Drift oscillations of an inhomogeneous plasma in a quantizing magnetic field

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The quantum theory of the electrical-conductivity tensor is used to investigate the drift oscillations of an inhomogeneous plasma in a quantizing magnetic field. Additional drift-oscillation branches due to the quantum nature of the orbital motion of the carriers are found. Both the potential and nonpotential quantized drift oscillations, the transparency regions, and the stability conditions are investigated in detail.

1. INTRODUCTION

The investigation of the electromagnetic oscillations of an inhomogeneous plasma located in a magnetic field is important for the solution of many problems in plasma physics itself, as well as in solid state physics, since a real plasma is, to a certain extent, always inhomogeneous. It is known, for example, that the presence of inhomogeneities leads to the appearance of additional oscillation branches (the so-called drift oscillations), which play an important role in the problem of magnetic confinement of plasmas.

Under conditions when the quantization of the motion of the carriers becomes essential (e.g., in quantizing magnetic fields), we can expect the appearance of new effects in a homogeneous plasma. In particular, additional drift-oscillation branches should appear in a quantizing magnetic field as a result of the quantized nature of the orbital motion of the particles. The existence of these additional oscillation branches will be demonstrated by us in the investigation of the electrical-conductivity tensor of a slightly inhomogeneous plasma (Sec. 2). The frequency of these new oscillations, given by $\omega_d^q = \hbar\Omega\omega_d^c/T$ (Ω is the cyclotron frequency), is higher than the frequencies ω_d^c of the ordinary classical drift oscillations, since $\hbar\Omega > T$. In contrast to classical drift oscillations, these quantized oscillations can exist in a cold plasma (for $T \to 0$) also, which is fully explicable, since, according to quantum theory, there is definite random particle motion even at zero temperature. The third, and last, section of the paper is devoted to a detailed analysis of the drift oscillations of a plasma located in a strong quantized magnetic field, the main attention being given to the investigation of quantized drift oscillations.

2. THE ELECTRICAL-CONDUCTIVITY TENSOR OF AN INHOMOGENEOUS PLASMA LOCATED IN A MAGNETIC FIELD

To consider the natural oscillations of a plasma, we must know the explicit expressions for the components of the electrical-conductivity tensor. We shall limit ourselves to the case of an inhomogeneous collisionless plasma located in a uniform magnetic field $\mathbf{H} = (0, 0, H)$, the inhomogeneity being assumed to be one-dimensional and directed along the x axis, i.e., perpendicular to the direction of the magnetic field.

The solution of the quantum-mechanical problem of the interaction of such a spatially inhomogeneous plasma containing one kind of carriers with an electromagnetic wave ($\sim e^{i\omega t}$) leads to the following expression for the electrical conductivity tensor $\sigma_{\alpha\gamma}(\mathbf{q}, \mathbf{q}', \omega)$ (cf., for example, with the results of Appendix A in [1]):

$$\sigma_{\alpha\gamma}(\mathbf{q},\mathbf{q}',\omega)=\frac{e^2}{mi\omega}\delta_{\alpha\gamma}\delta(q_y+q_y')\delta(q_z+q_z')\sum_{nk_yk_z}f_{nk_z}(X_0)I_{nn}(q_x+q_x')$$
$$+\frac{e^2}{i\omega}\delta(q_y+q_y')\delta(q_z+q_z')\sum_{\substack{nn'\\k_yk_z}}\frac{f_{n',k_z+q_z}(X_0+\lambda^2q_y/2)-f_{nk_z}(X_0-\lambda^2q_y/2)}{(\varepsilon_{n',k_z+q_z}-\varepsilon_{nk_z}-\hbar\omega+is)} \tag{1}$$
$$\times(\mathbf{V}_\mathbf{q}{}^\alpha)_{n'n}(\mathbf{V}_{\mathbf{q}'}{}^\gamma)_{nn'}.$$

Here (n, k_y, k_z) are the quantum numbers characterizing the state of the electron, whose wave function is

$$|n,k_y,k_z\rangle=\exp(ik_yy+ik_zz)\,\Phi_n\left(\frac{x-X_0}{\lambda}\right),$$

$\Phi_n(x)$ is the normalized oscillator function,

$$X_0=\lambda^2k_y,\quad \lambda^2=c\hbar/eH,\quad \varepsilon_{nk_z}=\hbar\Omega(n+{}^1\!/_2)+\hbar^2k_z{}^2/2m,$$

$\Omega = eH/mc$ is the cyclotron frequency, $f_{nk_z}(X_0)$ is the eigenvalue of the local-equilibrium statistical operator,

$$I_{n'n}(q_x)=\langle n'|\exp(iq_xx)|n\rangle,$$
$$(\mathbf{V}_\mathbf{q}{}^\alpha)_{n'n}={}^1\!/_2m^{-1}\langle n'|[(\mathbf{p}-e\mathbf{A}_0/c)^\alpha,\exp(iq_xx)]_+|n\rangle$$

The vector potential of the magnetic field is taken in the form $\mathbf{A}_0 = (0, Hx, 0)$.

If the characteristic dimension of an inhomogeneity is significantly greater than the wavelength, then it is often convenient to go over to a mixed representation and introduce the electrical conductivity tensor $\sigma_{\alpha\gamma}(\mathbf{q}, \omega, \mathbf{r})$ of the slightly inhomogeneous plasma:

$$\sigma_{\alpha\gamma}(\mathbf{q},\omega,\mathbf{r})=\int d\mathbf{Q}\,\sigma_{\alpha\gamma}(\mathbf{q},\mathbf{q}',\omega)e^{-i\mathbf{Q}\mathbf{r}}. \tag{2}$$

Restricting ourselves to the geometrical-optics approximation, we can write the matrix elements $(\mathbf{V}_{\mathbf{q}'})_{nn'}$ and $I_{nn}(q_x + q_x')$ in the form

$$(\mathbf{V}_{\mathbf{q}'})_{nn'}=e^{iQX_0}(\mathbf{V}_\mathbf{q})^*_{n'n},\quad I_{nn}(q_x+q_x')=e^{iQX_0}, \tag{3}$$

where $Q = q_x + q_x'$. In consequence, the tensor $\sigma_{\alpha\gamma}(\mathbf{q}, \omega, x)$ can be expressed as follows:

$$\sigma_{\alpha\gamma}(\mathbf{q},\omega,x)=\frac{N(x)e^2}{im\omega}\delta_{\alpha\gamma}+\frac{e^2}{i\omega}\frac{1}{2\pi^2\lambda^2}$$
$$\times\sum_{nn'}\int dk_z\frac{f_{n',k_z+q_z}(x+\lambda^2q_y/2)-f_{nk_z}(x-\lambda^2q_y/2)}{(\varepsilon_{n',k_z+q_z}-\varepsilon_{nk_z}-\hbar\omega+is)}(\mathbf{V}_\mathbf{q}{}^\alpha)_{n'n}(\mathbf{V}_\mathbf{q}{}^\gamma)^*_{n'n}. \tag{4}$$

Here N(x) is the particle density:

$$N(x)=\frac{1}{2\pi^2\lambda^2}\sum_n\int dk_zf_{nk_z}(x).$$

Outwardly, the expression (4) looks like the analogous formulas for the homogeneous plasma [2-5]. There are, however, differences, the principal one of which consists in the fact that the distribution function has become dependent on the space coordinate. Furthermore, this expression manifests an additional dependence on λ^2q_y, this dependence being responsible for the Larmor drift oscillations of an inhomogeneous plasma[6]. This is easily verified by considering the quasi-classical approximation. For this purpose, we must expand $f_{n',k_z+q_z}(x)$, assuming it is a Maxwellian distribution function, in a power series in q_z and $(n'-n)$. The components of the vector $(\mathbf{V_q})_{n'n}$ should be replaced in this expansion by their asymptotic expressions for large n, and, using the recursion relations for the Bessel functions, we can show that

$$\sigma_{\alpha\gamma}(\mathbf{q},\omega,x)=\left\{1-\frac{\lambda^2 q_y}{\hbar\omega}\frac{\partial}{\partial x}T(x)\right\}\tilde{\sigma}_{\alpha\gamma}(\mathbf{q},\omega,x), \tag{5}$$

where $\sigma_{\alpha\gamma}(\mathbf{q},\omega,x)$ is a tensor that coincides in form with the corresponding tensor of the homogeneous plasma [2,7], but in which the concentration and temperature are functions of the space coordinate x.

This is a well known result, which consists in the fact that the classical electrical-conductivity tensor of an inhomogeneous magnetoactive plasma can, in the zeroth geometrical-optics approximation, be obtained from the corresponding expression (in which N and T are functions of the space coordinate x) for the homogeneous plasma by applying to it the operator [6]:

$$T^{-1}(x)\left\{1-\frac{\lambda^2 q_y T(x)}{\hbar\omega}\frac{\partial}{\partial x}\right\}T(x). \tag{6}$$

In deriving this result we took into account only the terms that are linear in $\lambda^2 q_y$.

Returning to the case of quantitizing magnetic fields, and restricting ourselves besides to the linear approximation in the wave vector $\mathbf{q}$, we obtain from the formula (4) after simple transformations carried out under the conditions $\omega \gg q_z v_T$, $\lambda q_y \ll 1$ the expressions for the non-vanishing components of the electrical conductivity tensor of a slightly inhomogeneous Maxwellian plasma:

$$\begin{gathered}\sigma_{xx}=\sigma_{yy}=\frac{Ne^2\omega}{mi(\omega^2-\Omega^2)}\left\{1-\frac{\lambda^2 q_y\Omega}{2\omega}\frac{1}{N}\frac{\partial}{\partial x}\left[N\operatorname{cth}\left(\frac{\hbar\Omega}{2T}\right)\right]\right\},\\ \sigma_{xy}=-\sigma_{yx}=\frac{Ne^2\Omega}{m(\omega^2-\Omega^2)}\left\{1-\frac{\lambda^2 q_y\Omega}{2\omega}\frac{1}{N}\frac{\partial}{\partial x}\left[N\operatorname{cth}\left(\frac{\hbar\Omega}{2T}\right)\right]\right\}\\ \sigma_{zz}=\frac{Ne^2}{mi\omega}\left\{1-\frac{\lambda^2 q_y}{\hbar\omega}\frac{1}{N}\frac{\partial}{\partial x}(NT)\right\}.\end{gathered} \tag{7}$$

In the case of strong quantizing magnetic fields, when $\hbar\Omega \gg T$, these expressions go over into the following:

$$\begin{gathered}\sigma_{xx}=\sigma_{yy}=\left(1-\frac{\lambda^2 q_y\Omega}{2\omega}\frac{\partial}{\partial x}\right)\frac{Ne^2\omega}{mi(\omega^2-\Omega^2)},\\ \sigma_{xy}=-\sigma_{yx}=\left(1-\frac{\lambda^2 q_y\Omega}{2\omega}\frac{\partial}{\partial x}\right)\frac{Ne^2\Omega}{m(\omega^2-\Omega^2)},\\ \sigma_{zz}=\frac{1}{T}\left(1-\frac{\lambda^2 q_y T}{\hbar\omega}\frac{\partial}{\partial x}\right)T\frac{Ne^2}{mi\omega}.\end{gathered} \tag{8}$$

It is evident from this that there arise in a quantizing magnetic field the characteristic drift frequencies:

$$\omega_d^c\sim\frac{\lambda^2 q_y T}{\hbar L_0}=\frac{q_y v_T^2}{\Omega L_0},\quad \omega_d^q\sim\frac{\lambda^2 q_y\Omega}{L_0}, \tag{9}$$

which can respectively be regarded as the classical and quantum frequencies of the Larmor drift of the carriers.

The presence of the new drift frequency in a quantizing magnetic field is connected with the difference in the particle motion along and across the magnetic field. The nature of the motion along the field remained as before—classical. In consequence, the longitudinal component σ_{zz} includes a term containing the classical drift frequency. The nature of the particle motion across the magnetic field, on the other hand, becomes quantized, and, as a result, there appear in all the transverse components of the electrical conductivity tensor terms connected with the quantum drift frequency. It follows from (9) that the quantized drift oscillations lie in a region of higher frequencies than the classical frequencies for the same magnetic-field intensities, since

$$\omega_d^q=\hbar\Omega\omega_d^c/T$$

and $\hbar\Omega > T$.

If, on the other hand, the energy of the Larmor quantum is comparable to the energy of the thermal motion of the particles, then the expressions for the transverse components of the electrical conductivity tensor are representable in the form

$$\begin{aligned}\sigma_{xx}=\sigma_{yy}&=\frac{Ne^2\omega}{mi(\omega^2-\Omega^2)}\left\{1-\frac{\lambda^2 q_y T}{\hbar\omega}\frac{\partial\ln(NT)}{\partial x}\right.\\&\left.-\frac{\hbar\Omega}{12T}\frac{\lambda^2 q_y\Omega}{\omega}\frac{\partial}{\partial x}\ln\frac{N}{T}\right\},\\ \sigma_{xy}=-\sigma_{yx}&=\frac{Ne^2\Omega}{m(\omega^2-\Omega^2)}\left\{1-\frac{\lambda^2 q_y T}{\hbar\omega}\frac{\partial\ln(NT)}{\partial x}\right.\\&\left.-\frac{\hbar\Omega}{12T}\frac{\lambda^2 q_y\Omega}{\omega}\frac{\partial}{\partial x}\ln\frac{N}{T}\right\}.\end{aligned} \tag{10}$$

Thus, in this case there should exist in the plasma both classical and quantized drift oscillations, since the second terms in the curly brackets $\sim\omega_d^c/\omega$, while the last terms $\sim\omega_d^q/\omega$. However, until $\hbar\Omega \lesssim T$, the quantized oscillations manifest themselves weakly, becoming noticeable only when $\hbar\Omega > T$.

It should be added that it will be easiest to observe the quantized oscillations in a plasma consisting of light carriers, since it is easier to satisfy the conditions ($\hbar\Omega > T$) for a quantizing magnetic field in such a plasma, and, furthermore, the quantized-drift frequency is inversely proportional to the carrier mass. Further, the quantized drift oscillations are, in contrast to the classical oscillations, also possible in a cold plasma ($T \to 0$). This is connected with the fact that although the random thermal motion of the carriers, which is responsible for the existence of the classical drift oscillations, disappears as $T \to 0$, nevertheless from the point of view of quantum theory the presence in the system of the characteristic inhomogeneity dimension L_0 leads to an uncertainty in the velocity of a particle and, consequently, to a peculiar randomness in its motion even at $T = 0$. Therefore, it is quite natural for drift oscillations—albeit of a different nature—to exist in a cold plasma.

3. DRIFT OSCILLATIONS OF AN INHOMOGENEOUS PLASMA IN A QUANTIZING MAGNETIC FIELD

Let us now proceed to investigate the drift oscillations in a quantizing magnetic field. We shall restrict ourselves here to the limiting quantum case ($\hbar\Omega \gg T$) and to the low-frequency region ($\omega \ll \Omega$), since the drift oscillations usually lie precisely in this region; we shall however assume the particle density to be fairly high ($\bar{\omega}_0 > \Omega$, where $\bar{\omega}_0^2 = 4\pi\bar{N}e^2/m$ and $\bar{N}$ is the mean density). Further, using the fact that ω_d^q depends on the particle mass, we can restrict the analysis to only one type of carriers (light carriers), for which the quantization conditions are satisfied in the first instance.

It should be noted that the investigation of the classical drift oscillations in a plasma with one type of carriers is not always possible, since ω_d^c is not mass dependent. Therefore, we shall study only the quantized drift oscillations in a one-component plasma in the frequency region $\omega_d^c \ll \omega \lesssim \omega_d^q$, taking into account the fact that $\omega_d^q \gg \omega_d^c$ whenever $\hbar\Omega \gg T$. In this case the expressions for the nonvanishing components of the permittivity tensor $\epsilon_{\alpha\gamma}(\mathbf{q}, \omega, x)$ with allowance for damping can be represented as follows ($\omega \gg q_z v_T$):

$$\varepsilon_{xx}=\varepsilon_{yy}=\frac{\omega_0^2}{\Omega^2}\left(1-\frac{\hbar\Omega}{2T}\frac{q_y v_T^2}{\Omega\omega}\frac{\partial \ln N}{\partial x}\right)$$
$$+i\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_0^2}{\omega|q_z|v_T}\frac{\hbar\Omega}{T}\exp\left(-\frac{\Omega^2}{2q_z^2v_T^2}\right)\left[1-\frac{\Omega}{2\omega}\frac{q_y}{q_z^2}\frac{\partial\ln T}{\partial x}\right],$$
$$\varepsilon_{xy}=-\varepsilon_{yx}=i\frac{\omega_0^2}{\Omega\omega}\left(1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\right) \tag{11}$$
$$+\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_0^2}{\omega|q_z|v_T}\frac{\hbar\Omega}{T}\exp\left(-\frac{\Omega^2}{2q_z^2v_T^2}\right)\left[1-\frac{\Omega}{2\omega}\frac{q_y}{q_z^2}\frac{\partial\ln T}{\partial x}\right],$$
$$\varepsilon_{zz}=-\frac{\omega_0^2}{\omega^2}+i\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_0^2\omega}{|q_z|^3v_T^3}\exp\left(-\frac{\omega^2}{2q_z^2v_T^2}\right)\left[1-\frac{\omega}{2\Omega}\frac{q_y}{q_z^2}\frac{\partial\ln T}{\partial x}\right].$$

Knowing the explicit expressions for the components of the permittivity tensor $\epsilon_{\alpha\gamma}(\mathbf{q}, \omega, x)$, and solving the eikonal equation:

$$\left|q^2\delta_{\alpha\gamma}-q_\alpha q_\gamma-\frac{\omega^2}{c^2}\varepsilon_{\alpha\gamma}(\mathbf{q},\omega,x)\right|=0, \tag{12}$$

we can find the spectrum of the oscillations of the inhomogeneous plasma, as well as the damping constant γ, from the quasi-classical Bohr-Sommerfeld "quantization" condition [6]:

$$\int dx\,\mathrm{Re}\,q_x(\omega,x)=n\pi,$$
$$\gamma=-\int dx\,\mathrm{Im}\,q_x(\omega,x)\Big/\int dx\frac{\partial}{\partial\omega}\mathrm{Re}\,q_x(\omega,x), \tag{13}$$

where n is an integer ($n \gg 1$).

We shall consider two limiting cases of the oscillations: a) potential drift oscillations for $c^2q_z^2 \gg \omega^2\epsilon_{xx}$, $\omega^2\epsilon_{xy}$ and b) nonpotential drift oscillations for $c^2q_z^2 \ll \omega^2\epsilon_{xx}$, $\omega^2\epsilon_{xy}$.

In the first of these cases the eikonal equation (12) goes over into

$$\varepsilon_{xx}q_\perp^2+\varepsilon_{zz}q_z^2=0, \tag{14}$$

and, taking account of the relations (13), we obtain the following dispersion equations for the determination of the spectrum of the potential drift oscillations of an inhomogeneous plasma in a strong quantizing magnetic field:

$$\int dx\,\mathrm{Re}\,q_x(x,\omega)$$
$$=\int dx\left\{-q_y^2+q_z^2\frac{\Omega^2}{\omega^2}\left(1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\right)^{-1}\right\}^{1/2}=n\pi,$$
$$\gamma=\frac{1}{2}\left(\frac{\pi}{8}\right)^{1/2}\left(\int dx\frac{\partial\,\mathrm{Re}\,q_x}{\partial\omega}\right)^{-1}\int\frac{dx}{\mathrm{Re}\,q_x}\frac{\omega\Omega^2}{|q_z|v_T^3} \tag{15}$$
$$\times\left(1-\frac{\omega}{2\Omega}\frac{q_y}{q_z^2}\frac{\partial\ln T}{\partial x}\right)\exp\left(-\frac{\omega^2}{2q_z^2v_T^2}\right)\left(1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\right)^{-1}.$$

It is important to note that the integrands in (15) should not have singularities in the entire transparency region of the plasma, i.e.,

$$1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\neq 0. \tag{16}$$

Let us consider the long-wave oscillations ($\omega \gg q_z v_T$) in the region of frequencies lower than the quantized-drift frequencies of the carriers, but significantly higher than the classical drift frequencies. Then it follows from Eqs. (15) that

$$\int dx\,\mathrm{Re}\,q_x(\omega,x)=\int dx\left\{-q_y^2-2q_z^2\frac{\Omega}{\omega}\left(\lambda^2q_y\frac{\partial\ln N}{\partial x}\right)^{-1}\right\}^{1/2}=n\pi,$$
$$\gamma=-\left(\frac{\pi}{2}\right)^{1/2}\int\frac{dx}{\mathrm{Re}\,q_x}\frac{\omega^4}{|q_z|^3v_T^3}\left(\frac{\partial\ln N}{\partial x}\right)^{-1} \tag{17}$$
$$\times\left(1-\frac{\omega}{2\Omega}\frac{q_y}{q_z^2}\frac{\partial\ln T}{\partial x}\right)\exp\left(-\frac{\omega^2}{2q_z^2v_T^2}\right)\left[\int\frac{dx}{\mathrm{Re}\,q_x}\left(\frac{\partial\ln N}{\partial x}\right)^{-1}\right]^{-1}$$

It can be seen that such oscillations are hydrodynamically stable starting from the transparency region limited by the condition

$$-q_y^2-2q_z^2\frac{\Omega}{\omega}\left(\lambda^2q_y\frac{\partial\ln N}{\partial x}\right)^{-1}\geqslant 0.$$

The attenuation of these oscillations is exponentially small. Under certain conditions the oscillations described by the expressions (17) can become kinetically unstable. From the expression for γ, we can find that for the instability to develop in the transparency region for the oscillations under consideration, it is necessary that

$$\partial\ln T/\partial\ln N<0.$$

The growth rate of these oscillations remains exponentially small, and only at the edge $\omega \sim q_z v_T$ of the frequency region under consideration can it attain values comparable to the frequency of the oscillations.

Generally speaking, the potential oscillations of an electromagnetic field in a magnetoactive plasma are not natural oscillations. Therefore, it is necessary to consider the general nonpotential oscillations that are described by the eikonal equation (12), which, for our approximations, has the following form:

$$q_\perp^4\varepsilon_{xx}+q_\perp^2\left[\left(q_z^2-\frac{\omega^2}{c^2}\varepsilon_{xx}\right)(\varepsilon_{xx}+\varepsilon_{zz})-\frac{\omega^2}{c^2}\varepsilon_{xy}^2\right]$$
$$+\varepsilon_{zz}\left[\left(q_z^2-\frac{\omega^2}{c^2}\varepsilon_{xx}\right)^2+\frac{\omega^4}{c^4}\varepsilon_{xy}^2\right]=0.$$

It is easy to see that in the limit $c^2q_z^2 \gg \omega^2\epsilon_{xx}$, $\omega^2\epsilon_{xy}$, this equation goes over into the eikonal equation (14) for the potential oscillations. In the opposite case, when $c^2q_z^2 \ll \omega^2\epsilon_{xx}$, $\omega^2\epsilon_{xy}$, the eikonal equation determines two functions $q_x^2(\omega, x)$ corresponding to two oscillation branches of the magnetoactive plasma—the ordinary and extraordinary waves:

$$q_x^2(\omega,x)=-q_y^2+\frac{\omega^2}{c^2}\varepsilon_{zz},\qquad q_x^2(\omega,x)=-q_y^2+\frac{\omega^2}{c^2}\left(\varepsilon_{xx}+\frac{\varepsilon_{xy}^2}{\varepsilon_{xx}}\right). \tag{18}$$

The oscillation spectrum described by the first of the Eqs. (18), which contains no quantum drift waves, has been thoroughly investigated [6]; therefore, we shall study in detail only the second of these equations. In doing this, we need, generally speaking, to take into account terms of second order in the density and temperature gradients, and therefore the expressions (11) cannot justifiably be used to find the spectrum of such oscillations. Nevertheless, in our approximation the results obtained using the more general expressions (4) without expanding them in powers of λ^2q_y coincide with the results of the linear theory, and we arrive at the following dispersion relations for the extraordinary wave:

$$\int dx\,\mathrm{Re}\,q_x(\omega,x)=\int dx\left\{-q_y^2-\frac{\omega_0^2}{c^2}\left(1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\right)\right\}^{1/2}=n\pi,$$
$$\gamma=\left(\frac{\pi}{2}\right)^{1/2}\int\frac{dx}{\mathrm{Re}\,q_x}\frac{\Omega\omega_0^2}{|q_z|v_T}\frac{\Omega}{T}\left[1-\frac{q_yv_T^2}{\Omega\omega}\cdot\right.$$

$$\times\left(\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T}}+\frac{\Omega^2}{2q_z^2v_T^2}\frac{\partial\ln T}{\partial x}\right)\Big]$$

$$\times\exp\left(-\frac{\Omega^2}{2q_z^2v_T^2}\right)\left(\int\frac{dx}{\mathrm{Re}\,q_x}\frac{\omega_0^2}{T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}\right)^{-1}. \quad (19)$$

It follows from the first of the Eqs. (19) that these oscillations are hydrodynamically stable, and that the frequencies of the oscillations are of the order of the quantized-drift frequencies:

$$\omega\sim\omega_d^q\sim\frac{\hbar\Omega}{T}\frac{q_yv_T^2}{\Omega L_0}.$$

Taking into account the fact that the transparency region of the plasma with respect to such oscillations is, according to the first of the Eqs. (19), given by

$$1-\frac{\hbar\Omega}{2T}\frac{q_yv_T^2}{\Omega\omega}\frac{\partial\ln N}{\partial x}<0,$$

we find from the expression for γ the necessary condition for the kinetic instability of the inhomogeneous plasma with respect to these oscillations:

$$1-\frac{2T}{\hbar\Omega}\left[1+\frac{\Omega^2}{2q_z^2v_T^2}\frac{\partial\ln T}{\partial\ln N}\right]>0, \quad (20)$$

which is certainly satisfied when $\partial\ln T/\partial\ln N<0$ in the transparency region. However, the growth rate of the quantum drift oscillations remains exponentially small even when the growth rate of the classical drift oscillations can attain values comparable to the frequency, since $\Omega\gg\omega$. On the other hand, the attenuation of the classical drift oscillations becomes appreciable when $\omega\sim q_zv_T$, whereas the quantized-drift oscillations still do not experience noticeable damping in this frequency region.

In conclusion, let us note that the results of our investigation can be extended to the case of several types of carriers (electron-ion or electron-hole plasmas). It may then turn out that the motion of one type of carriers (the heavy ones) obeys classical laws, while that of the light carriers is described by quantum laws, which then introduces additional changes into the spectrum of the drift oscillations of such a plasma.

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[1]V. M. Eleonskiĭ, P. S. Zyryanov, and V. P. Silin, Zh. Eksp. Teor. Fiz. **42**, 896 (1962) [Sov. Phys.-JETP **15**, 619 (1962)].

[2]P. S. Zyryanov and V. P. Kalashnikov, Zh. Eksp. Teor. Fiz. **41**, 1119 (1961) [Sov. Phys.-JETP **14**, 799 (1962)].

[3]H. N. Spector, Phys. Rev. **132**, 522 (1963).

[4]J. J. Quinn and S. Rodriguez, Phys. Rev. **128**, 2487 (1962).

[5]V. V. Kolesov, Phys. Stat. Sol. **50**, 471 (1972).

[6]A. A. Rukhadze and V. P. Silin, Usp. Fiz. Nauk **82**, 499 (1964); **96**, 87 (1968) [Sov. Phys.-Uspekhi **7**, 209 (1964); **11**; 659 (1969)]; V. L. Ginzburg and A. A. Rukhadze, Volny v magnitoaktivnoĭ plasme (Waves in a Magnetoactive Plasma), Nauka, 1970, Secs. 10 and 12.

[7]A. G. Sitenko and K. N. Stepanov, Zh. Eksp. Teor. Fiz. **31**, 642 (1956) [Sov. Phys.–JETP **4**, 512 (1957)].

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