Transport theory of the interaction of ultrasound with conduction electrons

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A theoretical analysis is made of the interaction of ultrasound with electrons in solids in a wide range of ultrasound frequencies and, in particular, at intermediate frequencies too high for the phenomenological approach but sufficiently low for the scattering of electrons to remain important. It is shown that at these intermediate frequencies the theoretical formulas contain, in principle, the fullest information on the scattering mechanism. The relaxation of the anisotropic part of the electron momentum distribution function can be described with the required accuracy by a finite number of different time constants. Very simple approximations of this kind are used to derive for the first time the formulas for the electronic absorption coefficient of sound and for the acoustoelectric current, which are valid at intermediate frequencies in the case of an arbitrary electron statistics and a fairly wide range of scattering mechanisms.

1. INTRODUCTION

The interaction of ultrasound with electrons in solids has been investigated for many years. The problem was first formulated in 1938.[1] In some cases, use was made of a phenomenological approach valid at sufficiently low frequencies of sound.[2] In other cases a transport equation without the collision integral was employed.[3,4] The latter approach is valid at sufficiently high acoustic frequencies and is analogous to the Landau absorption of plasma waves. It would be interesting to develop a theory valid over a wide range of frequencies from which these two approaches could be deduced as the limiting cases. This theory should be based on transport equations for plasma particles with a full allowance for collisions. One variant of such a theory is put forward in the present paper.

Although the need for a theory valid over a wide range of frequencies has been recognized for some time (see, for example, reviews in [3,4]), a satisfactory solution has not yet been obtained. In most cases, the collision integral in the transport equation of electrons has been used in the form

$$-\frac{1}{\tau}[f(\mathbf{p},\mathbf{r},t)-f_e(\varepsilon,\mathbf{r},t)], \tag{1}$$

where $f(\mathbf{p},\mathbf{r},t)$ is the distribution function of the electron moment $\mathbf{p}$, $\mathbf{r}$ is the radius vector, t is the time, $f_e(\epsilon,\mathbf{r},t)$ is the local equilibrium distribution function, ϵ is the electron energy, and τ is the relaxation time. The value of τ is usually assumed to be constant[4-7] or dependent on ϵ.[8,9] The main objection is that the replacement of the true collision integral with Eq. (1) is sometimes too approximate. For example, the criterion for going over to the collisionless absorption of sound is not always correct if we use Eq. (1). We shall discuss this point in detail in Sec. 3.

A correct allowance for not one, as in Eq. (1), but two different (energy and momentum) relaxation times was made in [10,11] but only at low frequencies within the framework of the phenomenological approach. An attempt to go outside the approximation (1) in dealing with intermediate frequencies of ultrasound was made in [12]. By definition, at intermediate frequencies the electrons which interact effectively with sound include those for which $ql(\epsilon)\sim 1$, where q is the wave number of sound and $l(\epsilon)$ is the mean free path of an electron of energy ϵ. If $ql \gtrsim 1$, the coefficients in the expansion of $f(\mathbf{p})$ in terms of spherical harmonics do not decrease with increasing order of the terms.[13-15] However, only the zeroth and first-order harmonics are included in [12], which is not justified.

Thus, we can see that there are as yet no published general formulas for the electronic absorption coefficient α and the acoustoelectric current j_{ac} in the $ql\sim 1$ range. In fact, in this range we can use only the expressions for α derived in [7] for a totally degenerate electron gas and one special scattering mechanism. We shall try to avoid incorrect assumptions in the derivation and solution of the transport equation for electrons. This should make it possible to obtain for the first time the formulas for α and j_{ac} correct for $ql\sim 1$ in the case of an arbitrary statistics of electrons and a fairly wide range of scattering mechanisms.

2. RELAXATION OF ANISOTROPIC PERTURBATIONS OF THE ELECTRON DISTRIBUTION FUNCTION

An acoustic wave and an external electric field perturb the distribution of the electron momentum. The relaxation of these perturbations can be described by the classical transport equation

$$\frac{\partial f}{\partial t}+\frac{p_x}{m}\frac{\partial f}{\partial x}+F\frac{\partial f}{\partial p_x}=I[f_0]-\sum_{n=1}^{\infty}\frac{f_n(\varepsilon)P_n(\cos\theta)}{\tau_n(\varepsilon)}, \tag{2}$$

where F is the force exerted by the acoustic wave and the static electric field on an electron. It is assumed that sound travels along the x axis and that the force F is applied in the same direction; $\epsilon=p^2/2m$; m is the effective mass. The right-hand side of Eq. (2), which represents the collision integral, can be expanded in terms of the Legendre polynomials $P_n(\cos\theta)$, where θ is the angle between $\mathbf{p}$ and the x axis; $I[f_0]$ is the isotropic part of this expansion and

$$f=\sum_{n=0}^{\infty}f_n(\varepsilon)P_n(\cos\theta). \tag{3}$$

This representation is valid under the quasielastic scat-

tering conditions [15] and will be useful in obtaining the solution.

It is evident from the structure of the right-hand side of Eq. (2) that the relaxation of the anisotropic perturbations f (the terms with $n \geq 1$) is complex and generally cannot be described by a single relaxation time. Each spherical harmonic in Eq. (3) corresponds to a relaxation time $\tau_n(\epsilon)$. The properties of a sequence of relaxation times $\{\tau_n(\epsilon)\}$ have not been analyzed in sufficient detail. Since all these relaxation times play an important role in the acoustoelectronic effects in the $ql \gtrsim 1$ range, we shall discuss their properties in greater detail.

The time $\tau_n(\epsilon)$ is expressed in terms of the cross section for the scattering $s(\xi)$ through an angle ψ ($\xi = \cos\psi$):

$$\frac{1}{\tau_n(\varepsilon)} = \frac{1}{\tau(\varepsilon)} - \frac{\lambda_n}{\tau(\varepsilon)}, \tag{4}$$

where

$$\lambda_n = \left(\int_{-1}^{1} s(\xi)\,d\xi\right)^{-1} \int_{-1}^{1} s(\xi) P_n(\xi)\,d\xi. \tag{5}$$

The first term on the right-hand side of Eq. (4) is independent of the number n and represents the contribution of the processes resulting in the "loss" of an electron from a state with given value of **p**. The second term represents the "gain" by the state **p**. The parameter λ_n in Eq. (5) is evidently the ratio of the loss and gain contributions. Since $|P_n(\xi)| \leq 1$, it follows from Eq. (5) that $|\lambda_n| \leq 1$. Since $P_n(\xi)$ oscillates in the range $-1 \leq \xi \leq 1$ (the number of sites is n), we find that an increase in n causes λ_n to approach zero and it makes the time $\tau_n(\epsilon)$ to approach $\tau(\epsilon)$. This property is the basis of the approximation which can be formulated generally as follows. There is always such a number ν that if $n > \nu$ we can satisfactorily assume that $\tau_n(\epsilon) = \tau(\epsilon)$. Therefore, Eq. (2) can be written as a system of equations for the first harmonics $f_0(\epsilon), f_1(\epsilon), \ldots, f_\nu(\epsilon)$ and the residue

$$\varphi = f - \sum_{n=0}^{\nu} f_n(\varepsilon) P_n(\cos\theta).$$

This final system of differential equations must now be solved. We shall demonstrate later that the differential equations frequently reduce to the algebraic form. There are no principal difficulties in solving these equations for any value of ν. However, the simplest cases are those for which the values of ν are small, for example, $\nu = 0$ or 1.

We shall now consider the scattering mechanisms for which

$$s(\xi) \sim \frac{(1-\xi)^{a-1/2}}{[1+\varkappa(\varepsilon)-\xi]^2}, \tag{6}$$

where $\kappa(\epsilon) = \hbar^2/4m\epsilon r_D^2 \ll 1$ and r_D is the screening radius. A cross section of the type (6) corresponds primarily to a power dependence $\tau_1(\epsilon) \propto \epsilon^a$ and typical values of a are $-1/2$, 0, $1/2$, and $3/2$. The results of calculations of the integrals (5) with the functions $s(\xi)$ in Eq. (6) are given in Table I. We can clearly see a general tendency for $|\lambda_n|$ to decrease with increasing n. It follows from Eq. (6) that the rise of a reduces the anisotropy and the importance of scattering through small angles. The greater the role of small angles in the integrals (5), the smaller is the difference between λ_n and unity because $P_n(\xi) \approx 1$ for $(1-\xi) < 1/n$. This explains why (see Table I) the value of $|\lambda_n|$ decreases with increasing κ (for a given a) and rises to unity with increasing a (for a fixed κ). The dependence of λ_n on the number n weakens somewhat at high values of a.

a	$\varkappa$	λ_1	λ_2	λ_3	λ_4	λ_5
$-1/2$	10^{-4}	$-0.7\cdot10^{-3}$	$-0.6\cdot10^{-3}$	$-0.6\cdot10^{-3}$	$-0.6\cdot10^{-3}$	$-0.5\cdot10^{-3}$
0		0.32	0.19	0.13	0.10	0.08
$1/2$		0.78	0.66	0.59	0.53	0.49
$3/2$	$5\cdot10^{-4}$	0.9991	0.9976	0.9957	0.9932	0.9903
$-1/2$		$-0.3\cdot10^{-2}$	$-0.2\cdot10^{-2}$	$-0.2\cdot10^{-2}$	$-0.2\cdot10^{-2}$	$-0.2\cdot10^{-2}$
0		0.31	0.17	0.11	0.08	0.06
$1/2$	-	0.73	0.59	0.50	0.44	0.38
$3/2$		0.9963	0.9906	0.9831	0.9742	0.9642
$-1/2$	10^{-3}	$-0.5\cdot10^{-2}$	$-0.4\cdot10^{-2}$	$-0.3\cdot10^{-2}$	$-0.3\cdot10^{-2}$	$-0.3\cdot10^{-2}$
0		0.30	0.16	0.10	0.07	0.05
$1/2$		0.70	0.55	0.45	0.38	0.33
$3/2$		0.9934	0.9832	0.9704	0.9553	0.9387
$-1/2$	$5\cdot10^{-3}$	-0.02	-0.01	-0.01	-0.01	$-0.7\cdot10^{-2}$
0		0.26	0.12	0.06	0.03	0.02
$1/2$		0.61	0.43	0.31	0.23	0.18
$3/2$		0.9750	0.9395	0.8981	0.8533	0.8069
$-1/2$	10^{-2}	-0.03	-0.02	-0.01	-0.01	$-0.9\cdot10^{-2}$
0		0.23	0.09	0.04	0.01	0.001
$1/2$		0.56	0.36	0.24	0.16	0.11
$3/2$		0.9567	0.8987	0.8342	0.7681	0.7024
$-1/2$	$5\cdot10^{-2}$	-0.09	-0.05	-0.03	-0.02	$-0.8\cdot10^{-2}$
0		0.12	0.005	-0.021	-0.024	-0.02
$1/2$		0.39	0.16	0.06	0.01	-0.01
$3/2$		0.86	0.71	0.57	0.45	0.35
$-1/2$	10^{-1}	-0.13	-0.06	-0.03	-0.01	$-0.5\cdot10^{-2}$
0		0.05	-0.03	-0.03	-0.03	-0.02
$1/2$		0.29	0.07	-0.0005	-0.02	-0.03
$3/2$		0.78	0.57	0.41	0.29	0.20

The lowest value $a = 3/2$ is obtained for the scattering by ionized impurities. For such values of a the parameter λ_1 is always close to unity, i.e., $\tau \ll \tau_1$. According to Table I, $\tau/\tau_1 \sim 10^{-1}$–10^{-3}. This means that the separate time constants in the sequence $\{\tau_n\}$ may differ from one another by several orders of magnitude ($\tau_n \to \tau$ for $n \to \infty$). Clearly, the approximations $\nu = 0$ or $\nu = 1$ with $a = 3/2$ are hardly justified even in rough estimates.

If $a = 1/2$ and $\kappa \geq 5\times10^{-2}$ and even for $a = 0$ and $\kappa \approx 10^{-2}$ or $\kappa \approx 5\times10^{-2}$ we have $\lambda_2 \ll \lambda_1$ and we can ignore all the parameters λ_n with $n \geq 2$, i.e., the approximation $\nu = 1$ is valid. If $a = -1/2$, we find that irrespective of the value of κ we obtain $|\lambda_n| \leq 1.3\times10^{-1} \ll 1$ ($n \geq 1$). This allows us to use the approximation $\nu = 0$. If an error of 30% can be tolerated, we may assume that $\nu = 0$ also for $a = 0$ and $\kappa \geq 10^{-4}$. If we take $\nu = 1$ for $a = 0$ and $\kappa \geq 10^{-4}$, we obtain a simple interpolation which allows for the difference between the relaxation times of the individual spherical harmonics to within 20%. This interpolation improves the agreement between the theory and experiment.[16] We shall derive concrete results for $\nu = 1$. Our formulas also include the approximation $\nu = 0$ as a special case.

3. ABSORPTION AND VELOCITY OF SOUND. ACOUSTOELECTRIC CURRENT

Equation (2) is nonlinear because of the term $F\partial f/\partial p_x$. The nonlinearity will be allowed for by iterations of the amplitude. We shall be interested in the linear effects (absorption α and correction to the velocity of sound $\Delta v_s/v_s$) and in the acoustoelectric current jac in the approximation linear with respect to the acoustic energy flux. The effects under consideration can be calculated using Eq. (2) if

$$\varepsilon\tau/\hbar \gg 1, \quad \hbar q \ll p = mv. \tag{7}$$

Then f and F can be represented in the form

$$\begin{aligned} f &= \bar{f} + \tilde{f} e^{i(qx-\omega t)} + \tilde{f}^* e^{-i(qx-\omega t)}, \\ F &= \bar{F} + \tilde{F} e^{i(qx-\omega t)} + \tilde{F}^* e^{-i(qx-\omega t)}, \end{aligned} \tag{8}$$

where $\omega = v_S q$ is the velocity of sound, and $\bar{F}, \bar{f}, \tilde{F}$ and $\tilde{f}$ may depend weakly on x and t, compared with the exponential functions in Eq. (8). Substituting Eq. (8) into Eq. (2), we obtain

$$-\bar{F}\frac{\partial \bar{f}}{\partial p_x} + I[\bar{f}_0] - \sum_{n=0}^{\infty} \frac{\bar{f}_n(\varepsilon) P_n(\cos\theta)}{\tau_n(\varepsilon)} = \frac{\partial}{\partial p_x}[\tilde{F}\tilde{f}^* + \tilde{F}^*\tilde{f}], \quad (9)$$

$$-i\left(\frac{qp_x}{m} - \omega\right)\tilde{f} + I[\tilde{f}_0] - \sum_{n=1}^{\infty} \frac{\tilde{f}_n(\varepsilon) P_n(\cos\theta)}{\tau_n(\varepsilon)} = \tilde{F}\frac{\partial \bar{f}}{\partial p_x} + \bar{F}\frac{\partial \tilde{f}}{\partial p_x}. \quad (10)$$

In Eq. (10) the function $\bar{f}$ is the electron distribution in an external static field $\bar{F}$. The equation for this function can be obtained from Eq. (9) by dropping the right-hand side. If $\bar{F}\tau_1/m \ll v$, the solution of this equation is well known and we shall use it later. The term $-iqp_x\tilde{f}/m$ in Eq. (10) ensures the "coupling" of the spherical harmonics so that if $ql(\varepsilon) \gtrsim 1$ the harmonics do not decrease with increasing n and the function $\tilde{f}$ is "elongated." The presence of an inhomogeneous term $\sim\tilde{f}$ on the right-hand side of Eq. (9) causes "elongation" of the smooth function $\bar{f}$.

We shall assume that $\nu = 1$ and use Eq. (10) to derive equations for $\tilde{f}_0$, $\tilde{f}_1$, and $\tilde{\varphi}$. In this way, we obtain

$$i\omega\tau_1(\varepsilon)\tilde{f}_0(\varepsilon) + \tau_1(\varepsilon) I[\tilde{f}_0] - \frac{1}{3}\left\{iql_1(\varepsilon)\tilde{f}_1(\varepsilon) + \frac{v_d(\varepsilon)}{v(\varepsilon)}\left[2\varepsilon\frac{\partial \tilde{f}_1(\varepsilon)}{\partial \varepsilon} + 2\tilde{f}_1(\varepsilon)\right]\right\} = -\bar{F}\tau_1(\varepsilon)\left\langle\frac{\partial \tilde{f}}{\partial p_x}\right\rangle_0, \quad (11)$$

$$[1 - i\omega\tau_1(\varepsilon)]\tilde{f}_1(\varepsilon) + \left\{iql_1(\varepsilon)\tilde{f}_0(\varepsilon) + \frac{v_d(\varepsilon)}{v(\varepsilon)} 2\varepsilon\frac{\partial \tilde{f}_0(\varepsilon)}{\partial \varepsilon}\right\} + \tilde{A}(\varepsilon) = -\tau_1(\varepsilon)\bar{F}\left\langle\frac{\partial \tilde{f}}{\partial p_x}\right\rangle_1, \quad (12)$$

$$\left[1 + i\left(\frac{p_x q}{m} - \omega\right)\tau(\varepsilon)\right]\tilde{\varphi} + \bar{F}\tau(\varepsilon)\frac{\partial \tilde{\varphi}}{\partial p_x} = \delta(\varepsilon)\tilde{A}(\varepsilon)P_1(\xi) - \tau(\varepsilon)\bar{F}\left[\frac{\partial \tilde{f}}{\partial p_x} - \left\langle\frac{\partial \tilde{f}}{\partial p_x}\right\rangle_1 P_1(\xi) - \left\langle\frac{\partial \tilde{f}}{\partial p_x}\right\rangle_0\right] - \frac{2}{3}P_2(\xi)\left\{iql(\varepsilon)\tilde{f}_1(\varepsilon) + \delta(\varepsilon)\frac{v_d(\varepsilon)}{v(\varepsilon)}\left[2\varepsilon\frac{\partial \tilde{f}_1(\varepsilon)}{\partial \varepsilon} - \tilde{f}_1(\varepsilon)\right]\right\}, \quad (13)$$

where

$$\tilde{A}(\varepsilon) = iql_1(\varepsilon)\langle\xi\tilde{\varphi}\rangle_1 + \frac{v_d(\varepsilon)}{v(\varepsilon)}\left[2\varepsilon\left\langle\xi\frac{\partial \tilde{\varphi}}{\partial \varepsilon}\right\rangle_1 + 3\langle\xi\tilde{\varphi}\rangle_1\right],$$
$$\xi = \cos\theta, \quad l_1(\varepsilon) = v(\varepsilon)\tau_1(\varepsilon), \quad l(\varepsilon) = v(\varepsilon)\tau(\varepsilon),$$
$$v(\varepsilon) = \sqrt{\frac{2\varepsilon}{m}}, \quad v_d(\varepsilon) = \frac{\bar{F}\tau_1(\varepsilon)}{m}, \quad \delta(\varepsilon) = \frac{\tau(\varepsilon)}{\tau_1(\varepsilon)}, \quad (14)$$
$$\langle(\ldots)\rangle_n = \frac{2n+1}{2}\int_{-1}^{1} d\xi P_n(\xi)(\ldots).$$

The following expression is obtained from Eq. (9) for the function $\bar{f}_1(\epsilon)$:

$$\bar{f}_1(\varepsilon) = -\tau_1(\varepsilon)\bar{F}\left\langle\frac{\partial \bar{f}}{\partial p_x}\right\rangle_1 - 2\tau_1(\varepsilon)\,\mathrm{Re}\left[\tilde{F}^*\left\langle\frac{\partial \tilde{f}}{\partial p_x}\right\rangle_1\right]. \quad (15)$$

We shall substitute an electron current $\tilde{j} = \sigma(\omega, q)\tilde{F}/e$ alternating at the frequency of sound ω into the system of standard Maxwell equations and of elasticity equations. Then, α and $\Delta v_S/v_S$ can be expressed, subject to the usual assumptions,[2-4] in terms of $\sigma(\omega, q)$:

$$\alpha = \frac{2\pi}{v_s\varepsilon_0}[\eta^2 + \chi^2]\frac{\mathrm{Re}\,\sigma(\omega, q)}{|1 + 4\pi i\sigma(\omega, q)/\omega\varepsilon_0|^2}, \quad (16)$$

$$\frac{\Delta v_s}{v_s} = \frac{2\pi}{\omega\varepsilon_0}[\eta^2 + \chi^2]\frac{\mathrm{Im}\,\sigma(\omega, q) - 4\pi|\sigma(\omega, q)|^2/\omega\varepsilon_0}{|1 + 4\pi i\sigma(\omega, q)/\omega\varepsilon_0|^2}, \quad (17)$$

where the electromechanical constants are $\eta^2 = 4\pi\beta^2/\epsilon_0\rho v_S^2$ and $\chi^2 = \Lambda^2 q^2\epsilon_0/4\pi e^2\rho v_S^2$, β is the piezoelectric modulus, Λ is the deformation potential, ϵ_0 is the permittivity of the crystal, and ρ is its density. The calculation of α and $\Delta v_S/v_S$ reduces to the calculation of $\sigma(\omega, q)$.

The equation of continuity yields $\tilde{j} = e\tilde{n}v_S$, where the alternating electron density is

$$\tilde{n} = \frac{2}{(2\pi\hbar)^3}\int d^3p\,\tilde{f}_0(\varepsilon). \quad (18)$$

It follows from Eq. (18) that in order to find $\tilde{n}$, $\tilde{j}$ and $\sigma(\omega, q)$ from Eqs. (11)–(14), it is sufficient to find only $\tilde{f}_0(\epsilon)$. When $\tilde{f}_0(\epsilon)$ is found, we can ignore the energy relaxation, i.e., we can drop the term $I[\tilde{f}_0]$. This can be done if

$$ql_0(\varepsilon) \gg 1 \quad (\text{for } ql(\varepsilon) \gtrsim 1) \quad (19)$$

and

$$q^2 l_1(\varepsilon) l_0(\varepsilon) \gg 1 \quad (\text{for } ql(\varepsilon) \ll 1), \quad (20)$$

where $l_0(\epsilon) = v(\epsilon)\tau_0(\epsilon)$. The conditions (19) and (20) have a clear physical meaning. If $ql \gtrsim 1$, an electron actually travels a distance $\sim l_0$ before its energy relaxes significantly. Therefore, l_0 should be compared with the wavelength in Eq. (19). If $ql \ll 1$, the collisions which change the momentum effectively reduce the distance traveled by an electron without energy relaxation to $\sim\sqrt{l_0 l_1}$. This quantity occurs in Eq. (20). The condition (20) is the lower limit of the frequency range in which our final expressions are valid.

The problem of the influence of electron-electron collisions (of frequency $\nu_{ee} \ll \tau_1^{-1}$) on $\tilde{f}_0(\epsilon)$ can be solved in a similar manner. These collisions are unimportant if the conditions (19) and (20) are satisfied and $l_0(\epsilon)$ is replaced with $l_{ee}(\epsilon) = v\nu_{ee}^{-1}$. Since $l_{ee} \gg l$, the electron-electron collisions may be important only at low frequencies subject to $ql \ll 1$. We shall therefore ignore electron-electron collisions in our analysis.

Equations (11)–(14) are easiest to solve if they can be reduced to linear algebraic forms. This can be done if the terms $\sim\bar{F}$ can be dealt with by the perturbation theory. In this case, the small parameters are

$$v_d(\varepsilon)/v(\varepsilon) \quad (\text{for } ql(\varepsilon) \gtrsim 1),$$
$$\frac{3\omega\tau_1(\varepsilon)}{q^2 l_1^2(\varepsilon)}\frac{v_d(\varepsilon)}{v_s} \quad (\text{for } ql(\varepsilon) \ll 1). \quad (21)$$

We must bear in mind that these parameters are, in fact, small for all the energies ϵ which contribute to the final integrals. It is worth noting that the linearization of Eqs. (11)–(14) with respect to d does not apply to the isotropic part of the static function $\bar{f}_0(\epsilon)$ which thus remains the distribution function of the hot electron gas.

In the zeroth approximation with respect to v_d we find from Eq. (13) that

$$\tilde{\varphi} = \tilde{A}(\varepsilon)\delta(\varepsilon)G(\varepsilon, \xi)P_1(\xi) - {}^2/_3 iql(\varepsilon)G(\varepsilon, \xi)P_2(\xi), \quad (22)$$

where

$$G(\varepsilon, \xi) = \{1 + i[ql(\varepsilon)\xi - \omega\tau(\varepsilon)]\}^{-1}.$$

Multiplying Eq. (22) by ξ and applying the operation $\langle(\ldots)\rangle_1$, we obtain the equation for $\tilde{A}(\epsilon)$. Eliminating from this equation and from Eqs. (11) and (12) the quantities $\tilde{A}(\epsilon)$ and $\tilde{f}_1(\epsilon)$, we arrive at the equation for $\tilde{f}_0(\epsilon)$. In solving the last equation, we must bear in mind that

$$(v_s/v(\varepsilon))^2 \ll 1 \quad (23)$$

for all the energies which contribute to the quantity $\sigma(\omega, q)$ being calculated. The validity of Eq. (23) is easily checked with the aid of the final formulas for $\sigma(\omega, q)$. Naturally, the solution method is basically similar to the next (first) approximation with respect to v_d. In this way, we find the following expressions for the electrical conductivity:

$$\mathrm{Re}\,\sigma(\omega,q)=-\frac{4\pi e^2 v_*(2m)^{3/2}}{q(2\pi\hbar)^3}\int_0^\infty \sqrt{\varepsilon}\,d\varepsilon\,\frac{\partial \bar f_0(\varepsilon)}{\partial\varepsilon}\,\frac{\Delta(\varepsilon)\mathcal{F}(\varepsilon)}{1+\Delta^2(\varepsilon)}, \quad (24)$$

$$\mathrm{Im}\,\sigma(\omega,q)=\frac{4\pi e^2 v_*(2m)^{3/2}}{q(2\pi\hbar)^3}\int_0^\infty \sqrt{\varepsilon}\,d\varepsilon\,\frac{\partial \bar f_0(\varepsilon)}{\partial\varepsilon}\,\frac{1}{1+\Delta^2(\varepsilon)}, \quad (25)$$

where

$$\Delta(\varepsilon)=\omega\tau(\varepsilon)\left[\frac{\mathrm{arctg}\,ql(\varepsilon)}{ql(\varepsilon)-\mathrm{arctg}\,ql(\varepsilon)}+\frac{3(\delta(\varepsilon)-1)}{q^2l^2(\varepsilon)}\right],$$

$$\mathcal{F}(\varepsilon)=1-\frac{v_d(\varepsilon)}{v_*}\left[1+\frac{2av_*}{ql_1(\varepsilon)v(\varepsilon)\Delta(\varepsilon)}\right].$$

The correction, proportional to v_d in $\mathrm{Im}\,\sigma$, is small compared with the expression given above if the small parameters of Eq. (21) are used. If $ql(\epsilon)\ll 1$ and $v_d=0$, Eqs. (24) and (25) reduce to the corresponding formulas given by Lipnik.[12]

It is now convenient to specify the statistics of electrons. Let us assume that the statistics is of the Boltzmann type and the electron temperature is T. If

$$\omega_D(T)\ll\omega, \quad (26)$$

where $\omega_D(\epsilon)=3v_S^2/v^2(\epsilon)\tau_1(\epsilon)$, the expression for $\mathrm{Im}\,\sigma(\omega,q)$ can be simplified by dropping $\Delta^2(\epsilon)$ from the integral in Eq. (25). Then, the denominator in Eqs. (16) and (17) becomes

$$\left|1+i\sigma(\omega,q)\frac{4\pi}{\omega\varepsilon_0}\right|^2=\left(1+\frac{1}{q^2r_D^2}\right)^2. \quad (27)$$

If the condition (26) is satisfied, the quantity $\Delta^2(\epsilon)$ can also be dropped from the denominator in the integrand of Eq. (24) for $\mathrm{Re}\,\sigma(\omega,q)$ provided the remaining integral converges and the main contribution to this integral comes from energies $\epsilon\sim T$. This is true for $a<1/2$. If $a=1/2$, the remaining integral diverges at the point $\epsilon=0$ as $\ln\epsilon$. The convergence is ensured by $\Delta^2(\epsilon)$. If $\epsilon\to 0$, we find that $\Delta^2(\epsilon)\to[\omega_D(\epsilon)/\omega]^2$ and this quantity rises, restricting the range of integration if $\epsilon\sim\epsilon_1\ll T$ ($[(\omega_D(\epsilon_1)/\omega]^2=1$). An analysis shows that the contributions of the thermal ($\epsilon\sim T$) and low ($\epsilon\sim\epsilon_1$) energies to the integral in Eq. (24) are in the ratio which is equal to $ql(T)$. In other words, if $ql(T)\lesssim 1$, the electrons with low energies $\epsilon\sim\epsilon_1$ may play an important role in the absorption of sound. It should be noted that the conditions of validity (7) of the transport equation are not yet violated for $\epsilon\sim\epsilon_1$. Using Eq. (27), we find that Eqs. (16) and (24) yield the final formula for α:

$$\alpha=-\frac{(\eta^2+\chi^2)(qr_D)^4}{(1+q^2r_D^2)^2}\frac{e^2(2m)^{3/2}}{\varepsilon_0 q\pi\hbar^3}\int_0^\infty \sqrt{\varepsilon}\,d\varepsilon\,\frac{\partial \bar f_0(\varepsilon)}{\partial\varepsilon}\,\frac{\Delta(\varepsilon)\mathcal{F}(\varepsilon)}{1+\Delta^2(\varepsilon)}. \quad (28)$$

We shall now consider the condition for going over to the collisionless regime. It is clear from Eq. (28) and from the expression for $\Delta(\epsilon)$ that if throughout the range of ϵ which is important in integration the condition $ql(\epsilon)\gg 1$ is satisfied, the value of α ceases to depend on the relaxation times τ and τ_1. This is the collisionless regime. The condition for going over to this regime can be written in the form

$$ql(T)\gg 1. \quad (29)$$

It is indeed found that if Eq. (29) is satisfied, only the energies $\epsilon\sim T$ contribute to the integral (28). It should be stressed that l in Eq. (29) is defined in terms of the "loss" relaxation time and not in terms of the total relaxation time which occurs in the expression for the static electrical conductivity.

A criterion of the type given by Eq. (29) was first derived by Akhiezer, Kaganov, and Lyubarskiĭ[5] and then confirmed by many others. In all these investigations use was made of Eq. (1) with τ independent of ϵ. It was shown in [8,9] that if Eq. (1) was used the dependence of τ on the energy resulted in the retention of the parameter $\omega\tau(T)$ in the expression for α applicable in the $ql(T)\gg 1$ range and the collisionless formula was obtained only for $\omega\tau(T)\gg 1$. It follows from our calculations that if the approximation represented by Eq. (1) is not invoked, the criterion (29) applies to τ independent of or dependent on ϵ.

It should also be noted that if $\tau=\mathrm{const}$, our final formulas for $\sigma(\omega,q)$ and α differ considerably from those obtained in the approximation of Eq. (1).[3] The difference is obviously due to the fact that $\tau_1\neq\tau$. However, even if we formally assume that $\tau_1=\tau$, we find that for an arbitrary electron statistics the structure of the expressions for $\sigma(\omega,q)$ obtained in the present paper differs from that given in [3]. Only the Spector formulas for $\sigma(\omega,q)$ and α obtained for a degenerate electron gas are fully justified (for $\tau_1=\tau$).

The results obtained are illustrated in Figs. 1 and 2. Figure 1 shows the dependence of the relative absorption coefficient α/α_0 on the parameter $ql(T)$ calculated using Eq. (28) subject to $v_d=0$. Here, α_0 is the absorption coefficient in the collisionless case. We can see that the collisionless formula is considerably in error not only for $ql(T)=1$ but also for $ql(T)\leq 10$. However, α can be smaller or greater than α_0. Figure 2 shows the dependence of the parameter $r=v_S/v_{dc}$ on $ql(T)$ for the case when $a=-1/2$ (here, v_{dc} is the critical drift velocity at which the absorption of sound changes to amplification). If $a=0$, we find that $r=1$ and if $a=1/2$ we obtain $r\approx 0.65$, which is practically independent of $ql(T)$.

We shall now consider the acoustoelectric current. The density of this current is given by

$$j_{ac}=\frac{2me}{3\pi^2\hbar^3}\int_0^\infty \varepsilon\bar f_1(\varepsilon)\,d\varepsilon. \quad (30)$$

Substituting as $\bar f_1(\epsilon)$ the second term from Eq. (15), we obtain, subject to the approximations (19)–(21), (23):

$$j_{ac}=-\frac{2me|F|^2v_*}{\pi^2\hbar^3q}\int_0^\infty d\varepsilon\,\tau_1(\varepsilon)\frac{\partial \bar f_0(\varepsilon)}{\partial\varepsilon}\,\frac{\Delta_1(\varepsilon)\mathcal{F}_1(\varepsilon)}{1+\Delta^2(\varepsilon)}, \quad (31)$$

where

$$\Delta_1(\varepsilon)=\frac{2a}{ql_1(\varepsilon)}+\Delta(\varepsilon)\frac{v(\varepsilon)}{v_*},\quad \mathcal{F}_1(\varepsilon)=1-\frac{v_d(\varepsilon)}{v_*}\left[1+\frac{2a}{ql_1(\varepsilon)}\,\frac{1+{}^3/_2a}{\Delta_1(\varepsilon)}\right].$$

The expression (31) is valid in a wide range of frequencies, including the intermediate frequencies $ql\sim 1$. If Eq. (29) is satisfied, the dependence on τ disappears and Eq. (31) reduces to the formula for the collisionless case.

4. CONCLUSIONS

The analysis given above has been stimulated by the need to have clear ideas about the acoustoelectronic interaction at all frequencies. At low and high frequencies this interaction can be described by the phenomenological and collisionless theories, respectively, but there has been no theory for the intermediate range of frequencies.

It is clear from our analysis that intermediate frequencies are of special importance from some points of view. At these frequencies the relaxation times $\tau_n(\epsilon)$ with all values of n occur in the theory. The set of these relaxation times gives the fullest information on the scattering processes. For example, Eqs. (28) and (31)

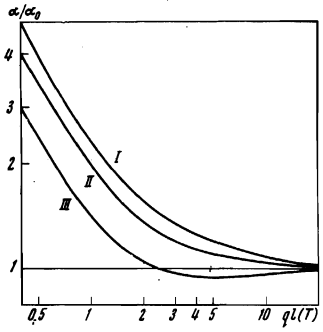

FIG. 1

FIG. 2

FIG. 1. Dependence of the absorption on the parameter ql(T): I—a = −½, δ = 1; II—a = 0, δ = 2/3; III—a = ½, δ = 1/3.

FIG. 2. Dependence of the amplification threshold on ql(T) for the case a = −½, δ = 1.

contain the "loss" relaxation time τ. This time τ is interesting because it represents the relaxation of localized (in the electron momentum space) perturbations of the statistical equilibrium state. These local perturbations appear not only as a result of interaction with ultrasound but also in other cases such as the application of strong electric fields when $v_d \gtrsim v(T)$. A comparison of Eqs. (28) and (31) with the experimental results can give information on the relaxation time τ. This has been demonstrated by recent experiments.[16]

Finally, it should be noted that the information on the relaxation times $\tau_n(\epsilon)$ with different values of n can be obtained from the interaction of electrons with any (not necessarily ultrasonic) short-wavelength ($ql \sim 1$) excitations in solids.

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