Electromagnetic detectors of gravitational waves

V. B. Braginskiĭ, L. P. Grishchuk, A. G. Doroshkevich, Ya. B. Zel'dovich, I. D. Novikov, and M. V. Sazhin

Moscow State University
State Astronomical Institute
Applied Mathematics Institute of the USSR Academy of Sciences
(Submitted May 28, 1973)
Zh. Eksp. Teor. Fiz. **65**, 1729–1737 (November 1973)

The detection of gravitational waves with the aid of electromagnetic resonators and waveguides is considered. The changes produced in the electromagnetic field of a resonator by passage of a gravitational wave are calculated, optimal methods of gravitational-wave detection are indicated, and the minimal observable gravitational-wave fluxes are estimated.

1. INTRODUCTION

The detection of gravitational waves (GW) is connected with conversion of their energy into other forms or with their action on other types of motion.

In the laboratory generation and detection problems it is possible to generate GW with a rigorously determined frequency and with a known phase. This advantage is fully utilized in resonant reception of GW. It is therefore natural to use as the receiver a system having a known frequency (or set of frequencies) and to consider the resonant oscillations exclusively. To receive high-frequency waves it is natural to attempt to employ electromagnetic (EM) oscillations in the resonator or in the waveguide. For low frequencies, on the contrary, mechanical systems are more convenient[1–3].

Thus, we consider a resonator in which a set of weakly damped electromagnetic oscillations can exist:

$$E_n(x, t)\sim e^{i\omega_n t}E_n(x),\qquad B_n(x, t)\sim e^{i\omega_n t}B_n(x),\tag{1}$$

where E_n and B_n are the intensities of the electric and magnetic fields. An arbitrary electromagnetic field can be represented in the form

$$A=\Sigma a_n(t)A_n(x),\tag{2}$$

where A stands for E (or B).

For $a_n(t)$ in the absence of GW, the following equations hold in an ideal resonator:

$$d^2a_n/dt^2+\omega_n^2a_n=0.\tag{3}$$

The action of GW, as will be shown later on, intermixes different oscillations:

$$d^2a_n/dt^2+\omega_n^2a_n=h(t)\,\omega_n^2C_{nm}a_m.\tag{4}$$

where $h=h_0e^{i\Omega t}$ (dimensionless correction to the metric), h_0 is the GW amplitude, and Ω is its frequency. The coefficients C_{nm} depend significantly on the form of the resonator and in certain cases (from symmetry considerations, etc.) we have $C_{mn}=0$, but the maximum value is $C_{nm}\approx r^2/\lambda^2$, where r is the dimension of the resonator and λ is the length of the GW[1)].

Obviously, for purposes of resonant reception of gravitational waves it suffices to consider a pair of oscillations with indices m and n such that

$$\omega_n\pm\omega_m=\Omega\tag{5}$$

and the right-hand side of the equation is resonant:

$$\frac{d^2a_n}{dt^2}+\omega_n^2a_n=\omega_n^2b_ne^{i\omega_n t},\qquad b_n\sim h_0a_m.\tag{6}$$

Then the additional amplitude $a_n^{(1)}$ due to the action of the GW increases linearly with time (gravitational-electromagnetic resonance):

$$a_n^{(1)}\sim\omega_n t b_n e^{i\omega_n t}.\tag{7}$$

This formula is the basis of the entire subsequent analysis, and includes the following different cases:

1. The initial amplitude of the n-th oscillation is equal to zero,

$$a_n=a_n^{(1)},\qquad \varepsilon_n=a_n^{(1)2}\sim t^2,\tag{8}$$

the n-th oscillation is generated, and the energy of the n-th oscillation (and the number of quanta of this oscillation) are proportional to t^2. Since the energy of the GW passing through the resonator is proportional to t, the coefficient α for the conversion of the GW energy into the energy of an electromagnetic wave (EMW) in the resonator is proportional to t and its order of magnitude is

$$\alpha\approx Gc^{-4}(E^2+B^2)\,lct,\tag{9}$$

where E^2+B^2 is the energy density of the m-th oscillation or of the constant field.

2. The initial amplitude $a_n^{(0)}$ is not equal to zero; by virtue of the linearity of the problem, the amplitudes are additive

$$a_n(t)=a_n^{(0)}+a_n^{(1)}.\tag{10}$$

Depending on the phase relations, we obtain here either (a) a change in the oscillation energy under the influence of the GW and

$$\Delta\varepsilon_n\sim|a_n|^2-|a_n^{(0)}|^2\approx a_n^{(0)}a_n^{(1)}\sim t,\qquad d\varepsilon_n/dt\sim(\varepsilon_m\varepsilon_n)^{1/2}|\dot h|\sin\varphi,\tag{11}$$

where ϵ_m and ϵ_n are the energies of the oscillations with frequencies ω_m and ω_n, or (b) a change in the phase (with the phase of $a_n^{(1)}$ shifted relative to $a_n^{(0)}$ by $\pi/2$) and

$$d\varphi_n/dt=(\varepsilon_m/\varepsilon_n)^{1/2}|\dot h|\cos\varphi.\tag{12}$$

Obviously, all the formulas are valid only at $t\leq\tau$, where τ is the characteristic damping time of the oscillations in the resonator as a result of the losses. It is precisely τ which limits the growth of the conversion coefficient α. Allowance for the finite Q of the resonator leads to a replacement of t at $t\gg\tau$ by Q/ω.

When comparing the variants 1 and 2, it must be borne in mind that although in the former case the effect is quadratic and in the latter it is linear in the small amplitude h of the GW, the advantage of the second variant is illusory: in the first variant a small ϵ_1 is obtained without a background, and in the second a large $\Delta\epsilon_n$ is obtained against a background of large $\epsilon_n^{(0)}$ and large fluctuations of $\epsilon_n^{(0)}$. Specific estimates of the value of the noise are given below. In practice, the first variant is preferable, and if it results in unacceptable characteristics (required dimensions, number of tests, etc.), then the second variant will likewise not do.

Different cases are also classified in accordance with the character of the field that is transformed under the influence of the GW into electromagnetic oscillations. We note two interesting cases:

1. The case $m=0$: transformation of a static time-independent electric or magnetic field into oscillations. Then

$$\omega_n=\Omega. \tag{13}$$

The advantages of this method are the absence of an alternating background and the absence of heating of the resonator by the m-th oscillation.

The maximal magnetic field that can be produced in a resonator is larger than the maximal electric field (10^5 G, which is equivalent to 3×10^7 V/cm), but the magnetic field nevertheless increases the high-frequency losses and decreases τ even in those cases when the superconductivity is not destroyed.

2. The case $m=n$, $\omega_n=2\Omega$—parametric resonance; the formulas become simpler and we are dealing with only one type of oscillation, in which the energy or the phase changes under the influence of the GW. Formulas (11)–(12) take the form

$$d\varepsilon_n/dt=\varepsilon_n|\dot{h}|\sin\varphi, \quad d\varphi_n/dt=|\dot{h}|\cos\varphi. \tag{14}$$

What remains in force, however, is everything said above concerning the difficulty of detecting the change of the energy or of the phase against the background of powerful initial oscillations, which introduce fluctuations, in comparison with detection of the production of quanta of an oscillation that is not excited in the initial state.

The most important experiment is one in which the gravitational waves are also generated in the laboratory (and not in outer space). Estimates of the possibility of laboratory generation of gravitational waves are given in [4].

2. DETECTION OF GRAVITATIONAL WAVES WITH THE AID OF WAVE PACKETS

Braginskiĭ and Menskiĭ[5] have proposed a method of detecting gravitational waves with the aid of a packet of electromagnetic waves moving in a circular resonator.

We shall first discuss the detection of GW with the aid of wave packets, inasmuch as in this case the geometrical-optics approximation is valid, the entire analysis becomes lucid, and there is no need to solve the wave equations.

We shall then discuss the general case of arbitrary electromagnetic waves in resonators.

When dealing with a gravitational-electromagnetic resonator, it is necessary to bear in mind that the speed of sound in the resonator material is much smaller than the speed of light, so that the resonator walls move in the passing gravitational wave like free particles. This makes it particularly convenient to use a synchronous reference frame[6] in which the resonator walls are at rest.

We consider an electromagnetic wave packet moving in a waveguide in the field of a gravitational wave. We consider a compact wave packet and therefore assume the geometrical-optics approximation. The change of the frequency of the waves making up the packet is proportional to the change of the length of the packet itself. On the other hand, the relative change $\Delta\epsilon/\epsilon$ of the energy of the electromagnetic wave packet is equal to the relative change of the wave frequency. Consequently, the relative change of the packet energy is

$$\Delta\varepsilon/\varepsilon=\Delta l/l, \tag{15}$$

where l is the length of the packet.

We begin with consideration of a packet in a ring resonator, as proposed in [5]. Let the GW propagate along the z axis and let it be circularly polarized. Then the metric in cylindrical coordinates takes the form

$$\begin{gathered} ds^2=c^2dt^2-(1+h)\,dr^2-r^2(1-h)\,d\varphi^2-dz^2+2rh'drd\varphi, \\ h=h_0\cos(\Omega(t-z)+2\varphi), \quad h'=h_0\sin(\Omega(t-z)+2\varphi), \end{gathered} \tag{16}$$

where Ω is the frequency of the wave.

Let a thin ring waveguide lie in the plane $z=0$ and have a radius $r=r_0$. The waveguide dimension is chosen such that the light packet travels through the ring in the absence of a GW with a frequency $\Omega=2c/r_0$. The equation of motion of any point of the packet in the waveguide is determined by the equality $ds=0$ $(dr=dz=0)$:

$$\frac{d\varphi}{dt}=\frac{c}{r_0}\left[1+\frac{1}{2}h_0\cos(2\varphi-\Omega t)\right]. \tag{17}$$

In the zeroth approximation $\varphi=\varphi_0+ct/r_0$. The solution of (17) accurate to first-order terms is

$$\varphi=\varphi_0+{}^1/{}_2\Omega t(1+{}^1/{}_2h_0\cos 2\varphi_0), \tag{18}$$

where φ_0 is the coordinate of the point at $t=0$.

With the aid of (18) we obtain the relative change $\Delta\varphi$ of the length of the packet along the φ coordinate:

$$\delta(\Delta\varphi)/\Delta\varphi={}^1/{}_2h_0t\Omega\sin 2\varphi_0. \tag{19}$$

We emphasize that the change of the length of the packet is connected with the constant difference between the coordinate velocities of the motion along φ between the leading and trailing boundaries of the electromagnetic packet.

With the aid of (18) we obtain for the change in the packet energy (we use the connection between h_0 and the energy flux density F in a gravitational wave):

$$\Delta\varepsilon/\varepsilon={}^1/{}_2h_0\Omega t\sin 2\varphi_0=t\sin 2\varphi_0\sqrt{64\pi GF/c^3}. \tag{20}$$

We consider now a packet in a straight waveguide with ideal mirrors at its ends. Assume that a plane gravitational wave is incident on such a detector, and let the waveguide be located in a plane perpendicular to the wave vector of the GW. In this case the metric along the straight waveguide is a function of the time only (and is furthermore periodic), and does not depend on the coordinate along the waveguide. There is no difference between the coordinate velocities of motion of the leading and trailing boundaries of the electromagnetic packet, as was the case in the circular waveguide, so that there is no systematic change in the length of the packet even in the case of synchronism, i.e., when the packet moves at double the frequency of the gravitational wave. There are only periodic increases and decreases of the packet length at the frequency of the gravitational wave [2)]. If, however, the mirrors are not strictly perpendicular to the wave vector, but at a certain angle to it (but not parallel), then again a difference appears between the coordinate velocities of the leading and trailing ends of the electromagnetic packet. If we now choose the waveguide length such that the packet returns to the ends of the waveguide in one and the same phase of the gravitational wave, then the length of the packet will vary systematically with time. The change of the length has numerically the same order of magnitude as in formula (19):

$$\delta(\Delta l)/\Delta l \approx h_0 \Omega t/2. \qquad (21)$$

The numerical factor is here smaller than (but of the order of) unity.

3. ELECTROMAGNETIC RESONATORS IN THE FIELD OF GRAVITATIONAL WAVES

We turn now to registration of gravitational waves with the aid of electromagnetic resonators in cases when the geometrical-optics approximation no longer holds [3].

Let us examine the possibility of registering the gravitational wave with the aid of a ring coaxial waveguide, in which a monochromatic electromagnetic wave propagates. The simplest problem is that of a circularly-polarized gravitational wave propagating along the z axis, and a fundamental electromagnetic wave propagating through a thin ring waveguide of radius r_0 located in the plane $z = 0$ (similar results can also be obtained for a different polarization of the GW, and also for a standing EMW). The waveguide is coaxial and the outside and inside radii of its cross section are R_2 and R_1. It is convenient to solve the problem in a "toroidal" coordinate system connected with the cylindrical coordinate system by the equation

$$r=r_0+R\cos\theta,\quad z=R\sin\theta,$$
$$R_1\leqslant R\leqslant R_2,\quad 0\leqslant\theta\leqslant 2\pi. \qquad (22)$$

The third coordinate is the azimuthal angle φ $(0 \le \varphi \le 2\pi)$. The condition that the waveguide be "thin" reduces to the requirement $r_0 \gg R_2$. It is convenient to choose as the unperturbed EMW a fundamental wave in which the electric field is aligned with the radius of the torus (the component $E_1 = E_R$), while the magnetic field is directed along the concentric circles (the component $H_3 = H_0$). In this case the resonant part of the perturbed field has only the same components. The unperturbed field of the n-th harmonic can be described by one component of the four-vector-potential A_i:

$$A_0=A_2=A_3=0,\quad A_1=-\frac{2E_0}{h\Omega}\frac{R_1}{R}\sin(n\psi+\varphi_0), \qquad (23)$$
$$E_1=F_{01}=\frac{\partial A_1}{\partial t}=E_0\frac{R_1}{R}\cos(n\psi+\varphi_0),$$
$$H_3=F_{12}=\partial A_1/\partial(r_0\varphi)=E_1,$$

where $\psi = \varphi - \Omega t/2$.

We seek the perturbations of the electromagnetic field by solving Maxwell's equations in curved space:

$$\frac{\partial}{\partial x^i}\sqrt{-g}\,F^{ik}=0, \qquad (24)$$

where g is the determinant of the metric tensor and F^{ik} is the electromagnetic tensor. The solutions should satisfy the usual boundary conditions on the walls of the resonator. Substitution in (24) of the metric of the gravitational wave (16) and of expressions (23) for the unperturbed electromagnetic wave leads to the following conclusions:

1. The resonant part of the perturbation can also be described only by the component A_1 of the four-potential in the form $\delta A_1 = (R_1/R)f(\varphi, t)$.

The form of the function $f(\varphi, t)$ is determined by the growing solution of the equation

$$\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}-\frac{1}{r_0^2}\frac{\partial^2 f}{\partial\varphi^2}=0.5E_0h_0\{(n+2)\sin[(n+2)\psi+\varphi_1]$$
$$+(n-2)\sin[(n-2)\psi+\varphi_2]\}, \qquad (25)$$

where φ_2 is the difference and φ_1 is the sum of the constant terms of the phase in (16) and (23), whence

$$f=0.25E_0th_0\{\cos[(n+2)\psi+\varphi_1]+\cos[(n-2)\psi+\varphi_2]\}. \qquad (26)$$

Expression (26) shows that interaction of a gravitational wave with a monochromatic electromagnetic wave of frequency $n\Omega/2$ produces new quanta, of frequency $(n \pm 2)\Omega/2$, which were not present in the initial electromagnetic wave. This circumstance is of particular importance for the detection of GW.

Let us determine the energy content $\Delta\epsilon$ in the produced new quanta of frequency $(n + 2)\Omega/2$.

Using (26), we obtain the intensity of the electric field and the energy density at the new frequency:

$$E=0.125E_0h_0t\Omega\frac{R_1}{R}\{(n+2)\sin[(n+2)\psi+\varphi_1]+(n-2)\sin[(n-2)\psi+\varphi_2]\},$$
$$\frac{\Delta\varepsilon}{\varepsilon}=\frac{(2\pm n)^2}{64}(\Omega th_0)^2=4(2\pm n)^2\pi Gt^2Fc^{-3}\sim(hQ)^2. \qquad (27)$$

It is seen from the last formula that the effect is quadratic in hQ.

The quantity $\Delta\epsilon$ can be linear in hQ only if the produced quanta have the same frequency as the initial quanta (in the unperturbed solution). This takes place for the fundamental harmonic, when $n = 1$ (case of parametric resonance).

In this case we obtain for the perturbed component of the electric field

$$E_1=E_0\frac{R_1}{R}[\cos(\psi+\varphi_0)+0.125th_0\Omega\sin(\psi+\varphi_2)] \qquad (28)$$

and an analogous expression for the intensity of the magnetic field (only the resonant terms are preserved!). Depending on the choice of the phase φ_2 (in comparison with φ_0), two different treatments of formula (28) are possible.

1. By choosing $\varphi_2 = \varphi_0 + \pi/2$, we obtain

$$E_1=(E_0R_1/R)(1+0.125h_0t\Omega)\cos(\psi+\varphi_0), \qquad (29)$$

which corresponds to an electromagnetic wave energy increment proportional to the time:

$$\Delta\varepsilon/\varepsilon=0.125th_0\Omega=t[16\pi GF/c^3]^{1/2}\sim hQ. \qquad (30)$$

2. By choosing $\varphi_2 = \varphi_0$, we are able to write (28) in the form

$$E_1=(E_0R_1/R)\cos[\psi+(\varphi_0+0.125th_0\Omega)], \qquad (31)$$

meaning that the phase shift of the electromagnetic wave is proportional to the time.

The choice of the particular condition for registration of GW is determined by the experimental technique.

Another variant, which makes $\Delta\epsilon$ linear in h_0, is the case when electromagnetic waves of two frequencies, ω_1 and $\omega_2 = \omega_1 \pm \Omega$, are present in the waveguide, and the intensity of one of the waves (for example ω_1) is large in comparison with the intensity of the other. In this situation, the interaction of the GW with the EMW of frequency ω_1 leads to the appearance of quanta of frequency ω_2, which can be registered when summed with the additional EMW of frequency ω_2.

In this case we obtain in place of (28)

$$E_1=E_0\cdot\frac{R_1}{R}\left\{\cos(\psi_2+\varphi_0)+0.25\frac{E_0}{E_0^*}h_0\omega_1 t\sin(\psi_2+\varphi_2)\right\},\tag{32}$$

where $\psi_2=\varphi-\omega_2 t$, the constants E_0 and E_0^* determine the amplitudes of the electromagnetic waves of frequency ω_1 and ω_2, and the phases of the two unperturbed waves are chosen to be equal. For the energy perturbation at the frequency ω_2 we obtain ($\varphi_2=\varphi_0+\pi/2$):

$$\frac{\Delta\varepsilon}{\varepsilon}=\frac{1}{2}h_0\omega_2 t\frac{E_0}{E_0^*}\sim\frac{E_0}{E_0^*}hQ,\tag{33}$$

which differs from (30) by the factors ω_2 (in place of Ω) and $E_0/E_0^*\gg 1$.

The choice $n=0$ leads to the problem of generation of EMW in the case of interaction of a gravitational wave with constant electric and magnetic fields. The electric field is directed along the radius of the torus and the magnetic field along concentric circles in the torus. As follows from (26), in this case there is produced an electromagnetic wave that coincides in frequency with the gravitational wave and has an amplitude that increases with time:

$$j=0.5E_0 th_0\cos(2\psi+\varphi_3).\tag{34}$$

The energy density of the electromagnetic wave in this problem is

$$\varepsilon=4GE_0^2t^2Fc^{-3}(R_1^2/R_2^2)\cos^2(2\psi+\varphi_3)\tag{35}$$

and for the total energy of the electromagnetic wave in the resonator we obtain

$$\varepsilon_\Sigma=(8\pi r_0R_2Ft)\left(\pi GE_0^2R_1^2R_2^{-1}tc^{-3}\ln\frac{R_2}{R_1}\right),\tag{36}$$

where the first factor is equal to the total energy flux of the gravitational wave through the resonator, and the second is equal to the coefficient of conversion of GW energy into energy of the electromagnetic oscillations.

Similar results can be obtained for rectangular resonators. The numerical results for rectangular resonators of finite dimensions are close to those given above (formulas (26)–(36)). Thus, for a rectangular resonator with dimensions l_1, l_2, and l_3, in which a homogeneous magnetic field is produced along the x axis (the GW propagates in the direction of the z axis), the perturbed vector potential can be expressed in the form

$$A_2=H_0z+H_0h_0tc\left(\frac{n}{\pi^2p^3}\frac{l_1^2}{l_3^2}\right)[\cos\Omega t-(-1)^n\cos(\Omega t-\Omega l_3)]\sin\frac{2\pi px}{l_1}\sin\frac{\pi nz}{l_3}\tag{37}$$

under the resonant condition $\Omega^2=4\pi^2p^2/l_1^2+n^2\pi^2/l_3^2$ (n and p are integers). For the total energy in the resonator (we take into account only terms that increase with time) we obtain

$$\varepsilon=0.25l_1l_2l_3\left(\frac{\Omega}{2}H_0h_0t\right)^2\left(\frac{n}{\pi^2p^3}\frac{l_1^2}{l_3^2}\right)^2\left[1-(-1)^n\cos\frac{\Omega l_3}{c}\right]$$
$$=(FSt)(8\pi GtH_0^2c^{-3})\left(\frac{n}{\pi^2p^3}\frac{l_1^2}{l_3^2}\right)^2\left[1-(-1)^n\cos\frac{\Omega l_3}{c}\right],\tag{38}$$

($S=l_1l_2$ is the area of the resonator); formula (38) differs from (36) only in the structure factors contained in the brackets.

4. DISCUSSION OF RESULTS

We present numerical estimates of the minimal possible fluxes that we can hope to register with the aid of electromagnetic resonators at the present time and in the nearest future. From the experimental point of view, as stated in the Introduction, the detection of GW by exciting electromagnetic oscillation modes in a resonator that were not present in the unperturbed electromagnetic field is probably most promising. The initial unperturbed field can be either alternating or constant.

In the former case, we use formula (30) for the estimates. We rewrite this formula in the form

$$\Delta\varepsilon=\varepsilon\tau(16\pi GF/c^3)^{1/2}.\tag{39}$$

Here τ is the maximum possible signal-accumulation time, equal to the damping time of the electromagnetic oscillation of given frequency in the resonator.

From signal-observation theory (see, e.g., [11]), it is known that to observe the signal we must have

$$\Delta\varepsilon\geqslant 2\sqrt{\varepsilon kT},\tag{40}$$

where k is Boltzmann's constant and T is the temperature. Formula (40) is valid at $kT\geq\hbar\omega$. From (39) and (40) we obtain

$$F_{min}\approx kTc^3/4\pi G\varepsilon\tau^2.\tag{41}$$

According to the experimental data [12] and the relations given there between the resonator Q and the frequency or the temperature, we can expect to be able in the nearest future, in principle, to obtain values $\tau\approx 3\times 10^4$ sec and $\epsilon\approx 10^9$ erg at a frequency $\omega\approx 10^8$ sec^{-1}. In this case, at $kT\approx\hbar\omega\approx 10^{-19}$ erg we obtain from (41)

$$F_{min}\approx 3\ \text{erg/sec}\cdot\text{cm}^2.\tag{42}$$

To estimate F_{min} in the case of a constant unperturbed electric field in a ring resonator, we use formula (36). We put $\Omega\approx 10^8$ sec^{-1}, and then the resonator radius is $r_0\approx 3\times 10^2$ cm. We assume the outside and inside cross-section radii of the waveguide to be $R_2=20$ cm and $R_1\approx 1$ cm, respectively. We then put $\tau=3\times 10^4$ sec and $E_0\simeq 3\times 10^7$ V/cm, and obtain under the condition $\epsilon_\Sigma=\hbar\omega$

$$F_{min}\approx 10^{-4}\ \text{erg/cm}^2\cdot\text{sec}.\tag{43}$$

It appears that the foregoing estimates are optimal for the prospects of detecting gravitational waves with the aid of the simplest electromagnetic resonators described in this article.

A review of the present status of the problem of detection of gravitational waves is also contained in a paper by Press and Thorne [3]

Note added in proof (27 September 1973). We present estimates pertaining to the joint operation of an emitter (see [4]) and a detector, which was discussed in the present article. The wavelength is 100 cm, the radiator measures $10^3\times 10^3\times 10^3$ cm, the oscillation-energy density in the emitter is 10^{10} erg/cm^3, corresponding to $H=3\times 10^5$ G and $E=10^8$ V/cm, producing a flux $q=10^{-11}$ erg/cm^2 sec on an area 100×100 cm^2. In a detector measuring $100\times 100\times 100$ cm with a field $H_0=10^6$ G, at $Q=10^{14}$, so that $\tau=10^6$ sec, the production of one photon has a probability on the order of 0.01. The thermal noise is equal to the signal at $T\sim 2\times 10^{-3}$ °K.

[1]The calculation of C_{nm} calls for the use of Maxwell's equations and the equations of wall motion; these calculations will be given later on.

[2]We emphasize that in vacuum without mirrors, the packet cannot acquire energy systematically from a plane gravitational wave, since the velocities of both the electromagnetic wave and the gravitational wave

are identical and in the case of motion at an angle there is no synchronism and there is no systematic change of l. In vacuum, a gravitational wave is not transformed into an electromagnetic wave, for when the directions do not coincide it is impossible to satisfy the energy and momentum conservation laws, and if the waves travel in the same direction the matrix element of the transformation is equal to zero, owing to the difference between the tensor dimensionalities of the gravitational and electromagnetic waves.

3)The conversion of a gravitational wave into an electromagnetic wave when the former passes through a constant magnetic (or electric field) is considered in [7–10].

1J. Weber, Phys. Rev. Lett. 22, 1302 (1969); 24, 276 (1970); 25, 180 (1970).

2V. B. Braginskiĭ, L. B. Manukin, E. I. Popov, V. N. Rudenko, and A. A. Khorev, ZhETF Pis. Red. 16, 157 (1972) [JETP Lett. 16, 157 (1972)].

3W. H. Press and K. S. Thorne, Ann. Rev. Astr. Astrophys. 10, 335 (1972).

4L. P. Grishchuk and M. V. Sazhin, Zh. Eksp. Teor. Fiz. 65, 441 (1973) [Sov. Phys.-JETP 38, 215 (1974)].

5V. B. Braginskiĭ and M. B. Menskiĭ, ZhETF Pis. Red. 13, 585 (1971) [JETP Lett. 13, 417 (1971)].

6E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk 80, 391 (1963) [Sov. Phys.-Usp. 6, 495 (1964)].

7M. E. Gertsenshteĭn, Zh. Eksp. Teor. Fiz. 41, 113 (1961) [Sov. Phys.-JETP 14, 84 (1962)].

8T. A. Lupanov, ibid. 52, 118 (1967) [25, 76 (1967)].

9D. Boccaletti, V. De Sabbata, P. Fortini, and C. Gualdi, Nuovo Cim., Ser. X, 70B, 129 (1970).

10V. K. Dubrovich, Soobshcheniya SAO, No. 6, 1972.

11V. B. Braginskiĭ, Fizicheskie éksperimenty s probnymi telami (Physical Experiments with Test Bodies), Nauka, 1970.

12W. H. Hartwing, Proc. IEEE 64, 58 (1973)

Translated by J. G. Adashko
178