much lower than the critical value at which superconductivity vanishes. The main effect is therefore the field-induced nonequilibrium distribution function.

1. THIN FILMS

We consider the case when the thickness of the irradiated superconducting film is less than the field penetration depth, so that the coordinate dependence of the considered quantities can be disregarded. The electrons absorb field quanta and go over far beyond the Fermi surface, after which they relax rapidly in energy and emit phonons. If the temperature is low enough, namely $T \ll \Delta$ (this is the case which will be investigated primarily), and the radiation intensity is not too large, then the probability of quasiparticle recombination through the gap is exponentially small like $e^{-\Delta/T}$.

The fact that the probability of simple scattering is larger than the recombination probability causes the excitations to accumulate directly above the threshold. By the same token, the principal part of the distribution function is concentrated in this region. This situation was considered by Owen and Scalapino [4].

To solve the problem, we use the Éliashberg kinetic equation [5] for the distribution function $n(\epsilon)$:

$$\begin{aligned}\frac{\alpha\omega_D^2}{\lambda}\Big[&\frac{[\varepsilon(\varepsilon+\omega)+\Delta^2]\theta(\varepsilon-\Delta)}{[(\varepsilon+\omega)^2-\Delta^2]^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}(n_{\varepsilon+\omega}-n_\varepsilon)\\&+\frac{[\varepsilon(\varepsilon-\omega)+\Delta^2]\theta(\varepsilon-\omega-\Delta)}{[(\varepsilon-\omega)^2-\Delta^2]^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}(n_{\varepsilon-\omega}-n_\varepsilon)\\&-\frac{\varepsilon(\omega-\varepsilon)-\Delta^2}{[(\omega-\varepsilon)^2-\Delta^2]^{1/2}}\frac{\theta(\varepsilon-\Delta)}{(\varepsilon^2-\Delta^2)^{1/2}}\theta(\omega-\varepsilon-\Delta)(n_\varepsilon+n_{\omega-\varepsilon}-1)\Big]\\=\int_\Delta^\infty&\frac{(\varepsilon\varepsilon'-\Delta^2)(\varepsilon-\varepsilon')^2}{(\varepsilon'^2-\Delta^2)^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}[n_\varepsilon(1-n_{\varepsilon'})(1+N_{\varepsilon-\varepsilon'})-n_{\varepsilon'}(1-n_\varepsilon)N_{\varepsilon-\varepsilon'}]d\varepsilon'\\+\int_\varepsilon^\infty&\frac{(\varepsilon\varepsilon'-\Delta^2)(\varepsilon'-\varepsilon)^2}{(\varepsilon'^2-\Delta^2)^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}[n_\varepsilon(1-n_{\varepsilon'})N_{\varepsilon'-\varepsilon}-n_{\varepsilon'}(1-n_\varepsilon)(1+N_{\varepsilon'-\varepsilon})]d\varepsilon'\\+\int_\Delta^\infty&\frac{(\varepsilon\varepsilon'+\Delta^2)(\varepsilon+\varepsilon')^2}{(\varepsilon'^2-\Delta^2)^{1/2}(\varepsilon^2-\Delta^2)^{1/2}}[n_\varepsilon n_{\varepsilon'}(1+N_{\varepsilon+\varepsilon'})-(1-n_\varepsilon)(1-n_{\varepsilon'})N_{\varepsilon+\varepsilon'}]d\varepsilon'.\end{aligned} \qquad (2)$$

Here $\omega_D = 2^{1/3}sp_0$ and $\alpha = 2^{4/3}\pi^{-1}D(e/c)^2A_\omega A_{-\omega}$, where $D = v^2\tau/3$ is the diffusion coefficient, A_ω is the vector potential of the field inside the film, s is the speed of sound, τ is the transport free path time of the elecron, and N_ϵ is the Planck distribution function of the honons.

It is seen from (2) that the recombination part of the ollision integral, which is described by the last term,

contains an additional exponential small factor $e^{-\Delta/T}$ in comparison with the principal terms of the scattering part.

The quasiparticle energy distribution function acquires a nonequilibrium increment $n'(\epsilon)=n(\epsilon)-n_F(\epsilon)$, which contains, besides the part $n_1(\epsilon)$ localized directly above the threshold, also a part $n_2(\epsilon)$ which is small in magnitude but decreases slowly with increasing energy. This last part is connected with the fact that at large energies there remain a certain number of quasiparticles, owing to the finite relaxation rate. Thus, $n'(\epsilon)=n_1(\epsilon)+n_2(\epsilon)$.

The function $n_1(\epsilon)$ is the principal part of the nonequilibrium increment to the distribution function, and should be obtained from the condition that the principal terms of the scattering part of the collision integral vanishes. Among the principal terms is the part linear in the electron distribution function:

$$\int_\Delta^\infty \frac{(\varepsilon\varepsilon'-\Delta^2)(\varepsilon-\varepsilon')^2}{(\varepsilon'^2-\Delta^2)^{1/2}}[n_{1\varepsilon}(1+N_{\varepsilon-\varepsilon'})-n_{1\varepsilon'}N_{\varepsilon-\varepsilon'}]d\varepsilon'$$

$$-\int_\Delta^\infty \frac{(\varepsilon\varepsilon'-\Delta^2)(\varepsilon'-\varepsilon)^2}{(\varepsilon'^2-\Delta^2)^{1/2}}[n_{1\varepsilon'}(1+N_{\varepsilon'-\varepsilon})-n_{1\varepsilon}N_{\varepsilon-\varepsilon'}]d\varepsilon'=0. \quad (3)$$

The solution is a Boltzmann function $n_1\epsilon=C\exp[-(\epsilon-\Delta)/T]$, since allowance for degeneracy yields the next order of smallness. The constant C includes the change of the chemical potential.

In accordance with the treatment given by Owen and Scalapino [4], the slow recombination causes first a temperature equilibrium to establish for the excess particles coming from the region of high energies, followed by a chemical equilibrium, so that a distribution with an effective chemical potential is established. However, the exact nonequilibrium distribution function will not have a Fermi form with effective chemical potential as assumed in [4]. Such a function would cause exact vanishing of only the scattering part of the collision integral, whereas the recombination part contains terms that are larger in order of magnitude than certain terms from the scattering part. The next orders of the expansion of the nonequilibrium distribution function in terms of the small quantity $e^{-\Delta/T}$ will therefore differ from the expansion of a Fermi function with an effective chemical potential. Only the first approximation, of the Boltzmann function, will be correct.

To find the normalization of the function $n_1(\epsilon)$, we integrate (2) with respect to ϵ:

$$\frac{\alpha\omega_D^2}{\lambda}\int_\Delta^{\omega-\Delta}\frac{\varepsilon(\omega-\varepsilon)-\Delta^2}{[(\omega-\varepsilon)^2-\Delta^2]^{1/2}}\frac{1-2n_\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}d\varepsilon$$

$$=\int_\Delta^\infty\frac{n_{1\varepsilon}\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\int_\Delta^\infty\frac{(\varepsilon\varepsilon'+\Delta^2)(\varepsilon+\varepsilon')^2}{(\varepsilon'^2-\Delta^2)^{1/2}}[n_{1\varepsilon}+2(n_{F\varepsilon'}+N_{\varepsilon+\varepsilon'})]d\varepsilon'. \quad (4)$$

This equation expresses the balance of the total number of particles and for this reason it does not contain the scattering part. In the case of interest to us we have in $n\ll 1$, so that n_ϵ in the left-hand side can be neglected. Recognizing also that n_ϵ is strongly localized over the threshold in a scale $T\ll\Delta$, we obtain for N the expression

$$N\equiv\int_\Delta^\infty\frac{n_{1\varepsilon}\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}=\frac{1}{(2\Delta)^{1/2}}\int_\Delta^\infty\frac{n_{1\varepsilon}\,d\varepsilon}{(\varepsilon-\Delta)^{1/2}}$$

$$=\sqrt{\frac{\pi T}{2\Delta}}e^{-\Delta/T}\left[\left(1+\frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\frac{2\Delta}{\pi T}e^{2\Delta/T}\right)^{1/2}-1\right]. \quad (5)$$

From (1) we obtain the small change of the gap

$$(\Delta_0-\Delta)/\Delta_0=2N, \quad (6)$$

where Δ_0 is the equilibrium value of the gap at zero pump. Thus, both the correction to the gap and the change of the equilibrium particle-number density $\rho=\Delta mp_0N/2\pi^2$ are expressed in terms of the quantity N.

As to the small and slowly-varying part of the nonequilibrium distribution function $n_2(\epsilon)$, it must be obtained from the inhomogeneous equation (2). The function $n_2(\epsilon)$ should be retained only in the principal part of the collision integral. At $\epsilon\gg\omega_D$, recognizing that the phonon energy does not exceed ω_D, we have

$$\frac{\omega_D^3}{3}n_2(\varepsilon)-\int_0^{\omega_D}\varepsilon'^2n_2(\varepsilon+\varepsilon')d\varepsilon'=\frac{\alpha\omega_D^2}{\lambda}n_F(\varepsilon-\omega). \quad (7)$$

At low temperatures, $n_F(\epsilon-\omega)$ can be replaced by $\theta(\omega-\epsilon)$, and then the solution of the integral equation with the source (7) takes the form

$$n_2(\varepsilon)=\frac{4\alpha}{\lambda\omega_D^2}(\omega-\varepsilon)\theta(\omega-\varepsilon),\quad \varepsilon\gg\omega_D. \quad (8)$$

In the energy interval $\Delta\ll\epsilon\ll\omega_D$ we have

$$\frac{\varepsilon^3}{3}n_2(\varepsilon)-\int_0^\infty\varepsilon'^2n_2(\varepsilon+\varepsilon')d\varepsilon'=\frac{\alpha\omega_D^2}{\lambda}, \quad (9)$$

and we can neglect the right-hand side, which plays the role of a source, since the latter is important only in the range of sufficiently high energies (a). At low energies, the arrival of phonons due to radiation is more effective than the direct injection of quasiparticles by the field quanta. The solution of the homogeneous equation (9) is

$$n_2(\varepsilon)\sim\alpha\omega\omega_D^2/\varepsilon^4\lambda, \quad (10)$$

and the coefficient is determined from the condition of matching to the solution (8). We obtain analogously the solution in the remaining regions

$$n_2(\varepsilon)\sim\frac{\alpha\omega\omega_D^2}{\lambda\Delta^4}\left(\frac{\Delta}{T}\right)^4\begin{cases}\left(\dfrac{\varepsilon-\Delta}{T}\right)^{-4}, & 1\ll\dfrac{\varepsilon-\Delta}{T}\ll\dfrac{\Delta}{T}\\ 1, & 0<\dfrac{\varepsilon-\Delta}{T}\ll 1\end{cases}. \quad (11)$$

Relation (5) makes it possible to find the normalization of the Boltzmann distribution function $n_1(\epsilon)$. We ultimately get

$$n(\varepsilon)=n_F(\varepsilon)+\left(\frac{2\Delta}{\pi T}\right)^{1/2}e^{-(\varepsilon-\Delta)/T}N+n_2(\varepsilon), \quad (12)$$

where $n_2(\epsilon)$ is given by formulas (8), (10), and (11).

We present an expression for the relative change of the gap when the field is turned on, in the limiting cases

$$\frac{\Delta_0-\Delta}{\Delta_0}=\frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\left(\frac{2\Delta}{\pi T}\right)^{1/2}e^{\Delta/T},\quad \frac{\alpha\omega\omega_D^2}{\lambda\Delta^4}\ll\frac{T}{\Delta}e^{-2\Delta/T}; \quad (13a)$$

$$\frac{\Delta_0-\Delta}{\Delta_0}=\left(\frac{\alpha\omega\omega_D^2}{2\lambda\Delta^4}\right)^{1/2},\quad \frac{\alpha\omega\omega_D^2}{\lambda\Delta^4}\gg\frac{T}{\Delta}e^{-2\Delta/T}. \quad (13b)$$

In the experiment, at a given radiation power, the temperature varies, and therefore relation (13) can be regarded as the dependence of the gap variation on the temperature. The transition from one limiting case to the other is effected at the temperature in which the number of excess excitations becomes comparable in order of magnitude with the equilibrium value. The first case corresponds to the linear approximation in the intensity, and the effect was proportional to the exponentially large recombination time. Formulas (13) agree with the experimental data [2].

The question of the limits of applicability of the obtained results calls for clarification. The splitting of

the collision integral into a large scattering part and a small recombination part is possible only at not too large values of the nonequilibrium increment to the distribution function. A simple analysis shows that the scattering is more effective than recombination if $\alpha\omega\omega_D^2/\lambda\Delta^4 \ll (T/\Delta)^7$. At these values of the intensity, the main contribution to the change of the gap and of the particle number is made by the function $n_1(\epsilon)$, with $n \ll (T/\Delta)^3 \ll 1$. The role of the function $n_2(\epsilon)$ will be made clear later on when electron-electron collisions are considered.

If we introduce the effective chemical potential, as is done in [4], namely $n_F + n_1 = \exp[-(\epsilon-\mu)/T]$, then

$$\mu = T\ln\left(1+\left(\frac{2\Delta}{\pi T}\right)^{1/2} e^{\Delta/T} N\right), \tag{14}$$

and this yields, unlike [4], a direct connection between μ and the field intensity and the temperature. We see therefore that when the foregoing limitations are imposed on the intensity we have $(\Delta-\mu)/T \ll 1$, and the situation is far from degeneracy.

2. THICK FILMS AND HALF-SPACE

When speaking of thick films, we have in mind primarily a case when the sample thickness is much larger than the depth of penetration of the electromagnetic field. Nonequilibrium carriers are produced within this depth and then penetrate by diffusion into the remaining volume of the sample. The distribution of the excitations over the thickness can in this case be quite complicated, since generally speaking the diffusion length depends on the energy of the excitations. To take into account the spatial inhomogeneity it is necessary to add to the right-hand side of (2) the diffusion term

$$\frac{2^{1/2}}{\pi\lambda}\omega_D^2 D_\epsilon k^2 \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}(n-n_F),$$

where D_ϵ is the energy-dependent diffusion coefficient [5,6]

$$D_\epsilon = \frac{(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon} D\theta(\varepsilon^2-\Delta^2), \quad D = \frac{v^2\tau}{3}.$$

Nonequilibrium excitations diffuse during their lifetime, and the characteristic distance to which they depart from the surface is proportional to the square root of this time. Since at low temperatures the lifetime with respect to scattering is much shorter than the lifetime with respect to recombination, the energy distribution function assumes its steady-state form at relatively short distances. As to the relaxation of the total excess number of particles, it takes place over a large length, where the distribution functions attune themselves "adiabatically" to the given number of particles at each point of space. Thus, at sufficiently large distances the distribution function contains a coordinate dependence in the form of an energy-independent factor. We emphasize that the possibility of such a factorization of the distribution function is due to the different scales of the scattering and recombination probabilities, and no longer exists when the temperature is increased.

Integration of the kinetic equation with respect to energy eliminates the large scattering terms

$$\int_\Delta^\infty \frac{n_{1\varepsilon}\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \int_\Delta^\infty \frac{(\varepsilon\varepsilon'+\Delta^2)(\varepsilon+\varepsilon')^2}{(\varepsilon'^2-\Delta^2)^{1/2}}[2(n_{F\varepsilon'}+N_{\varepsilon+\varepsilon'})+n_{1\varepsilon'}]\,d\varepsilon' - \frac{2^{1/2}}{\pi\lambda}\omega_D^2 D\frac{\partial^2}{\partial z^2}\int_\Delta^\infty n_{1\varepsilon}\,d\varepsilon = \frac{\alpha(z)\omega\omega_D^2}{\lambda}. \tag{15}$$

In accordance with the foregoing, we seek the correction to the distribution function in the factorized form $n_1 = C(z)\exp[-(\epsilon-\Delta)/T]$. The energy part has the same form as before, since it is determined from the scattering part without the coordinate dependence. The laser-radiation penetration depth is $\delta = c/\omega_p$, and therefore $\alpha(z) = \alpha\exp(-2z/\delta)$. In the interior of the sample, at distances larger than δ, we have

$$z_0^2\partial^2 C/\partial z^2 = C + {}^1/_2 e^{\Delta/T}C^2,$$
$$C(z) = 12e^{-\Delta/T}\frac{\nu\exp(z/z_0)}{[\nu\exp(z/z_0)-1]^2}, \quad z_0 \equiv \frac{\xi_0}{2^{3/4}\pi\sqrt{\lambda}}\frac{\omega_D}{\Delta}e^{\Delta/2T} \tag{16}$$

where $\xi_0 = (D/\Delta)^{1/2}$ is of the order of the coherence length. The constant ν must be determined from the normalization condition. The condition $\partial n_1/\partial z = 0$ should be satisfied on the boundaries of the sample.

Integrating relation (15) with respect to the coordinates in the case of a half-space, we obtain the necessary normalization condition

$$\frac{\nu(\nu+1)}{(\nu-1)^3} = \frac{1}{24}\frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\frac{2\Delta}{\pi T}e^{2\Delta/T}\frac{\delta}{2z_0}.$$

It is convenient to obtain the solution in limiting cases. Recognizing that

$$\frac{\Delta_0-\Delta(z)}{\Delta_0} = C(z)\left(\frac{2\pi T}{\Delta}\right)^{1/2},$$

we obtain from (16)

$$\frac{\Delta_0-\Delta(z)}{\Delta_0} = \frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\left(\frac{2\Delta}{\pi T}\right)^{1/2}e^{\Delta/T}\frac{\delta}{2z_0}\exp\left(-\frac{z}{z_0}\right) \tag{17a}$$

at

$$\frac{\alpha\omega\omega_D^2}{\lambda\Delta^4}\frac{\delta}{2z_0} \ll \frac{T}{\Delta}e^{-2\Delta/T},$$

and

$$\frac{\Delta_0-\Delta(z)}{\Delta_0} = 6^{1/3}\left(\frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\left(\frac{2\Delta}{\pi T}\right)^{1/2}e^{\Delta/2T}\frac{\delta}{2z_0}\right)^{2/3}\left(1+\frac{z}{z_1}\right)^{-2} \tag{17b}$$

at

$$\frac{\alpha\omega\omega_D^2}{\lambda\Delta^4}\frac{\delta}{2z_0} \gg \frac{T}{\Delta}e^{-2\Delta/T},$$

where

$$z_1 = 2\cdot 6^{1/3}z_0\left(\frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4}\frac{2\Delta}{\pi T}e^{2\Delta/T}\frac{\delta}{2z_0}\right)^{-1/3}.$$

We have expanded the exponential in the last expression, since the main change of the distribution function is concentrated in the region $z \sim z_1 \ll z_0$.

The obtained solution $n_1\epsilon(z) = C(z)\exp[-(\epsilon-\Delta)/T]$ is valid at all points of the sample. The increments to this function, which are localized near the boundary, ensure the satisfaction of the boundary condition $\partial n/\partial z = 0$, and are obtained from the scattering part, are small in comparison with n_1 in proportion to the depth of penetration.

The transition from one limiting case to the other in (17) occurs, just as in a thin film, at temperatures such that the number of excess quasiparticles becomes of the same order as the number of the equilibrium particles. The magnitude of the effect in a bulky sample is less than in a thin sample, since the quasiparticles go off into the volume. For example, in the limit linear in the intensity, this attenuation amounts to $\delta/2z_0$. The main feature of the solutions is the large penetration depth which, as follows from (16), exceeds ξ_0 by many times.

The penetration depth reaches a maximum value

$$z_{max} \sim \frac{\xi_0}{\sqrt{\lambda}} \frac{\omega_D}{\Delta} \exp\left(\frac{\Delta}{2T_0}\right),$$

when $T = T_0$ and

$$\frac{\alpha\omega\omega_D^2}{\lambda\Delta^4} \frac{\delta}{2z_0} \sim \frac{T}{\Delta} \exp^{-2\Delta/T}.$$

If we take a film with thickness d larger than z_{max} then we obtain on the non-irradiated boundary, in the respective limiting cases

$$\frac{\Delta_0 - \Delta(d)}{\Delta_0} = \frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4} \left(\frac{2\Delta}{\pi T}\right)^{1/2} e^{\Delta/T} \frac{\delta}{2z_0} \exp(-d/z_0), \qquad (18a)$$

$$\frac{\Delta_0 - \Delta(d)}{\Delta_0} = 6\left(\frac{2\Delta}{\pi T}\right)^{1/2} e^{\Delta/T} \left(\frac{2z_0}{d}\right)^2. \qquad (18b)$$

Unlike the thin film, the temperature dependence of the correction to the gap is different. The curve reaches a maximum at the temperature T_0.

The difference between the coordinate dependences of the two limiting cases in (17) is the consequence of the enhancement of the nonlinearity in (16), which sets in if the nonequilibrium increment to the distribution function exceeds n_F.

Just as for a thin film, it is necessary to indicate a criteria for the applicability of the derived formulas. The scattering remains more effective than recombination when

$$\frac{\alpha\omega\omega_D^2}{\lambda\Delta^4} \frac{\delta}{2z_0} \ll \left(\frac{T}{\Delta}\right)^{11/2} e^{-\Delta/2T}$$

If the film thickness is less than the characteristic length z_0, but nevertheless exceeds the depth of penetration of the radiation (this is precisely the case realized in [2]), then the excess number of particles is constant over the cross section. Integrating Eq. (15) with respect to the coordinate, we obtain the expression of the preceding section, the only difference being that α should be replaced by $\alpha\delta/2d$:

$$\frac{\Delta_0 - \Delta(d)}{\Delta_0} = 2\left(\frac{\pi T}{2\Delta}\right)^{1/2} e^{-\Delta/T} \left[\left(1 + \frac{\alpha\omega\omega_D^2}{8\lambda\Delta^4} \frac{2\Delta}{\pi T} e^{2\Delta/T} \frac{\delta}{2d}\right)^{1/2} - 1\right]. \qquad (19)$$

3. ROLE OF ELECTRON-ELECTRON INTERACTION AND SOME REMARKS

An excitation with energy ϵ can break up into three, knocking out at the same time a pair such as to satisfy the energy conservation law. For example, a quasiparticle with energy 3Δ is transformed into three particles of energy Δ each. The cause of such processes in which the number of quasiparticles is increased is electron-electron interaction. As will be shown below, allowance for these processes leads to a renormalization of the quantity α, corresponding, as it were, to an additional source of excitations.

Out of the entire electronic part of the collision integral [5,7], only the linear part responsible for the effects indicated above is of importance. This linear part comes into play starting with an energy 3Δ, and therefore does not influence the form of the main part of the distribution function. The same can be said also with respect to the form of the slowly-damped part of the distribution function, derived in Sec. 1, since the electronic integral contains a small effective coupling constant $\omega_D^2/\Delta\epsilon_F$. For this reason, only the integral contribution from the electron-electron integral is of importance; this contribution is determined by large values of ϵ. A new source of excitation is added to the field part of the kinetic equation, namely

$$\frac{\alpha\omega\omega_D^2}{\lambda} + \frac{a_1}{6\cdot 2^{2/3}} \frac{\omega_D^2}{\varepsilon_F} \int_0^\infty \varepsilon^2 n_2(\varepsilon)\, d\varepsilon = \frac{\alpha\omega\omega_D^2}{\lambda}\left[1 + \frac{a_1}{6\cdot 2^{2/3}} \frac{\omega}{\varepsilon_F}\left(\frac{\omega}{\omega_D}\right)^2\right]. \qquad (20)$$

Here a_1 is the ratio of the electron-electron and electron-phonon coupling constants, and was introduced earlier in [7]. The renormalization may turn out to be appreciable at optical frequencies. Expression (20) is valid both for thin films and for thick samples, since excitations with energies larger than ω_D relax over lengths shorter than the penetration depth. The correction (20) will therefore attune itself to $\alpha(z)$ at each point.

At optical frequencies, the condition $\omega\tau \ll 1$ may likewise not be satisfied (τ is the time of scattering by the impurities). For arbitrary $\omega\tau$, taking the foregoing correction into account, it is necessary to make in all the formulas the substitution

$$\frac{\alpha\omega\omega_D^2}{\lambda} \to \frac{\alpha\omega\omega_D^2}{\lambda} \frac{1}{1+\omega^2\tau^2}\left[1 + \frac{a_1}{6\cdot 2^{2/3}} \frac{\omega}{\varepsilon_F}\left(\frac{\omega}{\omega_D}\right)^2\right]. \qquad (21)$$

It was assumed everywhere above that the phonons are a thermal equilibrium with the thermostat. This will be indeed the case if the phonon relaxation occurs on the walls of the sample and not on the electrons. In the former case the corresponding mean free path is simply the film thickness, which is of the order of z_0. The mean free path connected with the electron-phonon interaction is equal to

$$\frac{v}{\omega} \frac{1}{\omega\tau} e^{\Delta/T}.$$

From this, at $\omega \sim T$, the condition that z_0 be relatively small becomes

$$(\Delta/T)^2 e^{\Delta/2T} \gg \omega_D\tau(\Delta\tau)^{1/2},$$

and is always satisfied. Simple estimates show also that the rates of phonon relaxation on the walls exceeds the rate of their emission by the quasiparticles. For this reason, the assumption that the phonons are at equilibrium is justified.

In concluding this section, let us discuss the case of a near-critical temperature. Let the thickness of the film be smaller than the field penetration depth. We then have in the approximation linear in the intensity

$$n_{1\varepsilon}\left(\frac{\varepsilon^3}{3T^3} + 2\int_0^\infty \frac{x^2\,dx}{e^x-1} + 2\int_{\varepsilon/T}^\infty \frac{x^2\,dx}{e^x+1} + 4\frac{\varepsilon}{T}\int_0^{\varepsilon/T} \frac{x\,dx}{e^x+1} + 2\frac{\varepsilon^2}{T^2}\int_{\varepsilon/T}^\infty \frac{dx}{e^x+1}\right)$$
$$+ \frac{2}{e^{\varepsilon/T}+1}\int_{\varepsilon/T}^\infty x^2 n_{1x}\,dx + \frac{4\varepsilon/T}{e^{\varepsilon/T}+1}\int_0^{\varepsilon/T} x n_{1x}\,dx + \frac{2\varepsilon^2/T^2}{e^{\varepsilon/T}+1}\int_{\varepsilon/T}^\infty n_{1x}\,dx$$
$$+ \int_{\varepsilon/T}^\infty \frac{x^2}{e^x-1} n_1\left(x - \frac{\varepsilon}{T}\right)dx - \int_0^\infty \frac{x^2 e^x}{e^x-1} n_1\left(x + \frac{\varepsilon}{T}\right)dx - \int_0^{\varepsilon/T} \frac{x^2}{e^x-1} n_1\left(\frac{\varepsilon}{T} - x\right)dx$$
$$= \frac{\alpha}{\gamma}\begin{cases} \operatorname{th}(\varepsilon/2T) - [e^{(\omega-\varepsilon)/T}+1]^{-1}, & \varepsilon < \omega \\ [e^{(\varepsilon-\omega)/T}+1]^{-1}, & \varepsilon > \omega \end{cases}. \qquad (22)$$

We have introduced here the probability of inelastic scattering $\gamma = \lambda T^3/\omega_D^2$. We have neglected the quantity Δ, inasmuch as $\Delta \ll T$, and the principal part of the distribution function is concentrated in the region $\epsilon \sim T$. The function $n_1(\epsilon)$ should satisfy the normalization condition (4)

$$\frac{\alpha}{\gamma}\frac{\omega}{T} = \int_0^\infty n_{1\varepsilon}\frac{d\varepsilon}{T}\left(2\int_0^\infty \frac{x^2\,dx}{e^x+1} + \int_{\varepsilon/T}^\infty \frac{x^2\,dx}{e^x-1} + 4\frac{\varepsilon}{T}\int_0^\infty \frac{x\,dx}{e^x+1} + 2\frac{\varepsilon^2}{T^2}\int_0^\infty \frac{dx}{e^x+1}\right). \qquad (23)$$

At $T \sim T_c$, the probability of recombination and of

scattering of the quasiparticles is of the same order, so that the method of solving the kinetic equation for low temperatures does not apply in this case. However, just as at low temperatures, most nonequilibrium increments to the distribution function are concentrated at $\epsilon \sim T$ and should be determined from the homogeneous equation (22), while the source in the right-hand side is needed only for the normalization. The right-hand side of (22) can be neglected at $\epsilon \ll \omega$. Then n_1 at $\epsilon \gg T$ is determined from the equation

$$\frac{\varepsilon^3}{3T^3} n_{1\varepsilon} - \int_{\omega}^{\infty} x^2 n_1 \left(x + \frac{\varepsilon}{T}\right) dx = 0, \tag{24}$$

and at $\epsilon \ll T$ we have

$$n_{1\varepsilon} \int_0^{\infty} \frac{x^2\,dx}{e^x+1} = \frac{\varepsilon}{4T} \int_0^{\infty} x^2 n_{1x}\,dx + \frac{\varepsilon}{2T} \int_0^{\infty} \frac{x^2}{\operatorname{th}(x/2)} \frac{\partial n_1}{\partial x}\,dx. \tag{25}$$

Therefore, using the normalization condition (23), we obtain

$$n_{1\varepsilon} \sim \frac{\alpha}{\gamma} \frac{\omega}{T} \begin{cases} \varepsilon/T, & \varepsilon \ll T, \\ (T/\varepsilon)^4, & \varepsilon \gg T. \end{cases} \tag{26}$$

The increment to the Ginzburg-Landau equation is expressed in terms of the distribution function

$$\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8\pi^2} \left(\frac{\Delta}{T}\right)^2 - U = 0, \quad U = 2 \int_{\Delta}^{\infty} \frac{n_{1\varepsilon}\,d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}. \tag{27}$$

Taking into account the renormalization of the field intensity (21) we get

$$U \sim \frac{\alpha}{\gamma} \frac{\omega}{T} \frac{1}{1+\omega^2\tau^2} \left[1 + \frac{a_1}{6 \cdot 2^{2/3}} \frac{\omega}{\varepsilon_F} \left(\frac{\omega}{\omega_D}\right)^2\right]. \tag{28}$$

Since the number of excess excitations increases at high pump frequencies, U is positive. This leads to a suppression of the superconductivity by the laser radiation, unlike the stimulation of the superconductivity by relatively small frequencies, when the quasiparticles were diverted to the high-energy region [8]. The form of $n_1(\epsilon)$ ensured in this case negative values of U.

As follows from the Testardi data [1], the film resistance vanished at temperatures lower than T_c in the presence of laser radiation. The lowering of the "transition temperature" became more effective with increasing power. It is perfectly possible that this is an effect of the type considered above.

4. DYNAMIC EFFECTS IN HIGH-FREQUENCY FIELDS

We consider the case when the field frequency is equal to the gap; to realize this case, millimeter waves are necessary [9]. Let us find the change of the equilibrium value of the gap under the influence of a weak alternating field of frequency $\omega/2 = \Delta$:

$$-\frac{\Delta_\omega(k)}{\lambda} = \int \frac{d\varepsilon}{4\pi i} \int d\xi \frac{dO_p}{4\pi} F_{\varepsilon,\varepsilon-\omega}(p, p-k).$$

This expression must be averaged over the impurities, and we can use for this purpose the procedure of Gor'kov and Éliashberg [6]. As a result we obtain

$$-\frac{1}{\lambda}\Delta_\omega(k) = \frac{3}{2} K\left(\frac{1}{2}\right) D\left(\frac{e}{c}\right)^2 A_{\omega/2}^2(k)$$
$$+\Delta_\omega(k) \frac{\tau}{4i} \int d\varepsilon \left\{ \operatorname{th}\frac{\varepsilon-\omega}{2T} \left[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon^R \xi_{\varepsilon-\omega}^R} + 1\right] \Pi_{\varepsilon,\varepsilon-\omega}^R \right.$$
$$- \operatorname{th}\frac{\varepsilon}{2T} \left[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon^A \xi_{\varepsilon-\omega}^A} + 1\right] \Pi_{\varepsilon,\varepsilon-\omega}^A +$$
$$\left. + \left(\operatorname{th}\frac{\varepsilon}{2T} - \operatorname{th}\frac{\varepsilon-\omega}{2T}\right) \left[\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon^R \xi_{\varepsilon-\omega}^A} + 1\right] \Pi_{\varepsilon,\varepsilon-\omega}^{(a)} \right.,$$

$$\Pi_{\varepsilon,\varepsilon-\omega}^{R(A)} = \left[kl \operatorname{arctg}^{-1} \frac{kl}{1 - i\tau(\xi_\varepsilon^{R(A)} + \xi_{\varepsilon-\omega}^{R(A)})} - 1\right]^{-1},$$

$$\Pi_{\varepsilon,\varepsilon-\omega}^{(a)} = \left[kl \operatorname{arctg}^{-1} \frac{kl}{1 - i\tau(\xi_\varepsilon^R + \xi_{\varepsilon-\omega}^A)} - 1\right]^{-1}$$

$$\xi_\varepsilon^R = -(\xi_\varepsilon^A)^* = \begin{cases} \operatorname{sign}\varepsilon\,(\varepsilon^2-\Delta^2)^{1/2}, & \varepsilon^2 > \Delta \\ i(\Delta^2-\varepsilon^2)^{1/2}, & \varepsilon^2 < \Delta \end{cases} \tag{29}$$

The case of low impurity concentration was considered in [10]. At high concentration $kl \ll 1$ we have

$$\Delta_\omega(k) = \frac{3}{\pi} K\left(\frac{1}{2}\right) D\left(\frac{e}{c}\right)^2 A_{\omega/2}^2(k) \left[\left(\frac{|\omega-2\Delta|}{\Delta}\right)^{1/2} \right. \tag{30}$$
$$\left. \times [\theta(2\Delta-\omega) - i\theta(\omega-2\Delta)] + \frac{Dk^2}{2\pi\Delta} \ln\frac{\Delta}{|\omega-2\Delta|}\right]^{-1}.$$

The term with Dk^2 was calculated with logarithmic accuracy. We see therefore that the harmonic of the gap at the frequency 2Δ has an anomalously large spatial-variation scale. For example, at $\omega < 2\Delta$ we have

$$\Delta_\omega(z) = \frac{3K(1/2)}{\sqrt{2}\pi} \ln^{-1/2}\frac{\Delta}{2\Delta-\omega} \left(\frac{\Delta}{2\Delta-\omega}\right)^{1/4} D\left(\frac{e}{c}\right)^2 \int_0^{\infty} A_{\omega/2}^2(z_1) e^{-z_1/L} \frac{dz_1}{\xi_0}, \tag{31}$$

$$L = \xi_0 \frac{1}{\sqrt{2}\pi} \ln^{1/2}\frac{\Delta}{2\Delta-\omega} \left(\frac{\Delta}{2\Delta-\omega}\right)^{1/4}.$$

The increase of L as the frequency approaches 2Δ is limited only by the value of the damping that must be introduced into the denominator of (30) [9]. At $T \sim \Delta$ we have $L_{max} \sim \xi_0(\omega_D/\Delta)^{1/2}$. As seen from (31), in addition to the anomalous spatial behavior, enhancement of the nonlinear properties takes place when the frequency of the external field is close to Δ.

It is of interest to consider the mixing of two laser frequencies, for example two neighboring modes. The shift frequency is then $\omega_0 \ll \Delta \ll \omega$, and the correction to the gap varies at the beat frequency

$$\Delta_{\omega_0} = \frac{\Delta}{\omega_0} D\left(\frac{e}{c}\right)^2 A_{\omega_0-\omega} A_\omega \left(\pi + 2i \ln\frac{2\omega}{\Delta}\right). \tag{32}$$

With decreasing frequency shift, $\Delta_{\omega 0}$ increases and the transition to the static case (13) takes place at very small ω_0, so that in practice it is impossible to observe the transition to the static limit. Nevertheless, an investigation of low-frequency gap harmonics is of interest from the point of view of the nonlinear properties, which become enhanced at low shift frequencies.

In conclusion, let us compare the experimental data of [2] with formula (19). The latter yields an exponential growth of the variation of the gap as a function of Δ/T, with subsequent saturation. The temperature at which the transition from one regime to the other takes place depends on the radiation intensity:

$$\frac{T}{\Delta} e^{-2\Delta/T} \sim \frac{\alpha\omega\omega_D^2}{\lambda\Delta^4} \frac{\delta}{2d} \frac{1}{1+\omega^2\tau^2} \left[1 + \frac{a_1}{6 \cdot 2^{2/3}} \frac{\omega}{\varepsilon_F} \left(\frac{\omega}{\omega_D}\right)^2\right].$$

At a radiation power on the order of several tenths of a watt, the transition frequency is $\Delta/T \sim 10$, which coincides in order of magnitude with the data of Parker and Williams [2].

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